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# ENGINEERING AERODYNAMICS

By

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## PREFACE

*Engineering Aerodynamics* has been written to meet a definite need for more practical information on aerodynamics, presented in forms suitable for direct application by aircraft designers and advanced students in aeronautical engineering. An endeavor has been made to include as much new material as possible while avoiding a needless repetition of well-known or easily obtained data. For this reason the conventional treatment of elementary aerodynamic considerations, such as airflow patterns and pressure distributions, is omitted, thus allowing more attention to be concentrated on the problems encountered in design.

A large amount of data has been analyzed and presented in the form of working diagrams and equations, while very little space has been devoted to the description of tests and test methods, or to the reproduction of undigested test data. Where given, derivations have been made as brief as practicable, but numerous references have been included for the benefit of those desiring more complete information. Many important calculations are illustrated by examples.

Most of the subject matter is original and much of this is now made available for the first time. A special effort has been made to present all data in the form best adapted to convenient application in design problems. This has led to the use of numerous working diagrams, or graphs, from which required factors, constants, or ratios, may be readily obtained.

The author deeply appreciates the encouragement received from many sources during the preparation of this volume.

WALTER S. DIEHL.

Washington, D. C.,

May 25, 1928.



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**ENGINEERING  
AERODYNAMICS**



## CHAPTER I

### WING SECTION DATA

The National Advisory Committee for Aeronautics<sup>1</sup> has compiled a series of technical reports giving wind tunnel test data on more than 600 wing sections. These reports are issued under the title "Aerodynamic Characteristics of Airfoils," and are numbered I to IV, as Report Nos. 93, 124, 182, and 244. Copies may be obtained directly from the Committee or from the Superintendent of Documents, Government Printing Office, Washington, D. C.

The Committee also has published technical reports and technical notes on full scale airfoil characteristics, as measured in the variable density wind tunnel. These are of great value to the engineer: Technical Report No. 221, "Model Tests with a Systematic Series of 27 Wing Sections at Full Reynolds Number," by Max M. Munk and Elton W. Miller (1925); Technical Report No. 233, "Aerodynamic Characteristics of Seven Frequently Used Wing Sections at Full Reynolds Number," by Max M. Munk and Elton W. Miller (1926).

In view of the availability of wing section data, this chapter will give only such data as are necessary to illustrate general wing characteristics and the selection of a section best suited for a general design.

**Coefficients.** Wing section characteristics are most conveniently expressed in the form of "coefficients." Practically all of the early work in this country was given in the form of coefficients having the dimensions of lbs./sq. ft. / (mi./hr.)<sup>2</sup>, using the symbols  $K_y$  for lift and  $K_x$  for drag. These coefficients were based on air of standard density and defined by the relations

---

<sup>1</sup> This expression frequently appears abbreviated as N. A. C. A.

$$\text{Lift} = L = K_y \left( \frac{\rho}{\rho_0} \right) S V^2$$

$$\text{Drag} = D = K_x \left( \frac{\rho}{\rho_0} \right) S V^2$$

where  $S$  is the area in square feet and  $V$  the relative speed in miles per hour.

About 1919 the National Advisory Committee for Aeronautics recommended the adoption of "absolute coefficients" having the same value in any consistent system of units. The coefficients tentatively adopted were defined by the relations

$$L = L_c \rho S V^2$$

$$D = D_c \rho S V^2$$

The coefficients  $L_c$  and  $D_c$  are the same as  $K_L$  and  $K_D$  used by the British.

With the improvement in theoretical aerodynamics it became evident that there were many advantages in the use of the absolute coefficients employed by Prandtl. Therefore, in 1921, the National Advisory Committee for Aeronautics decided to recommend the general use of the coefficients  $C_L$  and  $C_D$  defined by the relations

$$L = C_L \frac{\rho}{2} S V^2 = C_L q S \quad \text{and} \quad D = C_D \frac{\rho}{2} S V^2 = C_D q S$$

Where  $q$  is the dynamic pressure  $\frac{1}{2} \rho V^2$ . These absolute coefficients are now in extensive use.

The relations between the three types of coefficients are:

$$C_L = 2L_c = 391 K_y \quad K_y = .002558 C_L = .005116 L_c$$

$$C_D = 2D_c = 391 K_x \quad K_x = .002558 C_D = .005116 D_c$$

**Dynamic pressure.** The dynamic or impact pressure  $q = \frac{1}{2} \rho V^2$  is the pressure developed in bringing a moving perfect fluid to rest. Since the standard density is  $\rho_0 = .002378$ ,

$$\text{Curtiss} \quad q_0 = .001189 V^2$$

where  $q_0$  is in lbs./sq. ft., and  $V$  in ft./sec., or

$$q_0 = .002558 V^2$$

where  $q_0$  is in lbs./sq. ft. and  $V$  in mi./hr.

The value of  $q$  at any density other than the standard is obviously

$$q = q_0 \left( \frac{\rho}{\rho_0} \right)$$

In the metric system,  $V$  being in m./sec., and  $q$  in kg./sq. m.

$$q_0 = .0625 V^2$$

The relation between velocity in standard air and a given dynamic pressure in lbs. /sq. ft., is

$$V = 29.00 \sqrt{q} \text{ ft./sec.}$$

$$V = 19.77 \sqrt{q} \text{ mi./hr.}$$

In the metric system,  $q$  in kg./sq. m.

$$V = 4 \sqrt{q} \text{ m./sec.}$$

**Wing section characteristics.** Wing section characteristics are lift, drag, and moment coefficients as functions of angle of attack, Reynolds Number, absolute velocity ratio or any other non-dimensional ratio affecting these characteristics. Wing section characteristics as a function of angle of attack are usually determined by wind tunnel tests on an "airfoil" model at a constant air speed. This model is normally of aspect ratio 6, and in this country, of the dimensions 3" x 18", 5" x 30" or 6" x 36", the smaller dimensions being the chord, or length in the direction of the relative wind. The variation of wing section characteristics with Reynolds Number cannot be fully determined in the average atmospheric wind tunnel because the combination of the largest model which can be tested with the highest test speed available or attainable gives a Reynolds Number which is of the order of 5% to 10% of the full scale value. The variable density wind tunnel is the most satisfactory method yet found for obtaining full-scale data.

In comparing data from various wind tunnels, exact agreement must not be expected. Good agreement should always be obtained however, if the tests are carefully made, particularly

with regard to corrections for the model attachment, and if the data are corrected for wall interference. (See Chapter II.) It has been found by experience that the test speed should be not less than 30 nor more than 120 miles per hour in order to avoid critical values of Reynolds Number and the effects of compressibility. Model chords of less than 3" are also likely to show the effects of critical flow conditions, due perhaps to unavoidable spindle or clip interference.

**Methods of plotting wing section data.** Wing section characteristics in general may be plotted against angle of attack or lift coefficient. The former is convenient in presenting wind tunnel data, but the latter is more suitable for comparing various sections.

The conventional method of plotting characteristics against angle of attack makes use of the "geometrical" angle of attack measured from an arbitrary chord line, which is usually the line of tangency to the lower camber of the section. The engineer should understand that this arbitrary reference line is adopted for convenience of definition and not because it has any connection with lift and drag characteristics. If the characteristics are plotted against the "absolute" angle of attack measured from the angle of zero lift, the difference between various sections is found to be much reduced.

Moment coefficients should be plotted against lift coefficients, and center of pressure against speed range  $\frac{V}{V_s}$ , since in both cases the resulting curves will be approximately straight lines. The common plot of center of pressure against angle of attack is very difficult to read.

Profile drags should be plotted against either lift coefficient or speed range.

**Selection of a wing section.** The selection of the best wing section for any particular design depends upon a number of factors, the most important of which are:



Structural adaptability  
 Stable airflow characteristics  
 Low profile drag at all lift coefficients  
 Low minimum drag  
 High minimum lift  
 Moderate moments, or a reasonable center of pressure travel

By structural adaptability is meant the suitability of the section for the type construction under consideration. This usually sets a rather definite limit to the spar depths for an efficient structure. For example, in a biplane a much thinner section can be used than in a monoplane. The designer knows in advance the approximate spar depths desired and is able to set aside a group of wing sections for further investigation.

Certain wing sections are very sensitive to slight mutilations or variations from the specified curvatures. Other sections can be modified considerably without appreciable change in characteristics. The difference is probably due primarily to some instability in the air flow around sections of the first type, and may be caused by irregularities in the variation of radius of curvature along the upper camber. In some cases the unstable flow is apparent from the conventional wind tunnel test and it is usually indicated by irregularities in pressure distribution curves.

The requirement of either a low minimum drag or a high maximum lift serves to restrict the number of sections available for a given design. The final selection is then made on the basis of a low profile drag. A very convenient method of analyzing

section data is to plot  $\left(\frac{C_{D_0}}{C_{L_{max}}}\right)$  against  $\left(\frac{C_L}{C_{L_{max}}}\right)^{\frac{1}{2}}$ .

$$\left(\frac{C_{D_0}}{C_{L_{max}}}\right) = \left(\frac{D_0}{W}\right)$$

and hence is proportional to the total profile drag. This method, therefore, compares the sections on the basis of total profile drag for constant gross load and stalling speed. The section selected will vary with the major requirement as follows:

*Maximum speed.* Section having least value of

$$\left( \frac{C_{D_0}}{C_{L_{max}}} \right) \text{ at high speed ratios, } \frac{V}{V_s} > 2.5.$$

*Maximum climb and ceiling.* Section having least value

$$\text{of } \left( \frac{C_{D_0}}{C_{L_{max}}} \right) \text{ between } \left( \frac{V}{V_s} \right) = 1.10 \text{ and } \left( \frac{V}{V_s} \right) = 1.50.$$

*Maximum endurance.* Section having least value of

$$\left( \frac{C_{D_0}}{C_{L_{max}}} \right) \text{ at } \left( \frac{V}{V_s} \right) = 1.10.$$

*General performance.* Section having least average value

$$\text{of } \left( \frac{C_{D_0}}{C_{L_{max}}} \right) \text{ at all values of } \left( \frac{V}{V_s} \right).$$

A preliminary study of polar diagrams, moment curves and other data will ordinarily reduce to three or four of the number of sections to be analyzed for final selection. There is little to be gained by carrying out complete performance calculations in order to make a decision, but wind tunnel tests sometimes show a difference between two models which cannot be predicted from the wing section data. This difference is probably an interference effect which properly belongs under the heading of stable airflow characteristics. For example, certain sections such as the U.S.A.-27 and G-387 do not show up as well in an airplane model test as do the G-398 and its derivatives N-10 and N-22, or the R.A.F.-15 and Clark Y.

**Example of analysis of section data.** Values of  $\frac{C_{D_0}}{C_{L_{max}}}$  for the U.S.A.-27, Clark Y, G-398, R.A.F.-15, and N-22 sections are plotted against  $\frac{V}{V_s}$  on Figure 1. These data were obtained on 5" x 30" models at 40 miles per hour in the 8' x 8' Washington Navy Yard Wind Tunnel and have been corrected for wall interference. This figure brings out the superiority of the G-398 and N-22 sections (the latter is 90% of the G-398 reduced on the chord line). For climb and ceiling the N-22 is exceptionally

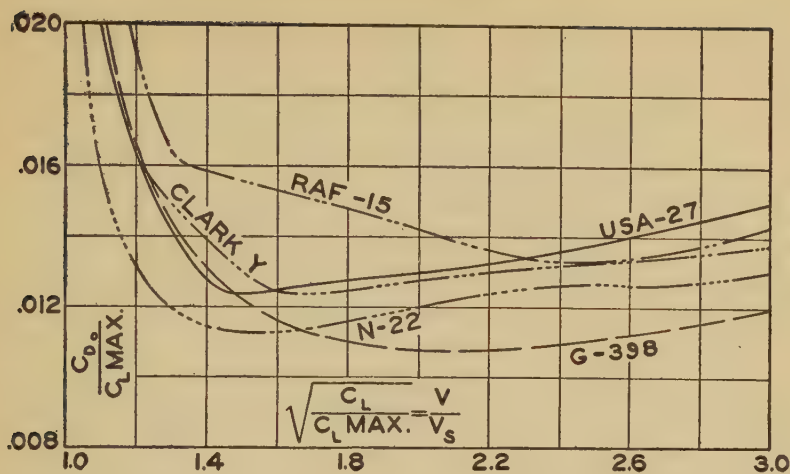


Figure 1. Variation of  $\frac{C_{D_0}}{C_{L \text{ MAX.}}}$  with  $\frac{V}{V_S}$  from Washington Navy Yard Tests

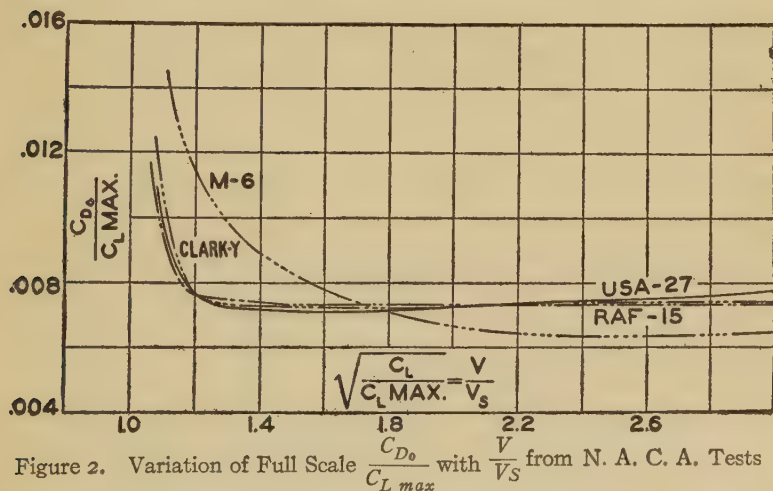


Figure 2. Variation of Full Scale  $\frac{C_{D_0}}{C_{L \text{ MAX.}}}$  with  $\frac{V}{V_S}$  from N. A. C. A. Tests

good, while for all-around performance the G-398 shows up well. The poor showing of the R.A.F.-15 is interesting in view of its general excellence on the basis drag per unit area.

This comparison must be made with full-scale data if the conclusions are to be reliable. Figure 2 is a plot of  $\frac{C_{D_0}}{C_{L \text{ MAX.}}}$  against

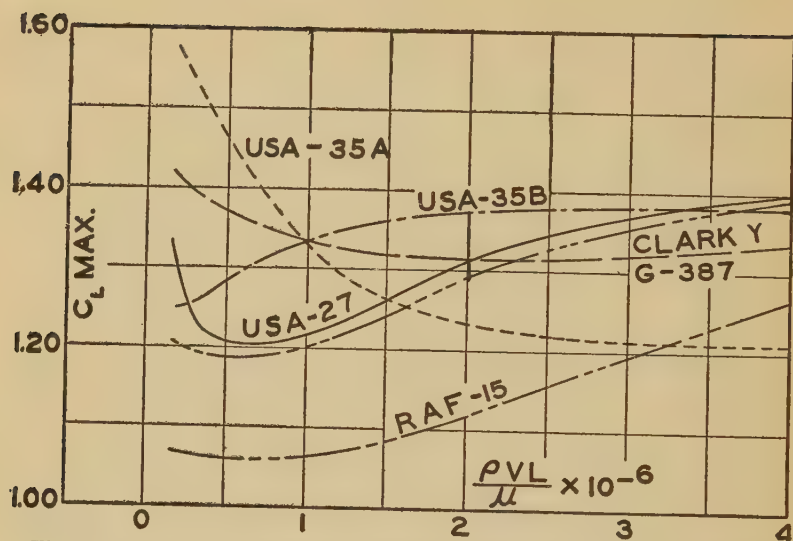


Figure 3. Effect of Reynolds Number on  $C_{L \max}$ . N. A. C. A. Tests

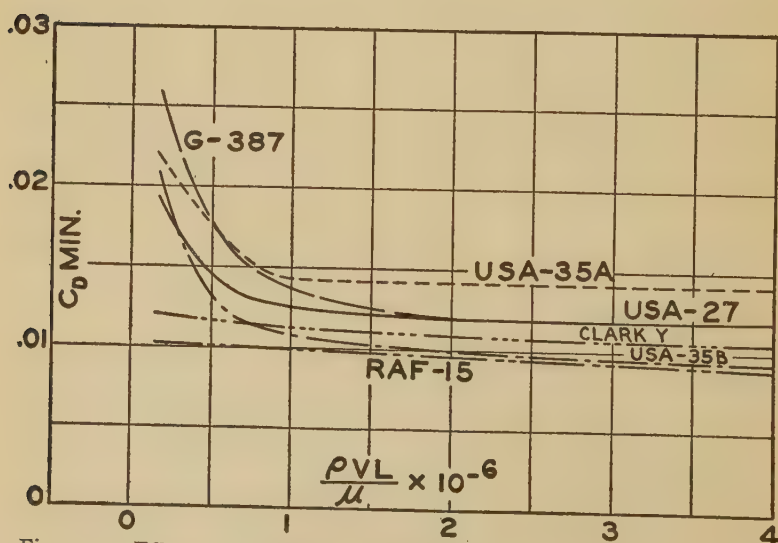


Figure 4. Effect of Reynolds Number on  $C_{D \min}$ . N. A. C. A. Tests

$\frac{V}{V_s}$  for the U. S. A.-27, Clark Y, R. A. F.-15, and M-6 sections, using data obtained in the N. A. C. A. variable density wind tunnel at approximately full-scale Reynolds Number. It appears that the first three sections are almost identical, but the M-6 is poor for climb and good for high speed. Unfortunately, similar data on the G-398 and N-22 are not now available so that the comparison can not be extended to these sections. However, preliminary tests indicate that the scale effect on the G-398 is similar to that on the Clark Y and flight tests indicate exceptionally good full-scale characteristics.

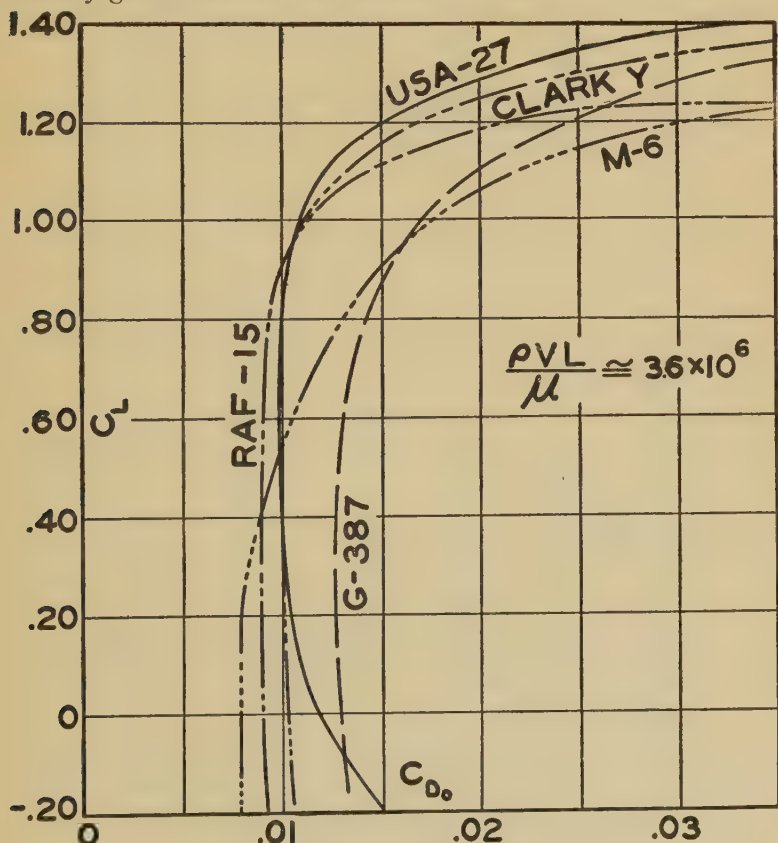


Figure 5. Full-Scale Profile Drag Coefficient. N. A. C. A. Tests

Variable density wind tunnel data. The principle of Munk's variable density method for securing data at high Reynolds

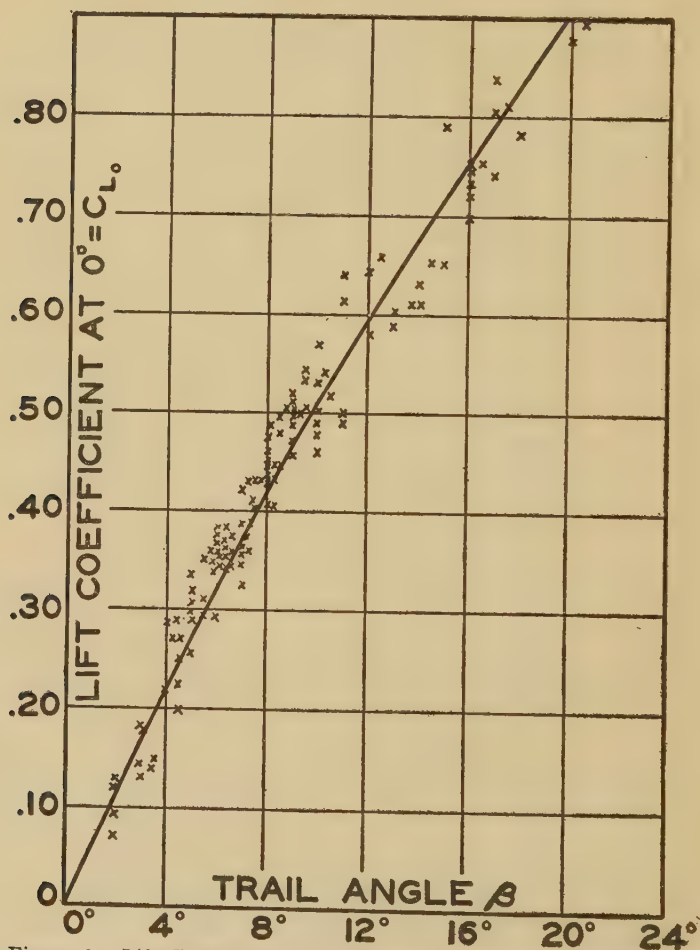


Figure 6. Lift Coefficient at  $\alpha = 0^\circ$  as a Function of Trail Angle  $\beta$

Number is well known. Comparison of flight test data with the results obtained in the N. A. C. A. variable density wind tunnel have shown such excellent agreement that there can be no doubt of the validity of this method.



Figure 3 gives the variation of  $C_{L\max}$  and Figure 4 gives the variation of  $C_{D\min}$  with Reynolds Number as obtained in the variable density tests on a series of popular wing sections. Figure 5 is a plot of profile drag  $C_{D_0}$  against lift coefficient for various sections, using test data at Reynolds Numbers of approximately  $3.6 \times 10^6$ .

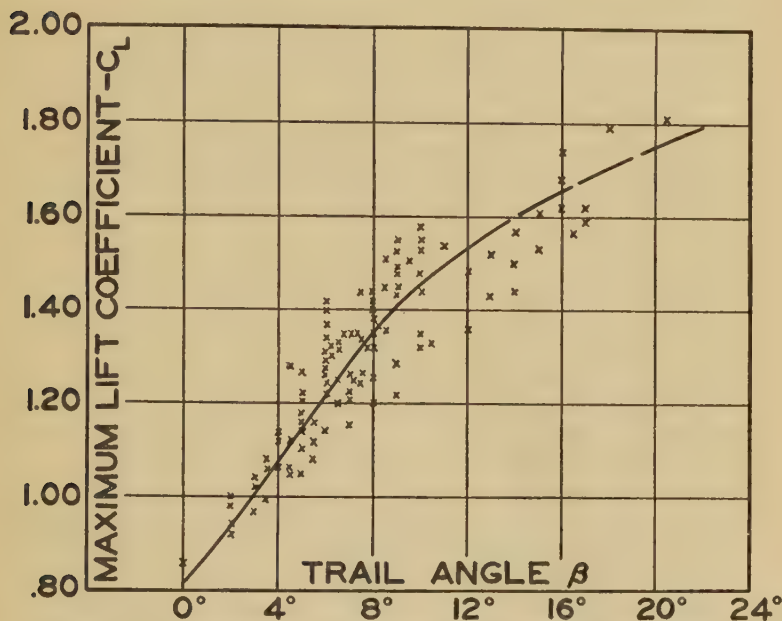


Figure 7. Maximum Lift Coefficient as a Function of Trail Angle  $\beta$

**Trail angle.** The acute angle which the tangent to the mean camber at the trailing edge of an airfoil section makes with the chord line of an airfoil is known as the trail angle. (See Figure 9.) This angle is a convenient substitute for the mean camber and certain characteristics may be referred to it for approximate estimates, etc. The effect of trail angle on lift at zero angle of attack (of the chord line) is given in Figure 6. The effect of trail on maximum lift coefficient is given in Figure 7, and the effect of trail angle on center of pressure travel is given in Figure

8. The center of pressure is given to a fair degree of approximation by the empirical equation:

$$C_p = .17 + .075 \left( \frac{V_M}{V_S} \right) + .00645 \left( \frac{V_M}{V_S} \right)^2 \beta \quad (1)$$

These data apply with a fair degree of accuracy to all normal wing sections, including those with flaps on the trailing edge.

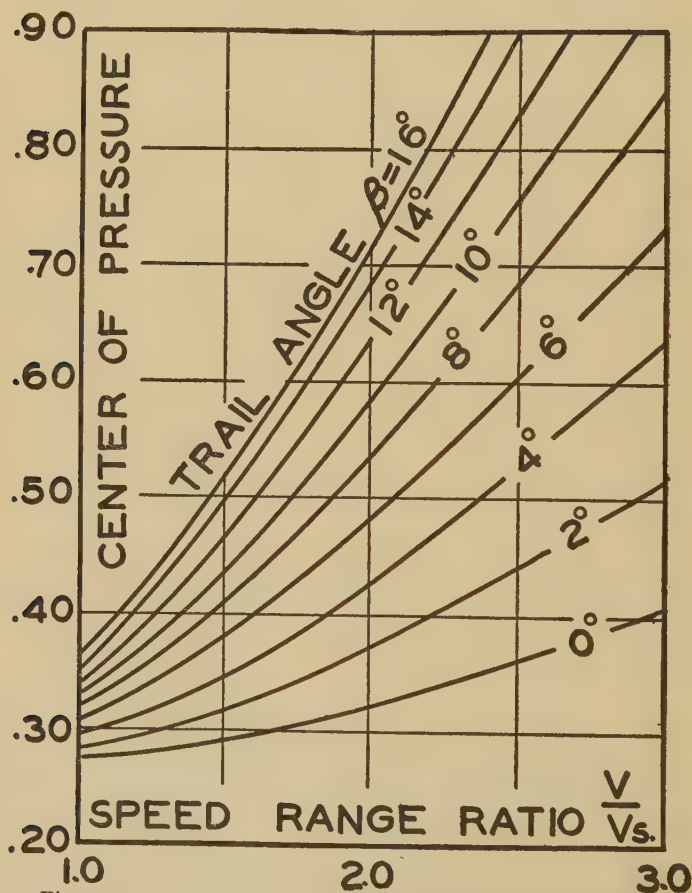


Figure 8. Variation of  $C_p$  with Trail Angle and Speed Range

**Mean camber.** The mean camber of an airfoil section lies midway between the upper and lower cambers. (See Figure 9.)

The mean camber has a greater effect on the aerodynamic characteristics of an airfoil than any other physical dimension. The general airfoil sections of the same mean camber have the same or very similar aerodynamic characteristics provided that the thickness of the section is not greater than 20% of the chord.

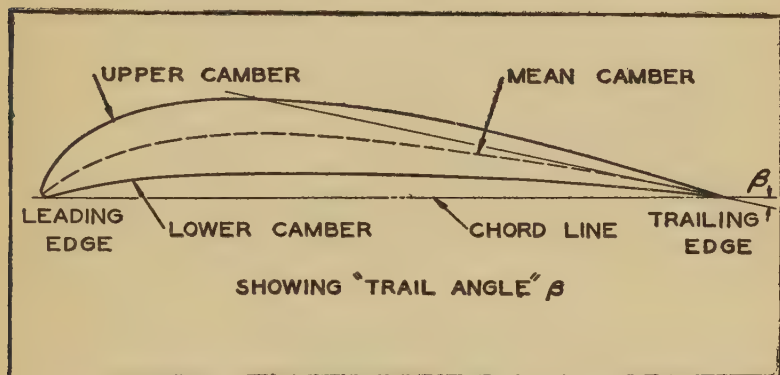


Figure 9. Illustrating Mean Camber and Trail Angle

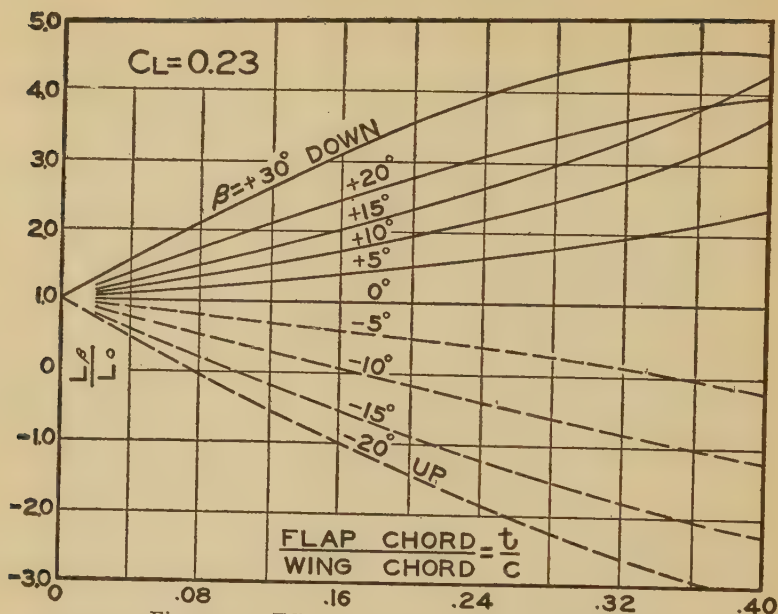
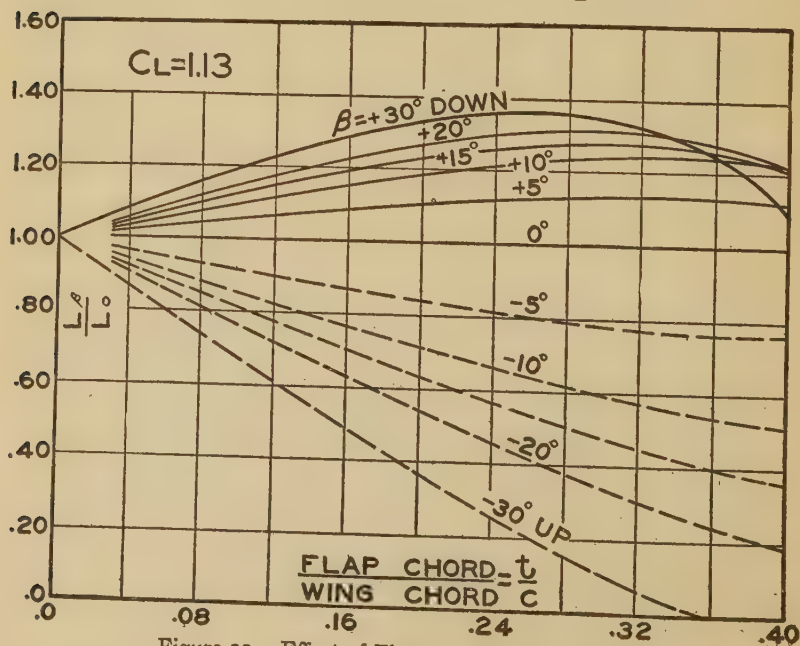
Wind tunnel tests at the Washington Navy Yard have shown that a thin section, like the well-known "Sloane," may be increased 60% in thickness in this manner without appreciably affecting the general aerodynamic characteristics.

**Wing flaps.** Munk has shown<sup>2</sup> that the effect of turning up or down a flap (or elevator) is

$$\Delta\alpha_e = K \frac{t}{c} \beta \quad (2)$$

where  $\Delta\alpha_e$  is the change in angle of attack of the entire surface which has the same effect as turning the flap (or elevator) to the angle  $\beta$  measured positive downward,  $\frac{t}{c}$  the ratio of the flap chord to the total chord, and  $K$  a constant which varies with the ratio  $\frac{t}{c}$ . In the original paper, Munk gives the derivation of  $K$  and a

<sup>2</sup> Max M. Munk "General Theory of Thin Wing Sections," N. A. C. A. Technical Report No. 142 (1922).

Figure 10. Effect of Flaps on Lift at  $C_L = 0.23$ Figure 11. Effect of Flaps on Lift at  $C_L = 1.13$

plot of  $K$  against  $\frac{t}{c}$ . Between  $\frac{t}{c} = .10$  and  $\frac{t}{c} = .60$ , which covers the whole practical range,  $K$  is given closely by

$$K = 1.90 - 0.57 \frac{t}{c} \quad (3)$$

Equation (2) may be written in the more convenient form:

$$\Delta C_L = \frac{dC_L}{d\alpha} K \frac{t}{c} \beta \quad (2a)$$

Analysis of a number of wind tunnel tests verifies Munk's equation for the small values of  $\beta$  on which it is based, but for large values of  $\beta$ ,  $K$  is considerably reduced. A plot of  $K$  against  $\beta$ , as determined from the experimental data in R. & M.<sup>3</sup> Nos. 110, 152, and 319, indicate that the variation of  $K$  with  $\beta$  is approximately linear and according to

$$K_a = K_t - .014 \beta^\circ = 1.90 - 0.57 \frac{t}{c} - .014 \beta^\circ \quad (3a)$$

where  $K_a$  is the actual value and  $K_t$  the theoretical value.

More recently Glauert<sup>4</sup> has derived some theoretical relations for lift and moment of wings with flaps, which are somewhat complicated but appear to give excellent results.

The engineer is interested primarily in flap hinge moments and the changes in lift and drag due to flaps. For this purpose a series of figures has been prepared from test data.<sup>5</sup>

The change in lift due to flaps of various chords set at various angles is shown on Figures 10 and 11. The former is for a lift coefficient  $C_L = 0.23$  and the latter for a lift coefficient of  $C_L = 1.13$ , corresponding to angles of attack of  $0^\circ$  and  $12^\circ$  on the R.A.F.-9 section. Figures 12 and 13 are companion curves showing the change in drag. These curves may be used to calculate rolling and yawing moments of ailerons, by assuming that only the wing area immediately forward of the aileron is affected together with the aileron.

<sup>3</sup> Reports and Memoranda, British Aeronautical Research Committee.

<sup>4</sup> H. Glauert, "Theoretical Relationships for an Aerofoil with Hinged Flap," Br. A.R. C. R. & M. No. 1095 (1927).

<sup>5</sup> Taken from R. & M. Nos. 110 and 319.

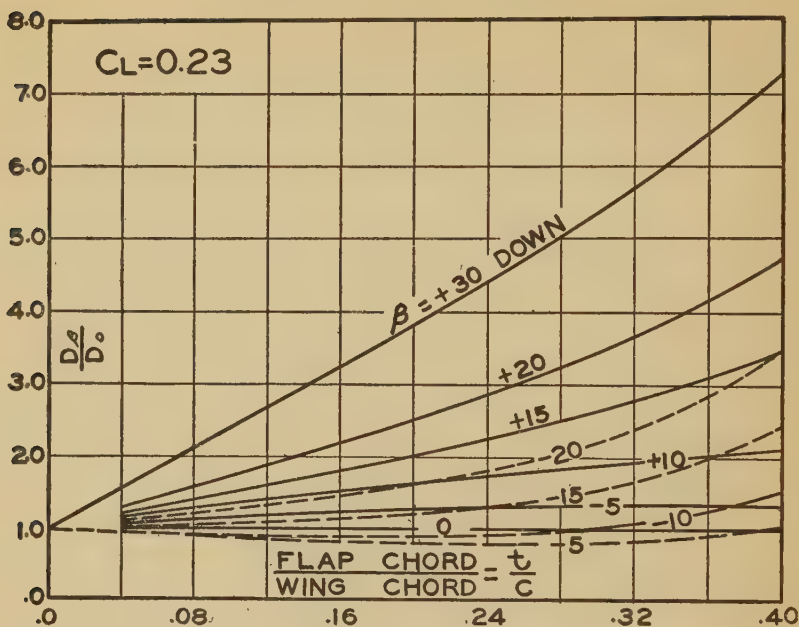


Figure 12. Effect of Flaps on Drag at  $C_L = 0.23$   
(0.0006)

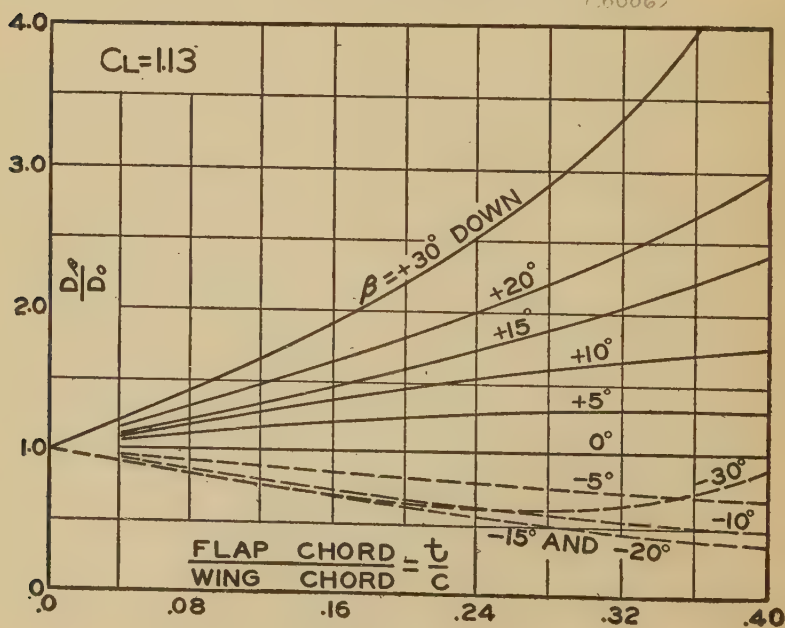


Figure 13. Effect of Flaps on Drag at  $C_L = 1.13$



Hinge moments are shown on Figures 14 and 15 for the same conditions applying to lift and drag. The hinge moment in lbs.-ft. is given by

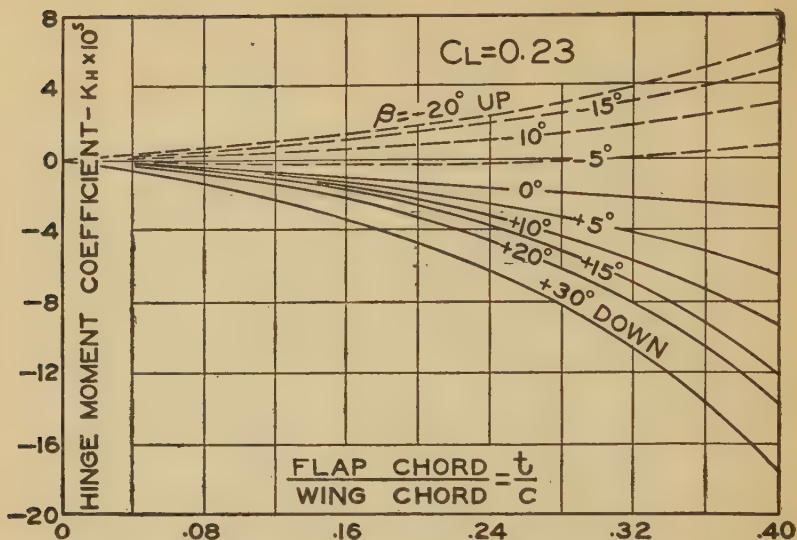
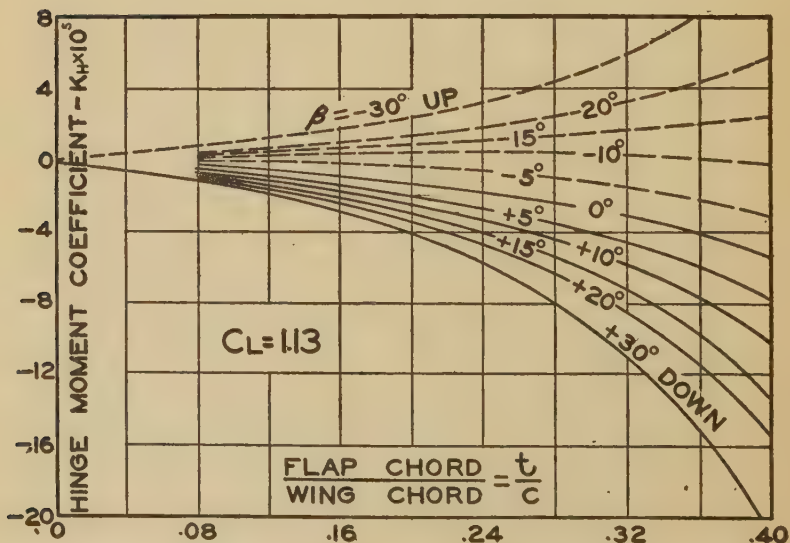
$$H = K_H \frac{\rho}{\rho_0} a c V^2 \quad (4)$$

where  $a$  is the flap area (sq. ft.),  $c$  the flap chord (ft.), and  $V$  the air speed in miles per hour.

These plots are of considerable interest in that they bring out the decrease in effectiveness of ailerons at large lift coefficients and show the origin of the yaw due to ailerons. They also show how rapidly the hinge moment increases with increase of flap chord.

The increase in maximum lift which can be secured by the use of flaps is practically independent of the original wing section, although it varies slightly with the camber ratio and  $C_{L\max}$ . It is approximately  $\Delta C_{L\max} = 0.70$  or  $\Delta K_{y\max} = .0018$  for a thin low-lift section, and  $\Delta C_{L\max} = 0.60$  or  $\Delta K_{y\max} = .0015$  for a thick high-lift section. Flaps therefore show up best on thin low-lift sections. The only practical use of flap is to increase the maximum lift and decrease the stalling speed. It is very unlikely that climb and ceiling can ever be appreciably improved by the use of flaps of the conventional form on a good wing section. However, there appears to be some field for the future use of very narrow balanced flaps extending over the entire span and fitted with a gear which permits differential operation as ailerons from any flap setting. The purpose of such an arrangement is to reduce the flap forces and weight, and the only gain to be anticipated is a decrease in stalling speed.

**Slots.** The Lachman-Handley Page slot is well known from the great amount of publicity that it has received as a means of increasing lift and preventing loss of control in a stall. The latest development is the automatic slot which opens without external control, by the reduction of pressure on the leading edge airfoil. This slot can be designed to open at an angle of attack just below the stall, and when fitted to the wing forward of the ailerons it gives remarkable lateral control at very high angles.

Figure 14. Flap Hinge Moment Coefficients at  $C_L = 0.23$ Figure 15. Flap Hinge Moment Coefficients  $K_H \times 10^5$  at  $C_L = 1.13$

As a means of increasing lift and reducing stalling speed, the slot is very effective, although somewhat disappointing in that the full-scale net gain is much less than might be expected from wind tunnel tests on airfoil models with a slot across the entire

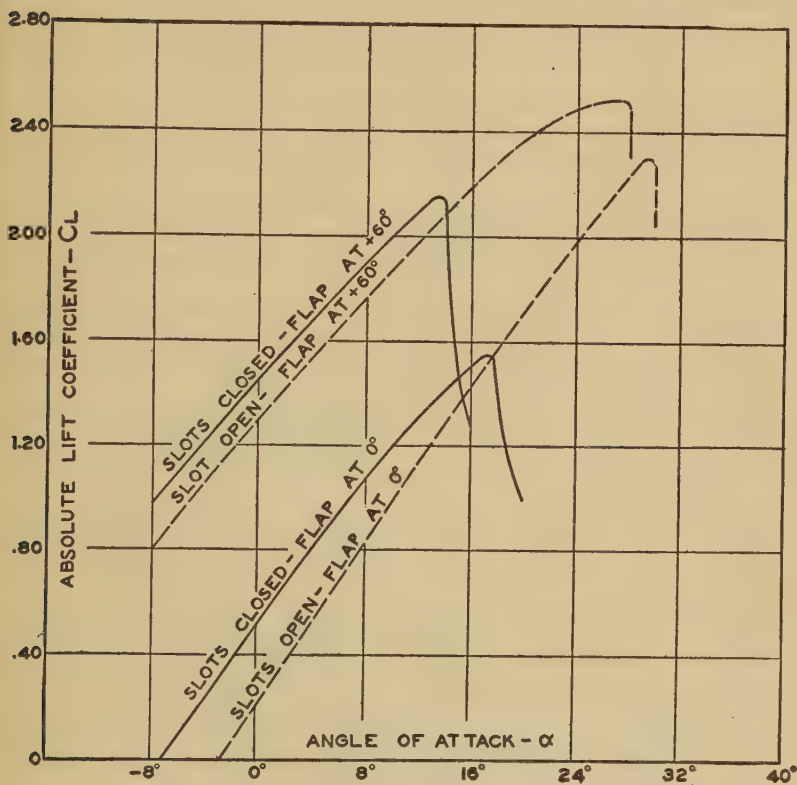


Figure 16. Effect of Slots and Flap on Lift Coefficient

span. The loss may be ascribed to two causes. In most practical applications it is not feasible to extend the slot across the entire span or to keep the slot clear of obstructions, and any interruption to the slot or to the flow through it causes a very marked reduction in lift. The second cause for failure to obtain the full benefit of the slot is found in the very high angles of attack required for maximum lift. The slot merely extends the

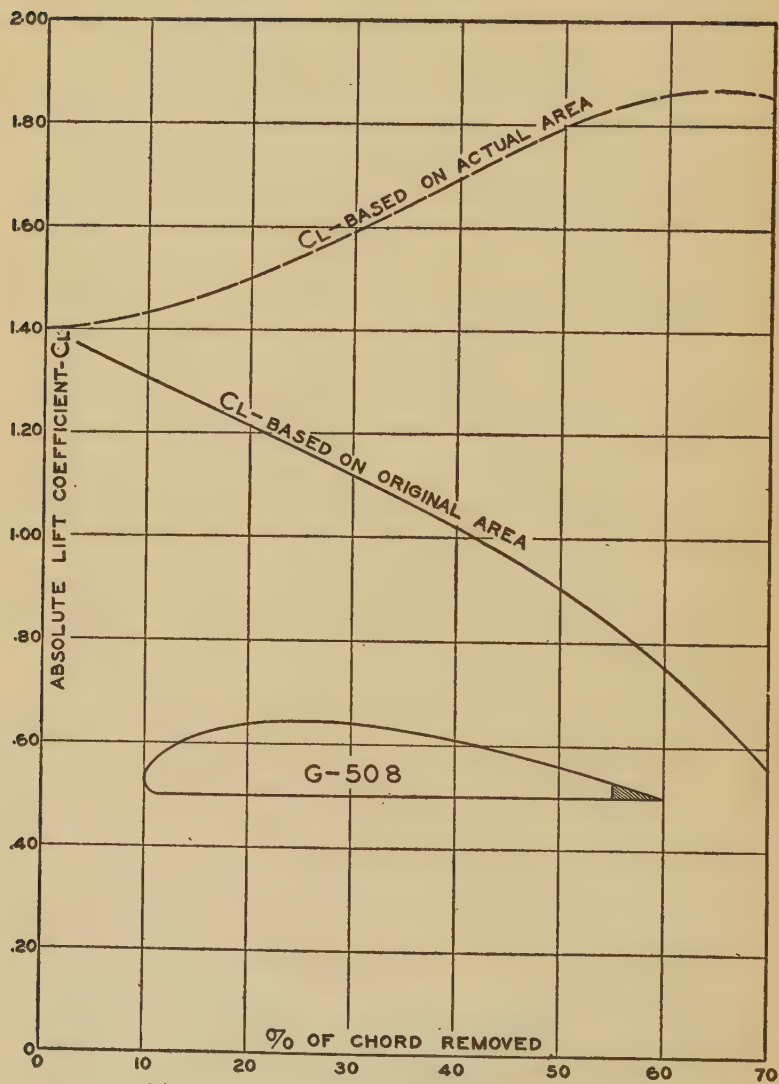


Figure 17. Effect of Cutting off Trailing Edge

lift curve against angle of attack without appreciably changing the slope, and maximum lift may occur between  $23^\circ$  and  $29^\circ$ .

Slots are most effective when used in conjunction with flaps as shown by Figure 16, which is based on wind tunnel tests on a model wing fitted with two slots and a flap. If the flap alone were used at an angle of attack of  $16^\circ$  there would actually be a decrease in lift coefficient from  $C_L = 1.50$  to  $C_L = 1.30$  approximately, but if slots are used with the flaps, the lift coefficient increases from  $C_L = 1.50$  to  $C = 2.18$ .

**Wing tips; cut-outs.** When the chord of a wing varies, the thickness of the section should vary in proportion, particularly at the tips. If the wing section is originally thick, an improvement is obtained by taper in camber as well as in chord, at the tips.

For cut-outs at the center section, very little is to be gained by making the sections geometrically similar, and a fairing to the trailing edge is sufficient. Figure 17 shows how little the lift is affected by cutting off the trailing edge of a wing. The section drag at all lift coefficients for any particular amount removed is increased by approximately  $1/10$  of the reduction in  $C_L$  based on the original area.

The effect of center section cut-outs on wing characteristics has been investigated at the Washington Navy Yard<sup>6</sup> and the conventional approximately semicircular form with rounded fairing into the normal trailing edge, was found quite satisfactory.

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<sup>6</sup>R. H. Smith, "Air Force and Moment for N-20 Wing with Certain Cut-Outs," N. A. C. A. Technical Report No. 266 (1927).

## CHAPTER II

### WING THEORY

**Hydrodynamical definitions.** Before giving an outline of some of the outstanding developments in theoretical aerodynamics, it is desirable to give a brief review of the hydrodynamical conceptions on which the subject of modern aerodynamics is based. It is not possible in a work of this type to give more than a definition of the more common terms, but every aeronautical engineer should have a clear conception of what is meant by potential flow, circulation, vorticity, etc.

For further information on theoretical hydrodynamics, reference should be made to Prandtl's "Application of Modern Hydrodynamics to Aeronautics" (N. A. C. A. Technical Report No. 1116), or to the following books:

- E. B. Wilson, "Aeronautics," John Wiley & Son (1920).
- "The Mechanical Properties of Fluids." A collective work. D. Van Nostrand (1924).
- H. Glauert, "The Elements of Aerofoil and Airscrew Theory." Cambridge University Press.
- C. Eberhardt, "Einführung in die Theoretische Aerodynamik." R. Oldenbourg, München (1927).
- H. Lamb, "Hydrodynamics." Cambridge University Press.

Each of these books contains much of value to the student but the last three are particularly valuable. The beginner who can read German will find Eberhardt's book very clear and instructive. Lamb's Hydrodynamics is the classical general treatment of this subject.

**Bernoulli's theorem.** Bernoulli's theorem states that the



total energy of a fluid particle is constant at all points on its path in a steady flow. Bernoulli's equation is

$$\frac{P}{g\rho} + \frac{1}{2} \rho V^2 + h = H \quad (5)$$

where  $\frac{P}{g\rho}$  is the "pressure head,"  $\frac{1}{2} \rho V^2$  the "velocity head,"  $h$  the potential head, and  $H$  a constant.

This equation is due to Daniel Bernoulli and dates from 1737. In the original form and as given it applies only to steady flow of an incompressible fluid but many of the more common problems of hydrodynamics are solved by its use, with the following restrictions:

For a general fluid in general motion,  $H$  is never constant.

For an incompressible fluid in general motion,  $H$  is constant for a given particle.

For an incompressible fluid in steady motion,  $H$  is constant for all particles along a streamline.

For an incompressible fluid in steady irrotational motion  $H$  is constant for all particles throughout the fluid.

**Stream function.** Consider the flow of an incompressible fluid across an element  $dS$  of any curve in the plane of a two-dimensional flow. The flow across  $dS$  must be equal to the sum of the flows across  $dx$  and  $dy$ . The flow across  $dx$  is  $v dx$  and that across  $dy$  is  $-u dy$ , hence the flow across the element  $dS$  is

$$d\psi = v dx - u dy$$

and the difference between the values of  $\psi$  at any two points along the curve is given by the integral

$$\int d\psi = \int (v dx - u dy) \quad \psi$$

$\psi$  is called the "stream function."

**Streamline.** Mathematically a streamline is defined by  $\psi = C$ , or  $d\psi = 0$ , since for this condition there is no flow across the curve so defined.

**Two-dimensional flow.** A flow which is two-dimensional in the plane of X-Y will be exactly similar in any parallel plane.

An example of such a flow is that around the center of a very long cylinder or strut. Two-dimensional flows can be completely investigated by considering the flow in a single plane.

The mathematical conditions for two-dimensional flow are: (1) no velocity component along the  $z$  axis, and (2) no  $x$  or  $y$  velocity gradients along the  $z$  axis. That is

$$V_z = 0$$

$$\frac{\partial V_x}{\partial z} = \frac{\partial V_y}{\partial z} = 0$$

**Three-dimensional flow.** The general motion of a fluid is three-dimensional, with acceleration and velocity components along all three axes.

**Velocity potential.** If a fluid flow is such that the circulation about every closed curve vanishes, then the flow may be represented by

$$u \cdot dx + v \cdot dy = - d\Phi$$

where  $\Phi$  is a function of  $x$  and  $y$ , known as the *velocity potential*. Any fluid motion for which there is a velocity potential is said to be *irrotational*, or a *potential flow*.

The velocity potential  $\Phi$  and stream function  $\psi$  occur in three-dimensional flow, in exactly analogous senses as in two-dimensional flow.

**Circulation.** Circulation in a fluid is determined by the line integral of the tangential velocity taken around any closed circuit. That is

$$\Gamma = \oint V_r \cos \theta \cdot dS$$

where  $V_r$  is the resultant velocity making an angle  $\theta$  with the elements  $dS$ . Resolving  $V_r$  into its components gives

$$V_r \cos \theta = \left( u \cdot \frac{dx}{dS} + v \cdot \frac{dy}{dS} \right)$$

hence

$$\Gamma = \oint \left( u \cdot \frac{dx}{dS} + v \cdot \frac{dy}{dS} \right) dS = \oint (u \, dx + v \, dy)$$

Many engineers instinctively associate circulation with lift. It should be understood that circulation in general is a type of fluid motion and that in any particular case it has the value given by the integral in the preceding paragraph.

The lift on an element of unit length of a wing of infinite span varies directly as the circulation around it. That is

$$L = \rho \Gamma V$$

where  $\rho$  is the density and  $V$  the relative velocity measured at a great distance from the element. This is the well-known Kutta-Joukowski equation.

For a wing of finite span, the lift and from it the induced drag can be calculated only when the distribution of circulation along the span is known. If the circulation is constant along the span, then  $\frac{d\Gamma}{db}$  is zero except at the tips where it equals  $\Gamma$ . This would correspond to a constant lift along the span, and there would be a vortex at each tip only. Actually there is a vortex at each tip and a gradient of  $\Gamma$  along the span from a maximum value at the center to zero at the tips. The vortices which peel off of the trailing edge vary in strength with  $\frac{\Gamma}{db}$ . Hence the variation in vorticity along the span is from a maximum positive value at one tip to an equal negative value at the other tip, passing through zero at the center.

**Vorticity-vortices.** The circulation around an infinitesimal area is proportional to the area, and the ratio of circulation to the area is the measure of the intensity of rotation or vorticity.

The so-called "Vortex Theory" of Prandtl, by means of which solutions have been made for many problems involving wing lift and drag, is perhaps better considered as a group of theorems explaining the action of vortices and the method of determining the resultant velocity at any point from a given distribution of vortices.

In this connection a *vortex* is a mathematical rather than a physical conception. The violent whirling motion in water,

which is called a vortex, certainly has some rotation although it may be irrotational. It would be *irrotational* if the velocity varied inversely as the radius, and it is *rotational* to the extent that the velocities deviate from the inverse law. The simplest case of rotation is a mass of fluid rotating as a rigid body, and for this case the *rotation* or *vorticity* is twice the angular velocity.

The mathematical vortex is a convenient method and not a necessity as Munk<sup>1</sup> has shown. The same results are obtained by kinetic energy relations.

**Induced drag.** In 1911, Dr. Prandtl and his assistants at Göttingen derived a relation between the circulation  $\Gamma$  and the vertical or downwash velocity component  $w$ , due to a lifting wing. At the same time it was proved that half of the final downwash velocity was acquired forward of the center of pressure, or in other words, a downward acceleration was imparted to the air at some distance forward of a lifting wing which therefore operated in a downwardly inclined airstream. The average downwash velocity at the center of pressure was found to be greater, the greater the lift, and the nearer to the center of the wing the main production of vortices.

As a result of the virtual inclination of the airstream through the angle  $\varphi = \tan^{-1} \left( \frac{w}{v} \right)$ , the wing "goes up a hill" having the slope  $\varphi$ . Consequently the lift, which is vertical to the relative wind, now has a rearward or drag component. Without going into the details of the derivation<sup>2</sup> it was shown that the inclination of the lift vector, and hence the drag, was a minimum when the downwash was constant along the span, and that this condition corresponded to a lift distribution proportional to the ordinates of an ellipse having the span as a diameter.<sup>3</sup> The constant value of the downwash velocity

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<sup>1</sup> Max M. Munk, "Elements of the Wing Section Theory and of the Wing Theory," N. A. C. A. Technical Report No. 191 (1924).

<sup>2</sup> See L. Prandtl, "Applications of Modern Hydrodynamics to Aeronautics," N. A. C. A. Technical Report No. 116.

<sup>3</sup> The mathematical proof was first given by Munk in his Göttingen Dissertation which was subsequently translated and published as N. A. C. A. Technical Report No. 121 "The Minimum Induced Drag of Aerofoils."

resulting from the elliptical lift distribution was shown to be

$$w = \frac{\Gamma_o}{2b}$$

where  $\Gamma_o$  is the circulation at the center of the span  $b$ . It may be shown that the value of  $\Gamma_o$  is

$$\Gamma_o = \frac{4L}{\pi\rho Vb}$$

where  $L$  is the lift and  $\rho$  the density. Hence the angle of downwash is

$$w = \frac{2L}{\pi\rho Vb^2}$$

Since the downwash is constant along the span the drag  $D$  is

$$D = L \times \frac{w}{v} = \frac{2L^2}{\pi\rho V^2 b^2} = \frac{L^2}{\pi q b^2} \quad (6)$$

It was found in 1913 that the actual measured drags were greater than this theoretical minimum but the wing sections investigated were very poor. Two years later, investigations on much better wing sections showed close agreement between the theoretical and the measured drag. The investigation was then continued on wings of various aspect ratios and a very important discovery made. At the same lift coefficient with the same wing section the difference between the theoretical and measured drag coefficients was always the same, within the experimental error of course, for any aspect ratio. The part of the total drag which does not change with aspect ratio is due to the shape of the wing section and hence it was given the designation "Profile Drag" or "Section Drag." The theoretical drag, varying only with lift and span, was given the designation "Induced Drag" from the analogy to electrical induction phenomena. This constituted one of the most important advances ever made in theoretical aerodynamics and opened an immense field of practical application.

Substituting  $C_L \frac{\rho}{2} S V^2$  for  $L$  and  $C_D \frac{\rho}{2} S V^2$  for  $D$  in equation



(6) gives the coefficient of induced drag

$$C_{Di} = \frac{C_L^2 S}{\pi b^2} \quad (7)$$

The section drag is  $C_{D_0} = C_D - C_{Di}$  and this is constant for any given section and lift coefficient. Hence at a constant value of  $C_L$  the relation between the drags for two aspect ratios is

$$C_{D_1} - \frac{C_L^2 S_1}{\pi b_1^2} = C_{D_2} - \frac{C_L^2 S_2}{\pi b_2^2}$$

or

$$C_{D_2} = C_{D_1} + \frac{C_L^2}{\pi} \left[ \frac{S_2}{b_2^2} - \frac{S_1}{b_1^2} \right] \quad (8)$$

which enables the drag to be calculated for any aspect ratio when the drag is known for one aspect ratio.

**Induced angle of attack.** Since the wing is operating in a vertical downwash velocity of

$$w = \frac{2L}{\pi \rho V b^2}$$

and a horizontal velocity of  $V$ , the relative wind direction is inclined downward to the rear of the wing by the angle  $\varphi$  having the value

$$\varphi = \tan^{-1} \left( \frac{w}{V} \right) = \tan^{-1} \left( \frac{2L}{\pi \rho V^2 b^2} \right)$$

$\varphi$  is always small so that  $\tan \varphi = \varphi$ , hence

$$\varphi = \frac{2L}{\pi \rho V^2 b^2} = \frac{C_L S}{\pi b^2} \quad (9)$$

$\varphi$  is the "induced angle of attack." It increases as the aspect ratio decreases. The physical significance is that as the aspect ratio is decreased the downwash increases and the wing must be turned up to a higher apparent or geometrical angle of attack in order to obtain a given lift coefficient.

**Munk's span factor.** Equation (7) was completed by Munk, who showed that in order to apply to biplanes (or multiplanes) the maximum span  $b$  must be replaced by  $kb$ , which is the span



of the monoplane having the same area and induced drag as the biplane (or multiplane). For a monoplane  $k = 1.00$ , but for a biplane  $k$  varies with the ratio of gap to span, the ratio of the spans and the proportional area in the two wings as will be shown later.

The introduction of the span factor was of great practical importance. With this factor equation (7) becomes

$$C_{Di} = \frac{C_L^2 S}{\pi (kb)^2} \quad (7a)$$

and equation (8) becomes

$$C_{D2} = C_{D1} + \frac{C_L^2}{\pi} \left[ \frac{S_2}{(k_2 b_2)^2} - \frac{S_1}{(k_1 b_1)^2} \right] \quad (8a)$$

In a similar manner, the induced angle of attack in radians is

$$\varphi = \frac{C_L S}{\pi (kb)^2} \quad (9a)$$

and in degrees

$$\alpha_2 = \alpha_1 + \frac{57.3 C_L}{\pi} \left[ \frac{S_2}{(k_2 b_2)^2} - \frac{S_1}{(k_1 b_1)^2} \right] \quad (10)$$

Equation (10) is only approximately correct. Munk completed it later<sup>4</sup> by dividing into three parts the angle of attack necessary to produce a given lift coefficient. These parts are as follows: (a) the intrinsic angle of attack for the given wing section and lift coefficient, (b) the additional induced angle of attack, and (c) the additional interference angle of attack. With this modification equation (10) becomes

$$\alpha_2 = \alpha_1 + \frac{57.3 C_L}{\pi} \left[ \left( \frac{S_2}{(k_2 b_2)^2} + I_2 \right) - \left( \frac{S_1}{(k_1 b_1)^2} + I_1 \right) \right] \quad (10a)$$

where  $I$  is the interference factor.  $I$  varies slightly with stagger and with wing section, and is less for a lift produced by curvature than for lift produced by angle of attack. However,  $I$  is approximately a function of gap chord ratio only, with the following average values according to Munk:

<sup>4</sup> Max M. Munk, "General Biplane Theory," N. A. C. A. Technical Report No. 151 (1922).

|                                            |      |      |      |      |      |      |      |
|--------------------------------------------|------|------|------|------|------|------|------|
| $\frac{\text{Gap}}{\text{Chord}} = \infty$ | 2.87 | 2.02 | 1.46 | 1.11 | .98  | .79  | .64  |
| $I = 0$                                    | .012 | .024 | .030 | .055 | .060 | .082 | .104 |

These values are plotted in Figure 18.

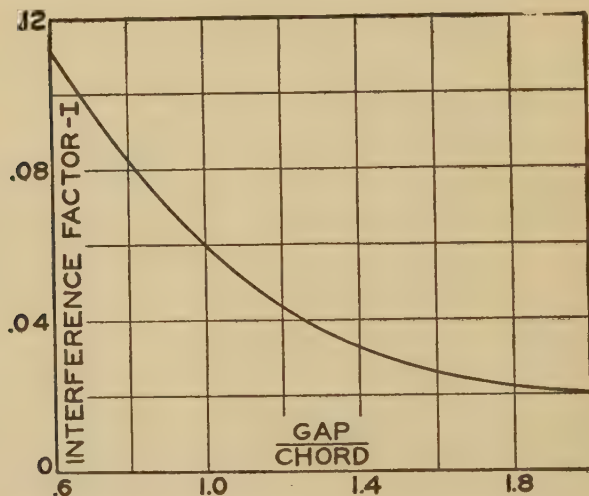


Figure 18. Munk's Interference Factor for Induced Angle of Attack

**Prandtl's interference factors.** Prandtl has shown<sup>5</sup> that the drag of one wing of a biplane in the presence of the other may be expressed in the form

$$D_{12} = \sigma \frac{L_1 L_2}{\pi q b_1 b_2} \quad (11)$$

where  $D_{12}$  is the drag of wing 1 (having lift  $L_1$  and span  $b_1$ ) in the presence of wing 2 (having lift  $L_2$  and span  $b_2$ ) and  $\sigma$  is an "interference factor" which varies with the ratios of gap to span and shorter span to longer span.

<sup>5</sup> "Technische Berichte," Vol. II No. 6, (N. A. C. A. Technical Note No. 182) and N. A. C. A. Technical Report No. 116.

Prandtl gives values of  $\sigma$  in terms of  $\frac{b_2}{b_1}$  and  $\frac{\text{gap}}{\text{average span}}$  or  $\left(\frac{2G}{b_1 + b_2}\right)$  as shown in Figure 19. These data have been con-

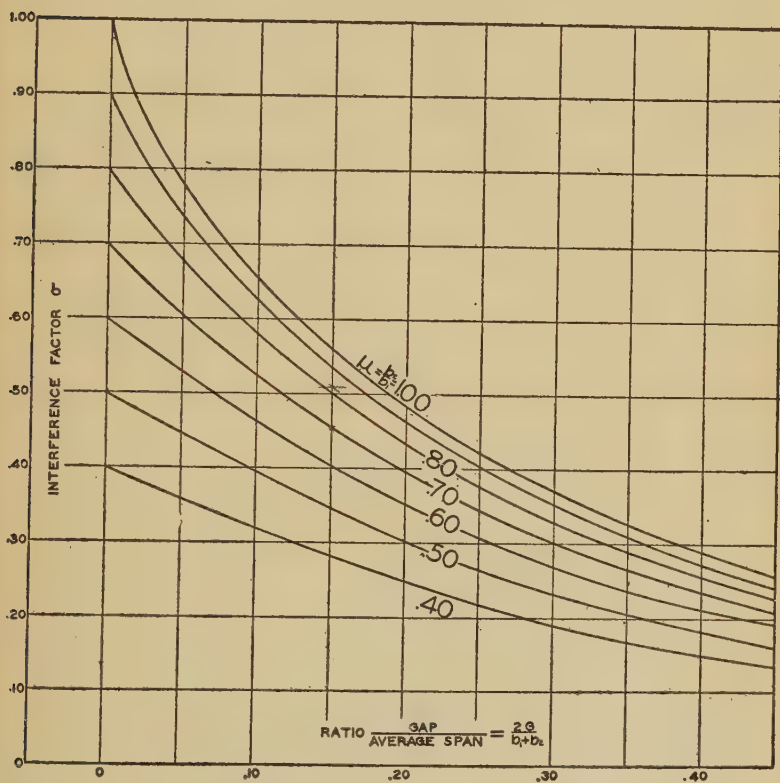


Figure 19. Prandtl's Drag Interference Factor for Biplanes in Terms of Average Span

verted and replotted in Figure 20, using the ratio of gap to maximum span, instead of the ratio, gap to average span.

**Induced drag of biplanes.** The interference factor  $\sigma$  may be used to calculate the value of the span factor  $k$  for any biplane, as shown by Prandtl. The method is as follows:

Let  $b_1$  = span of longer wing

$b_2$  = span of shorter wing

$\mu = \frac{b_2}{b_1}$  = ratio shorter span to longer

$L_1$  = lift on longer wing

$L_2$  = lift on shorter wing

$L = L_1 + L_2$  = total lift

Prandtl's equation for total induced drag is

$$D_i = \frac{1}{\pi q} \left[ \frac{L_1^2}{b_1^2} + 2 \sigma \frac{L_1 L_2}{b_1 b_2} + \frac{L_2^2}{b_2^2} \right] \quad (12)$$

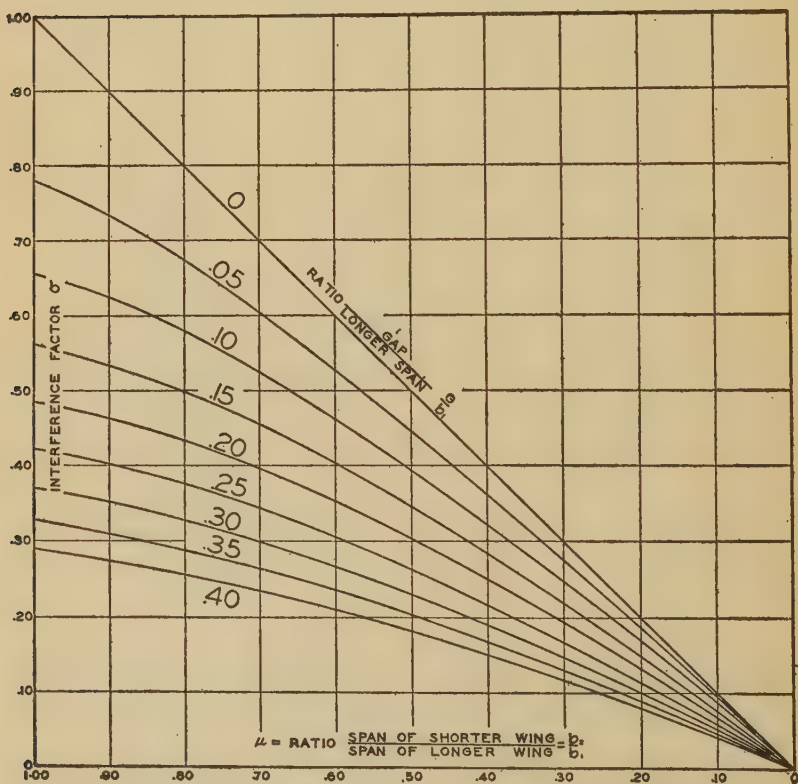


Figure 20. Prandtl's Drag Interference Factor for Biplanes in Terms of Maximum Span

This is a minimum when

$$\frac{L_2}{L_1} = \frac{\mu - \sigma}{\frac{1}{\mu} - \sigma}$$

and has the value

$$\text{Minimum } D_i = \frac{L^2}{\pi q b_1} \left[ \frac{1 - \sigma^2}{1 - 2\sigma\mu + \mu^2} \right] \quad (13)$$

Assuming the lift proportional to the area and setting  $S_1 = rS$ , it follows that  $S_2 = (1-r)S$ ,  $L_1 = rL$ , and  $L_2 = (1-r)L$ . The factor  $r$  is obviously the ratio of area (or lift) of the longer wing to the total area (or lift). Substituting these relations into equation (12) gives

$$D_i = \frac{L^2}{\pi q b_1} \left[ r^2 + \frac{2\sigma}{\mu} r (1-r) + \left( \frac{1-r}{\mu} \right)^2 \right]$$

from which the span factor is seen to be

$$k = \left[ \frac{\mu^2}{r^2 (\mu^2 - 2\mu\sigma + 1) + 2r (\mu\sigma - 1) + 1} \right]^{\frac{1}{2}} \quad (14)$$

Figures 21, 22, 23, 24, and 25 give the values of  $k$  against  $\mu$  and  $r$  for  $\frac{G}{b_1} = .05, .10, .15, .20$  and  $.25$ . This covers the extreme range ordinarily used. The variation of  $k$  is substantially linear with  $\frac{G}{b_1}$  between any two adjacent values of  $\frac{G}{b_1}$  and hence  $k$  may be obtained by interpolation. However, a great majority of biplane designs have wings which approximate either equal chords or equal aspect ratio. For this purpose Figures 26 and 27 have been prepared. These give the value of  $k$  directly in terms of  $\frac{b_2}{b_1}$  and  $\frac{G}{b_1}$ .

**Proportions of the most efficient biplane.** Figure 28 gives the proportions of the most efficient biplane as determined from Figures 21 to 25. From this diagram the best value of any one variable,  $\frac{C_2}{C_1}$ ,  $\frac{G}{b_1}$ , or  $\frac{b_2}{b_1}$ , is determined when the other two are assumed or known.

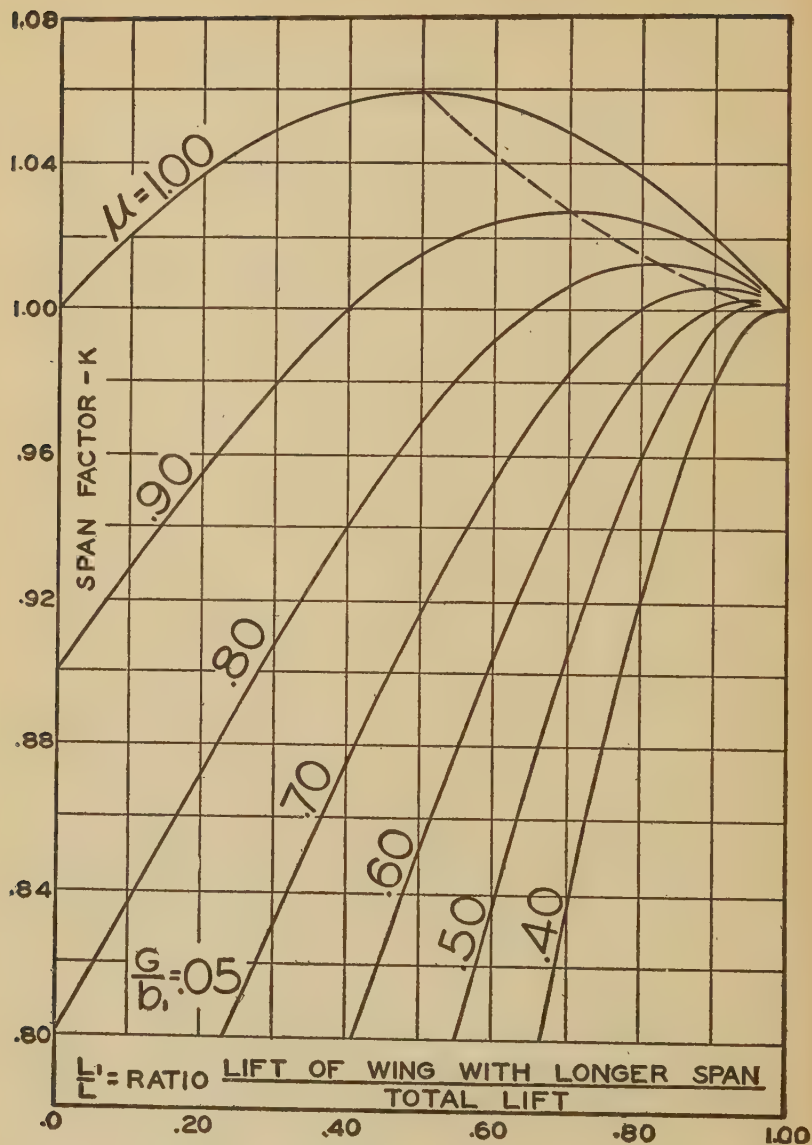


Figure 21. Munk's Span Factor  $k$  for  $\frac{G}{b_1} = .05$



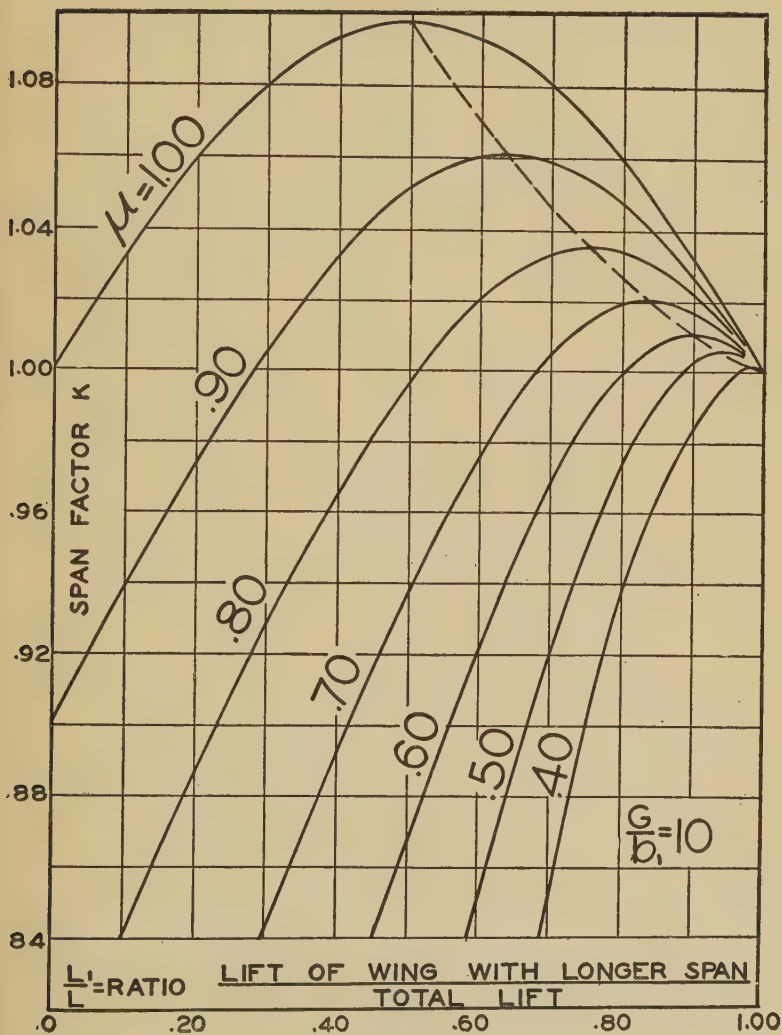


Figure 22. Munk's Span Factor  $k$  for  $\frac{G}{b_r} = 0.10$ .

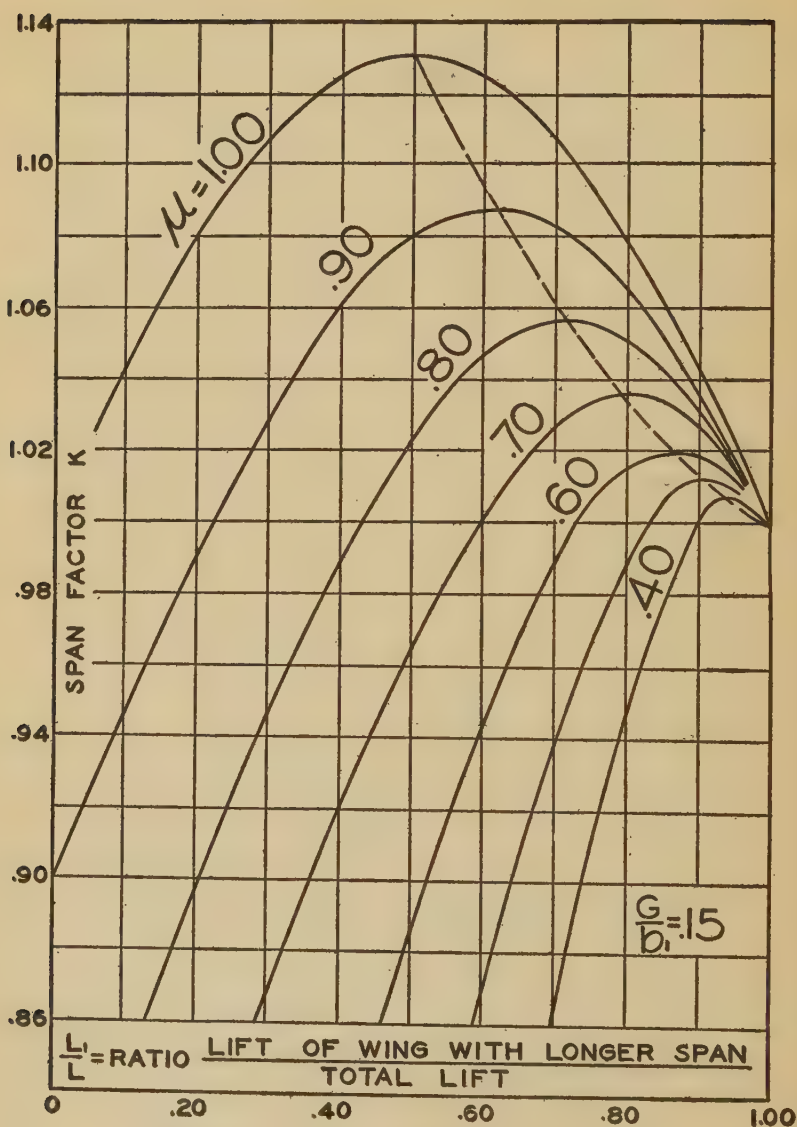


Figure 23. Munk's Span Factor  $k$  for  $\frac{G}{b_i} = 0.15$

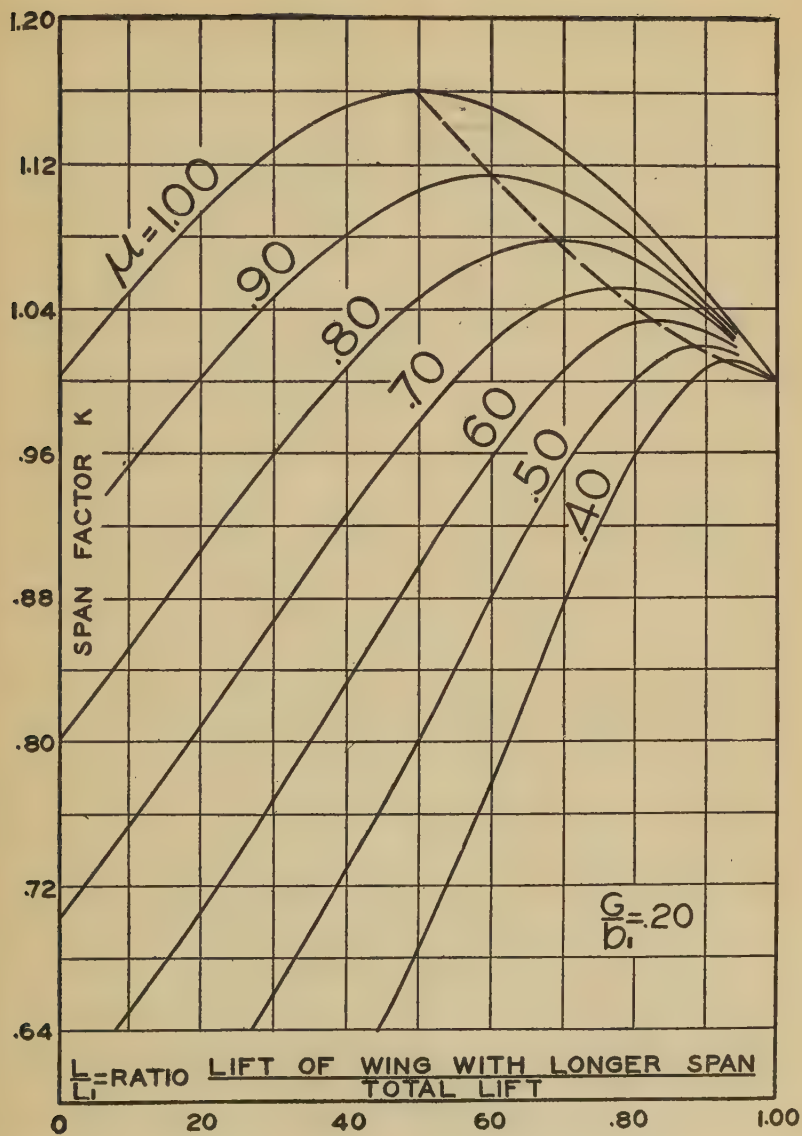


Figure 24. Munk's Span Factor  $k$  for  $\frac{G}{b_1} = 0.20$

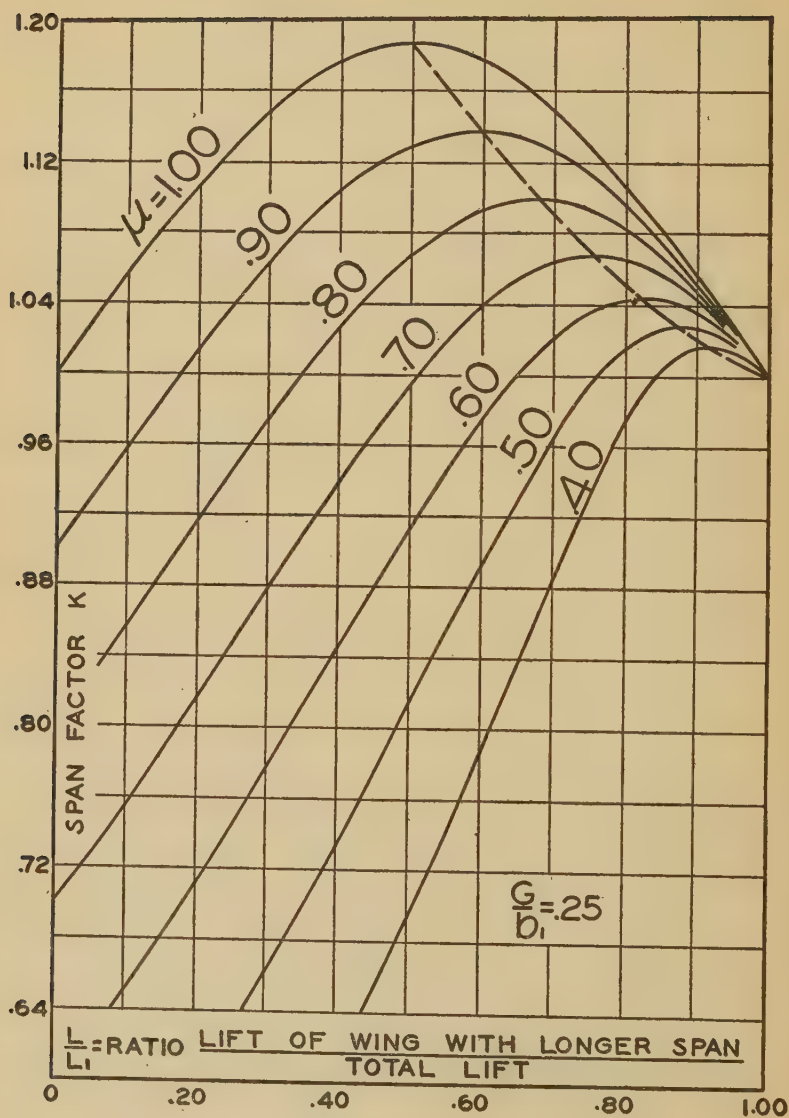
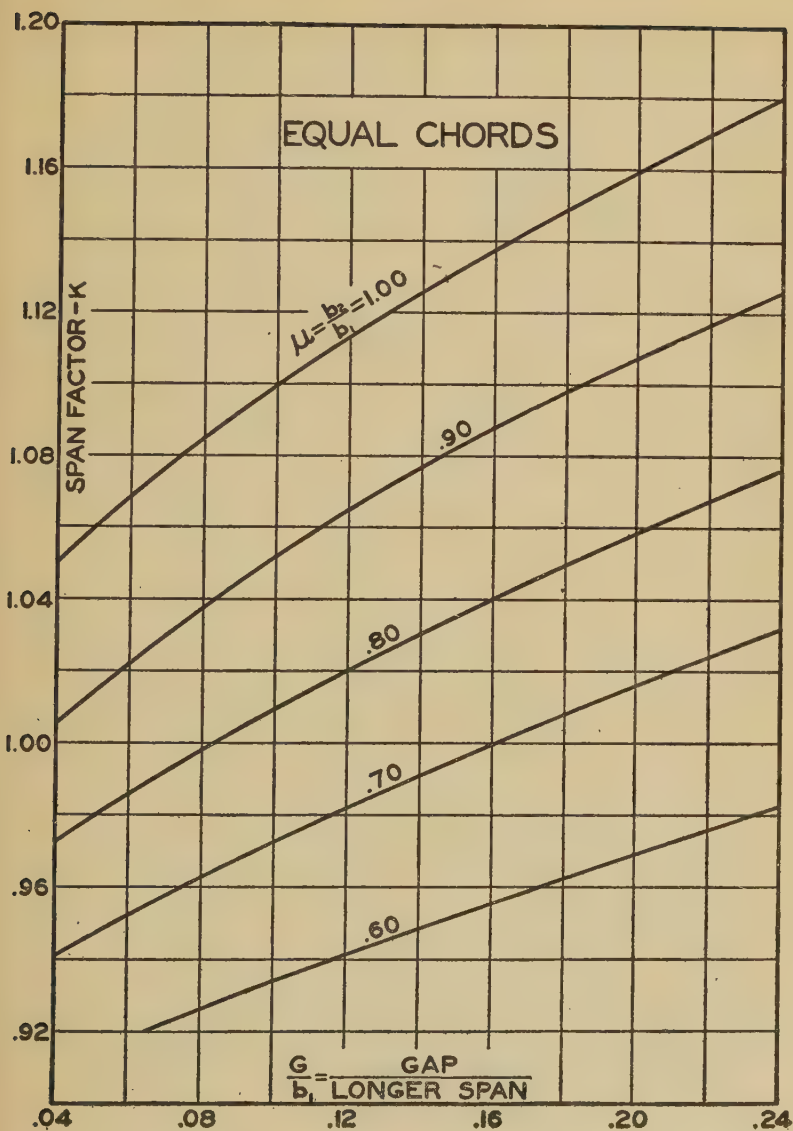


Figure 25. Munk's Span Factor  $k$  for  $\frac{G}{b_i} = 0.25$

Figure 26. Munk's Span Factor  $k$  for Biplanes with Wings of Equal Chords

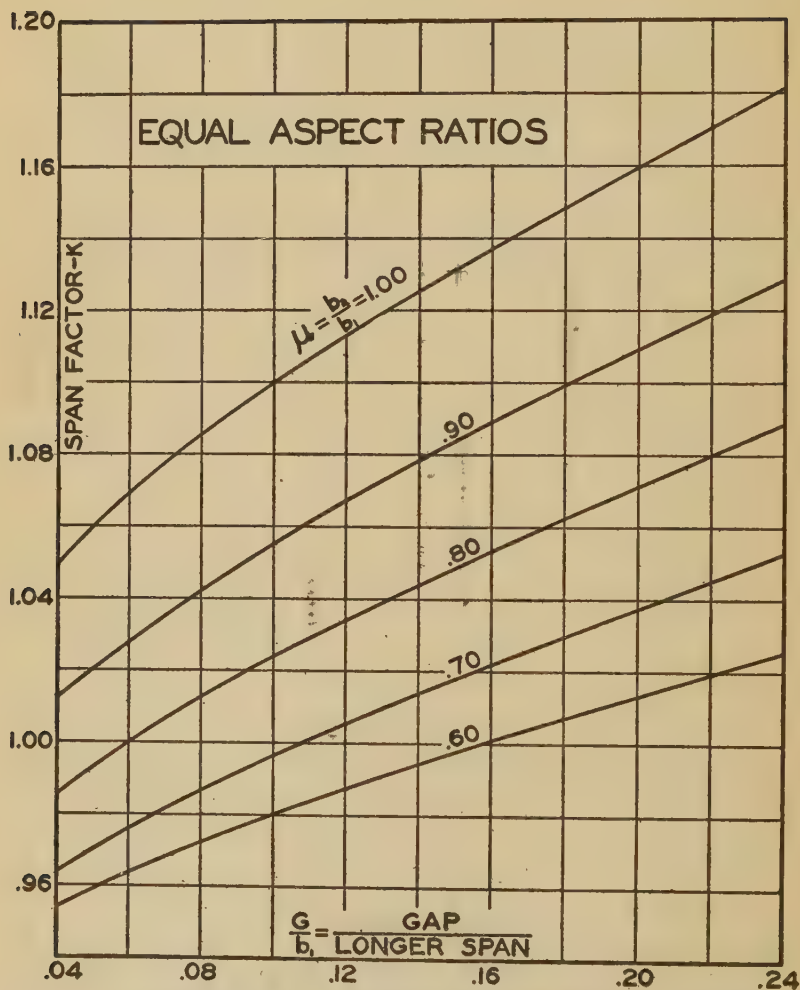


Figure 27. Munk's Span Factor  $k$  for Biplanes with Wings of Equal Aspect Ratio



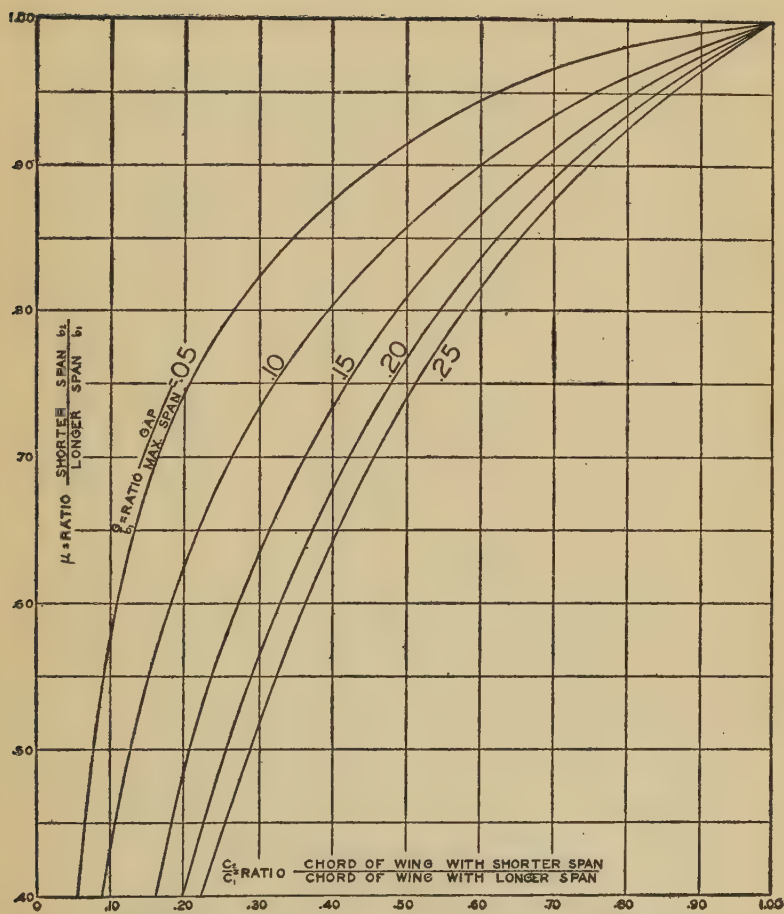


Figure 28. Proportions of the Most Efficient Biplane

**Induced drag charts.** The importance of obtaining a high aspect ratio is clearly brought out if  $K = \frac{C_{Di}}{C_L^2}$  be plotted against effective aspect ratio as in Figure 29. The change in induced drag in passing from aspect ratio  $n_1$  to  $n_2$  is

$$\Delta C_{Di} = (K_2 - K_1) C_L^2 = \Delta K \cdot C_L^2$$

The value of  $\Delta C_{Di}$  is readily found from the chart, Figure 30, in which  $\Delta K$  is plotted against  $n_1$  and  $n_2$ . Note that  $n_1$  and  $n_2$  are *effective* aspect ratios, i.e.,  $n_1 = \frac{(k_1 b_1)^2}{S}$  and  $n_2 = \frac{(k_2 b_2)^2}{S}$ .

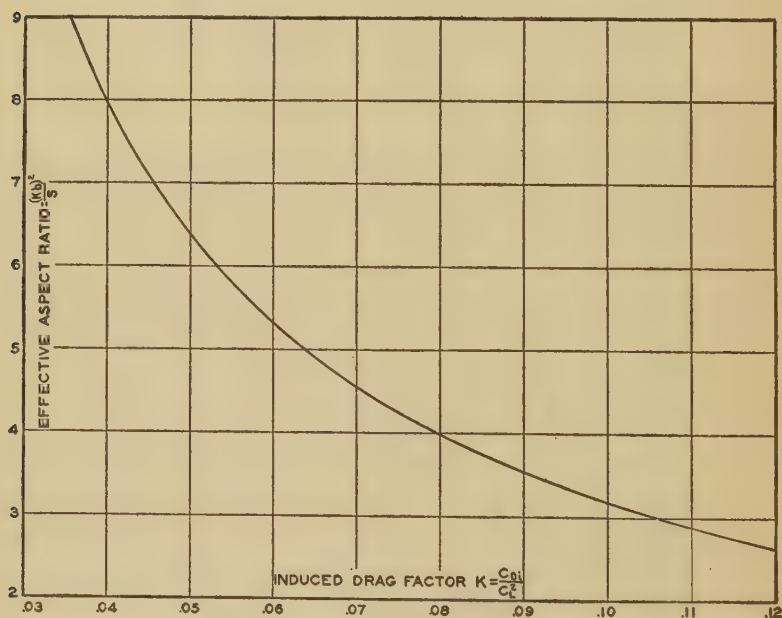


Figure 29. Induced Drag Factor  $K = \frac{C_{Di}}{C_L^2}$

**Practical applications of induced drag.** The division of wing drag into induced and section drags enables the value of a particular wing section to be appraised at a glance. This is particularly true if the section drag is plotted against lift coefficient.

Performance calculations are immensely simplified by the use of induced drag, as explained in Chapter VII. The effect of aspect ratio on performance is easily obtained. If the performance of an airplane is known with one wing section, the performance with another wing section is obtained by correcting for the difference in profile drag.

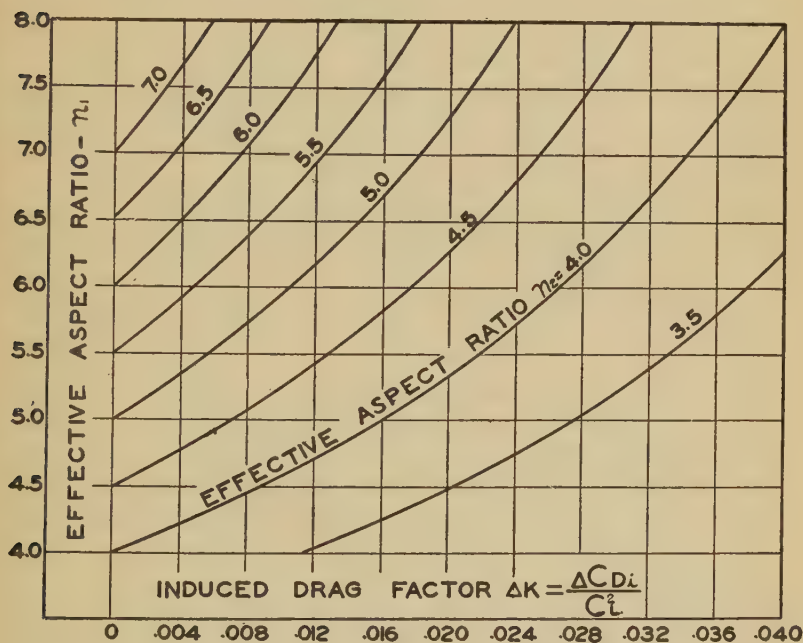


Figure 30. Change in Induced Drag Factor with Change in Aspect Ratio

**Center of pressure.** Munk has shown in a remarkable paper<sup>6</sup> that the classical treatment of wing theory by means of vortices may be replaced by kinetic energy considerations. In this paper he shows that the lift due to the curvature of a wing acts at 50% of the chord while lift due to angle of attack acts at 25% of the chord. Consequently the moment about a point on the chord 25% aft of the leading edge is constant. This

<sup>6</sup> Max M. Munk, "Elements of the Wing Section Theory and of the Wing Theory," N. A. C. A. Technical Report No. 191 (1924).

relation has been amply verified by wind tunnel tests. It is of considerable value in the solution of a number of practical problems, enabling the center of pressure to be estimated with considerable accuracy. The center of pressure is readily found from the moment coefficient about the 25% point, by the relation

$$C_p = 0.25 - \frac{C_M}{C_N}$$

where  $C_N$  is the normal force coefficient ( $C_N = C_L \cos \alpha + C_D \sin \alpha$ ). For all but very small lift coefficients and very large angles of attack  $C_L$  may be used for  $C_N$ .

**Zero lift—zero moment.** Munk has shown<sup>7</sup> that the angle of attack for zero lift is given approximately by the line drawn through the trailing edge and a point located on the *mean camber* at 50% of the chord. The exact angle of zero lift is determined as follows: pass a straight line  $AB$  through the trailing edge  $A$  and a point on the mean camber 11% of the chord aft of the leading edge, pass a second straight line  $AC$  through the trailing edge and a point on the mean camber 89% of the chord aft of the leading edge. The line  $AD$  bisecting the angle  $BAC$  is the direction of zero lift. These constructions are shown on Figure 31.

The angle for zero moment about the leading edge is found by passing a straight line through two points on the mean camber. The first point is at 4¼% of the chord and the second is at 95¾% of the chord measured from the leading edge. This construction is also shown on Figure 31.

If the wing section has a large leading edge radius  $r$ , the mean camber curve should pass through the center of this arc and be prolonged to a point  $P$ , which is one-half  $r$  distant from the center. The chord length should then be measured from the point  $P$  instead of the actual leading edge.

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<sup>7</sup> Max M. Munk, "The Determination of the Angles of Attack of Zero Lift and Zero Moment, Based on Munk's Integrals," N. A. C. A. Technical Note 122 (1923).

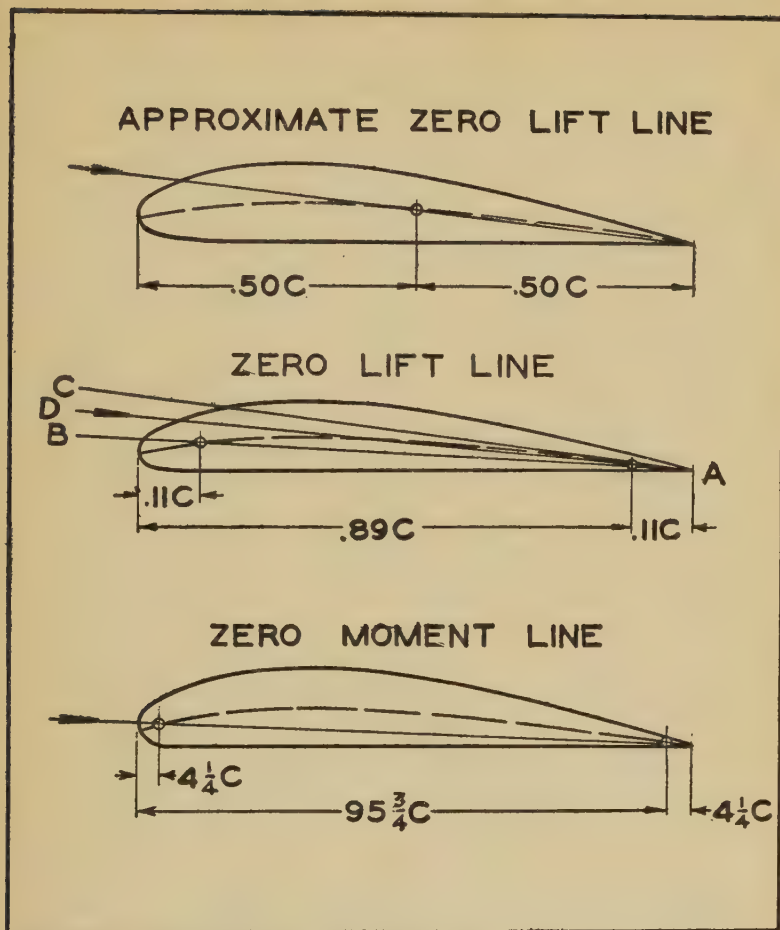


Figure 31. Munk's Methods for Finding Zero Lift and Zero Moment Lines

**Wing with a fore-and-aft slot.** Prandtl gives the theoretical solution<sup>8</sup> for the effect of a fore-and-aft slot in a wing, such as that existing at a panel joint. This solution is of great practical value in that it indicates the necessity for avoiding any kind of "leakage" joints in wing construction.

<sup>8</sup> Due to Grammel and Pohlhausen. See N. A. C. A. Technical Report No. 116.

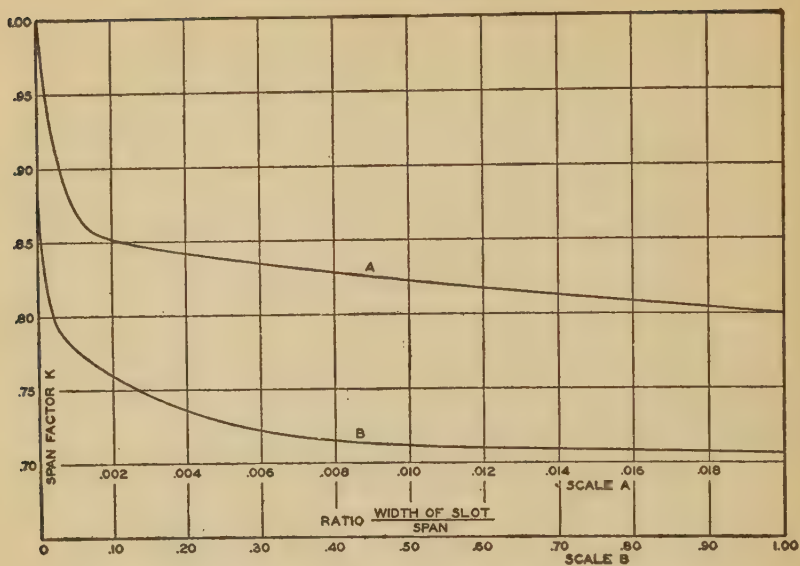


Figure 32. Munk's Span Factor for Wing with Fore-and-Aft Slot

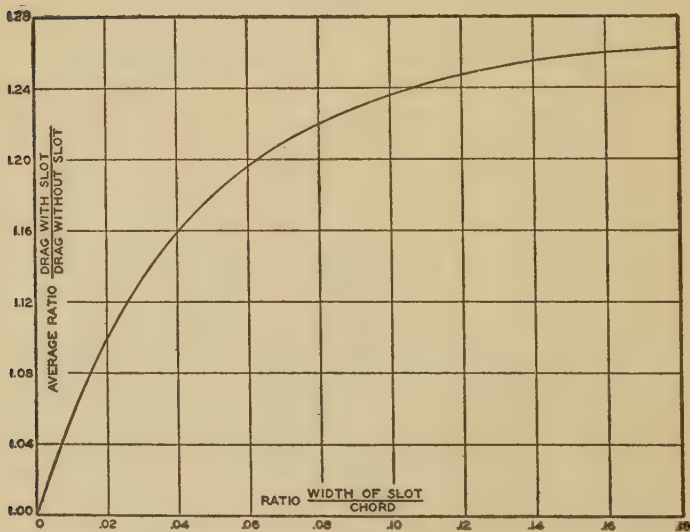


Figure 33. Observed Increase in Drag Due to Fore-and-Aft Slot



For a monoplane with a fore-and-aft slot, Munk's span factor  $k$  is no longer unity, but much less than unity, even for a quite narrow slot. The variation of  $k$  with the width of the slot is given on Figure 32. The curve marked  $A$  on this figure is an enlargement for small slot widths. A slot width of  $.001 \times$  span reduces  $k$  to 0.87, which is equivalent to a reduction of about 24% in aspect ratio. This effect has long been known from Munk and Cario's wind tunnel tests.<sup>9</sup>

The average increase in drag due to various slot widths, as found in these tests, is given in Figure 33. A slot width of 1% of the chord on a wing of aspect ratio 6 increases the drag about 6%.

**Correction for wall interference.** Prandtl has shown<sup>10</sup> that the boundary of any finite wind stream, in either a closed or an open-working section wind tunnel, restricts the flow past any object under test. The boundary conditions of constant pressure for the free jet and zero normal velocity for the closed tube are satisfied analytically by the introduction of a series of images which give an induced velocity at the model under test. This induced velocity is equivalent to a downwash for an open section or an upwash for a closed section. This causes the angle of attack and the drag, as measured, to be higher in a free jet and lower in a closed tube, than in an infinite air stream by the amounts

$$(\Delta\alpha)^{\circ} = \frac{57.3 C_L S}{2\pi D^2} = \frac{7.16 C_L S}{A} \quad (15)$$

$$\Delta C_D = \frac{C_L^2 S}{2\pi D^2} = \frac{C_L^2 S}{8A} \quad (16)$$

Where  $S$  is the model wing area,  $D$  the jet diameter, and  $A$  the jet area. The corrections must be added to the readings obtained in wind tunnels having an enclosed test section. The corrections are subtracted when the jet is of the open type.

<sup>9</sup> Technische Berichte I-6.

<sup>10</sup> Göttingen Nachrichten 1919, also N. A. C. A. Technical Report No. 116 (1921).

Glauert has shown<sup>11</sup> that the corrections for a square jet are about 10% greater than for a circular jet.

**The general drag equation.** The chief variables which affect the air force on any aerodynamic object, such as a wing, are: (1) angle of attack  $\alpha$ , (2) relative wind velocity  $V$ , (3) scale of the object  $L$ , (4) density of the fluid  $\rho$ , (5) compressibility of the fluid, (6) gravity  $g$ , (7) surface roughness, and (8) texture of the air flow. From the theory of dimensions it may be shown that the general drag equation, for example, is

$$D = \rho V^2 L^2 \cdot \varphi \left[ \alpha, \frac{\rho VL}{\mu}, \frac{V}{a}, \frac{V^2}{gL}, \frac{l}{L}, \frac{v}{V} \right] \quad (17)$$

We are accustomed to the use of a drag coefficient  $C_D$  instead of the function in the brackets, but in so doing, sight must not be lost of the fact that  $C_D$  may and does vary with a number of non-dimensional terms, and that this variation must in general be determined experimentally. The purpose of the ordinary wind tunnel test is to determine  $\varphi_1(\alpha)$  or the variation of  $C_D$  with angle of attack. The purpose of testing at various

speeds is to determine  $\varphi_2 \left( \frac{\rho VL}{\mu} \right)$  over a part of its range. Propeller

designers understand the effect of  $\varphi_3 \left( \frac{V}{a} \right)$  on propeller characteristics in general, if not in particular, when they try to hold the tip speeds well below the velocity of sound. Model tank experiments are usually made on the basis of  $\frac{V^2}{gL}$  because this term brings in the gravity effects associated with wave making.

If tests on two models do not agree, the surface roughness  $\frac{l}{L}$  may not bear the same relation in the two cases, and this is often observed in skin friction tests. Finally, if two wind tunnels do not agree in tests on the same model, it may be due to a

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<sup>11</sup> "The Interference of Wind Channel Walls on the Aerodynamic Characteristics of an Airfoil," Br. A. R. C. R. & M. No. 867.

difference in turbulence represented by the ratio of the average lateral turbulence velocity to the measured axial velocity.

These terms do not have the same weight. In aeronautical engineering, angle of attack is considerably more important than turbulence or surface effect. Next to angle of attack in importance is either the Reynolds Number  $\frac{\rho VL}{\mu}$  or the compressibility effect  $\frac{V}{a}$ . So long as the relative speed is low the compressibility effects are negligible, but as the speed increases they become increasingly more important. It is for this reason that wind tunnel tests at very high speed do not agree with tests at the same Reynolds Number at a lower speed. Such tests may include both  $\varphi_2 \left( \frac{\rho VL}{\mu} \right)$  and  $\varphi_3 \left( \frac{V}{a} \right)$ .

**Determination of Reynolds Number.**  $\frac{\rho VL}{\mu}$  will have the same value in any consistent system of units. If the units for  $\rho$ ,  $V$ , and  $L$  are respectively lbs. mass per cubic foot ( $\rho = \frac{w}{g}$ ), ft./sec., and ft., then  $\mu$  must have the dimensions of  $\frac{M}{LT}$ . In the c. g. s. system the value of  $\mu$  is given for air by Holman's formula

$$10^7 \mu = 1,715.5 (1 + .00275 t - .00000034 t^2) \quad (18)$$

In ft.-lb.-sec. units  $\mu_e$  is

$$10^{10} \mu = 3,582.9 (1 + .00275 t - .00000034 t^2) \quad (18a)$$

$t$  being in  $^{\circ}\text{C}$ .

The deviation of  $\mu$  from a straight-line function of  $t$  is less than 1 part in 3,000 over the usual working range and one may write

$$10^7 \mu = 3,408. + 5.483 t (^{\circ}\text{F.}) \quad (19)$$

$$= 3,583. + 9.870 t (^{\circ}\text{C.}) \quad (19a)$$

for use in determining the value of  $\frac{\rho VL}{\mu}$ . The value of  $\mu$  at

$15^{\circ}\text{C.}$  or  $59^{\circ}\text{F.}$  by the approximate formula is  $3,731. \times 10^{-10}$  while Holman's formula gives  $3,730. \times 10^{-10}$ . Figure 34 has been prepared for use in reading the value of  $\frac{\rho}{\mu}$  directly, or

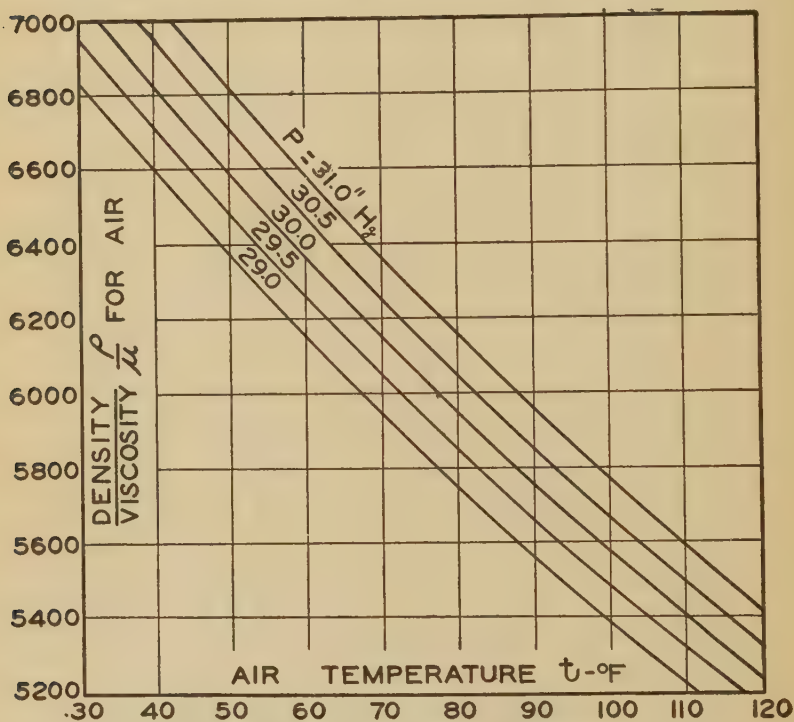


Figure 34. Reynolds Number Coefficient

for checking calculations. Reynolds Number is obtained by multiplying the value of  $\frac{\rho}{\mu}$  from this figure by  $V$  in ft./sec. and  $L$  in ft. The engineer will find it convenient to remember that in standard air a wing of 1.0 ft. chord at 100 mi./hr. gives a Reynolds Number of 925,000, or approximately one million.

## CHAPTER III

### AIRPLANE MODEL TESTS

The purpose of a routine wind tunnel test on an airplane model is to determine performance, stability, and control, with *reasonable* engineering accuracy, and the advantage of a wind tunnel test over any other method lies chiefly in its quick and definite indications when reasonable deviations from mathematical exactitude are allowed. Routine testing of airplane models cannot well be justified on any other basis.

It is an easily demonstrated, but not a widely recognized fact, that any model test is a compromise. There are always some conditions of similitude which conflict or cannot be met, but these conditions do not all have the same weight in determining air forces. Since there is no known theoretical method of assigning the proper relative values to the various conditions, a practical solution based, for example, on experience or on trial and error must serve instead. The principle of dimensional homogeneity leads to an equation for air force in the form

$$F = \rho V^2 L^2 \cdot \varphi \left[ \alpha, \frac{\rho V L}{\mu}, \frac{V}{a}, \frac{V^2}{gL}, \frac{l}{L}, \frac{v}{V}, \frac{b}{C} \right] \quad (17a)$$

where  $\alpha$  is the angle of attack,  $\frac{\rho V L}{\mu}$  the well-known Reynolds Number,  $\frac{V^2}{gL}$  the "wave making" or gravitational term,  $\frac{V}{a}$  the ratio of the relative wind to the velocity of sound,  $\frac{l}{L}$  a measure of surface roughness,  $\frac{v}{V}$  a measure of turbulence in the air, and  $\frac{b}{C}$  the aspect ratio. These are by no means all of the dimensionless combinations which may be written, but they comprise what are



usually considered the most important ones. It is desired to point out three facts: (1) theoretically no model test can completely represent a full-scale condition unless *all* of the dimensionless ratios are held constant; (2) it is impossible to hold all of these ratios constant at the full-scale value in a model test, for some of them are contradictory; and (3) the dimensionless ratios may not have equal weight but theory does not show it. Experience tells us, however, that the angle of attack  $\alpha$  is the most important of all the terms given, and that Reynolds Number probably comes second in importance. The "wave making" term  $\frac{V^2}{gL}$ , which is so important in testing ship models,

is negligible in air work. The term  $\frac{V}{a}$  does not have any great importance at speeds less than about 300 ft./sec., where compressibility effects begin to appear. The surface roughness has a very great effect under many conditions and cannot be neglected. Turbulence is known to have a fairly large influence, particularly on critical flow conditions. The large effect of aspect ratio is well known, and so on through the list of minor unconsidered terms.

The only major term which cannot be satisfactorily met is the Reynolds Number  $\frac{\rho VL}{\mu}$ . If the value of the function was the same for all airplanes, or even for all parts of an airplane, the problem would be capable of a satisfactory solution. Actually the effect of Reynolds Number is different on each item; wings, fuselage, struts, wires, etc. Furthermore, the effect is not the same on all wings or on all struts but varies widely in each group.

As a concrete example, the drag coefficient on streamline struts may be four times as great on a 1/24 scale model as on the full-size airplane; streamline wire may show even greater scale effects. This is very much greater than any scale effect on wings or fuselage. The conclusion is inevitable, that reasonable engineering accuracy is best secured by constructing a model



airplane with no streamline struts, wires, fittings, or minor details, and correcting the measured model drag for the scaled-down, calculated full-scale drag of the omitted parts. The wings of the simplified model can be held together and the landing gear attached by means of a simple system of round brass wire struts (about  $3/32$ " diameter), threaded right-hand on one end and left-hand on the other; the minimum length of wire necessary to insure rigidity being used. This method has three very important advantages.

The drag of the round wire struts is quite definite and can be determined very accurately from runs with an exact duplicate set of struts attached symmetrically and in such a manner as to avoid interference while duplicating each item. If  $\Delta D$  be the drag of one set of struts, it is obvious that

$$D_o = (D_o + \Delta D) - [(D_o + 2\Delta D) - (D_o + \Delta D)] \quad (20)$$

where  $D_o$  is the drag of the model without struts, wires, fittings, or miscellaneous omitted parts. The full-scale drag of these parts can be calculated with considerable accuracy. Assume that the value so calculated at a speed  $V$ , be  $d$ . Then assuming  $d$  to vary as  $(VL)^2$

$$d_m = d \cdot S^2 \left( \frac{V_m}{V} \right)^2 \quad (21)$$

where  $S$  is the (fractional) model scale ratio (for example,  $1/20$ ),  $V$  the model test speed, and  $d_m$  the correction to be applied to the model test. The correct model drag will be

$$D = D_o + d_m \quad (22)$$

Lift and moment corrections are also obtained from the "duplicate strut" runs. An additional moment correction due to the omitted parts is obtained by calculating the line of action of  $d$  (and therefore  $d_m$ ) so that the distance of this line of action from the moment axis is known. The table shown in Figure 35 is a very convenient form for calculating simultaneously the drag correction and its line of action. A typical lift wire calculation is given to illustrate the steps followed.



To sum up briefly, it may be stated that in order to obtain good results in an airplane model test it is necessary to omit all minor parts which show excessive scale effects, such as struts, wires, fittings, small attachments, etc. A drag correction which includes the resistance of *all omitted parts* must be calculated and added to the basic model drag.

The objections to this method are usually based on the two arguments: (1) that the principle of dynamic similarity is violated, and (2) that interference effects are neglected. These objections cannot be maintained since it has been shown that the principle of dynamic similarity does not apply for an exact scale model and since the interference effects of a strut, for example, certainly cannot be more than a small fraction of the four-fold resistance which this method corrects. On the basis of results, the method is quite satisfactory. The high speed predicted by a careful wind tunnel test on a simplified model normally differs less than about 3% from that obtained in flight tests, and it is not unusual to obtain less than 1% difference.

For additional detail on model construction and test corrections see N. A. C. A. Technical Notes No. 82, "Notes on the Construction and Testing of Model Airplanes," W. S. Diehl (1922), and No. 254, "Method of Correcting Wind Tunnel Data for Omitted Parts of Airplane Models," R. H. Smith (1927).

**Pitching moments.** Information concerning the longitudinal static stability and control is given by pitching moment curves for three or more elevator settings at a selected stabilizer setting. This part of the tests and the lift and drag data should be considered of equal importance.

In general, the pitching moments will be measured about an axis which is parallel to, but considerably displaced from, the gravity axis. Since the final moments must be referred to the c.g., either a graphical or an analytical method must be used in the conversion. The graphical method is simple, quick, and accurate if care is used in drawing the vector diagram. This diagram once drawn is always available for rapid calculations of the pitching moment about any new c.g. While an

experienced engineer can obtain an excellent idea of stability characteristics directly from the vector diagram, the slope of the moment curve is the only reliable criterion on which to base definite conclusions regarding static stability.

All airplane model tests should include pitching moment curves about the center of gravity for at least two different stabilizer settings with elevators at  $0^\circ$ . The purpose of this being to determine the moment change due to a definite increment in stabilizer angle, so that settings required for balance, or moments for new conditions, may be accurately calculated from the test data. It is easily shown that changing the stabilizer setting or elevator angle does not appreciably affect the *slope* of the moment curve, and merely shifts it up or down by a definite increment.

The elevator angles usually investigated are  $0^\circ$ ,  $\pm 5^\circ$ ,  $\pm 10^\circ$ ,  $-15^\circ$ ,  $-20^\circ$ , and  $-30^\circ$  where the  $+$  sign refers to "down" elevator. With the exception of the  $0^\circ$  and  $\pm 10^\circ$  settings, it is unnecessary to cover a greater angular range than that required to give the zero moment, or balance, condition. In general, the moment curve is not satisfactory unless it has a negative slope over the entire range of flight angles. Slight waves in the curve giving zero or slight positive slopes are not actually dangerous, but they are indicative of some very undesirable condition. The desirable slope of the moment curve varies with the type and purpose of the airplane. The slope of the full-scale moment curve should be either

$$\frac{dM}{d\alpha} = K_q WC \quad (23)$$

or

$$\frac{dM}{d\alpha} = K_1 q SC \quad (24)$$

where  $\frac{dM}{d\alpha}$  is the slope (in lbs.-ft. per degree),  $q$  the dynamic pressure  $\frac{1}{2} \rho V^2$ ,  $W$  the gross weight (lbs.),  $C$  the wing chord (ft.), and  $S$  the total wing area (sq. ft.). The value of  $K$  should lie

between .0006 and .0010 according to the stability desired. The corresponding values for  $K_r$  are .006 and .010, but  $K_r$  is not necessarily equal to  $10K$ . Some additional data on slope of the pitching moment curve may be found in Chapter V.

Whether or not the elevators give adequate control is usually quite obvious from an inspection of the moment curves or vector diagrams. It is a common rule to assume that  $20^\circ$  up elevator should balance the airplane at the angle of attack where maximum lift is obtained, assuming that balance with neutral elevators is in the normal range between  $0^\circ$  to  $6^\circ$  angle of attack.

**Rolling moments.** Rolling moments need not be measured unless there is considerable doubt in regard to lateral control. Under these conditions it is advisable to measure both rolling moments and yawing moments due to the ailerons. The rolling moment due to yaw is not often measured except in stability investigations.

Experience has shown that satisfactory lateral control is assured when the full-scale aileron rolling moment is equal to or greater than

$$L = .0060 qWb \quad (25)$$

for  $10^\circ$  setting at  $0^\circ$  angle of attack.  $W$  is the gross weight,  $q$  the dynamic pressure, and  $b$  the maximum span. This is based on satisfactory airplanes, and will assure adequate rolling moments up to stalling angles for a normal design.

**Yawing moments.** The conventional tests to determine directional stability and control consist of three parts as follows:

Yawing moments are measured with fixed rudder setting and angle of attack. Usual range  $\pm 20^\circ$  in yaw.

Yawing moments due to various rudder settings are measured with body at  $0^\circ$  yaw and fixed pitch. Usual range of rudder angles  $0^\circ$  to  $20^\circ$ .

Angles of yaw held by various rudder settings up to  $20^\circ$ .



While it is desirable that these tests be made at several angles of attack, the time required is often an important factor. If the stability and control shown by the first tests are fully satisfactory, little is to be gained by tests at other angles of pitch unless the lateral stability is being investigated completely. The chief criterions for a satisfactory condition in yaw are:

Definite slopes to the yawing moment curves. Absence of either irregularities in the curves or unsteadiness in test readings on balance, particularly at small angles of yaw.

The slope of the curve of angle of static yaw held by the rudder when plotted against rudder angle must be definite, and the curve must be free from irregularities. The ratio of static yaw angle to rudder angle should be greater than 0.5 and less than 1.0. A value less than 0.5 may indicate either too much directional (weather-cock) stability or deficient rudder control. A value greater than 1.0 may indicate either too little directional stability or too much rudder control.

The slope to the curve of yawing moments with rudder neutral may be written in the form

$$\frac{dN}{d\psi} = KqWb \quad (26)$$

in which  $\frac{dN}{d\psi}$  is the slope (lbs.-ft. per degree),  $W$  the gross weight (lbs.), and  $b$  the span (ft.). An analysis of 61 airplane model tests in comparison with reported full scale characteristics shows that  $K$  should be between .000040 and .000080 for a normal airplane. Values slightly lower or higher are not entirely objectionable, but the most favorable comments have been made on airplanes for which  $K$  is within the limits given.

**Calculation of performance from model tests.** Assuming that the air forces vary as  $(Vl)^2$ , the relations between model and full scale are readily obtained. Designating the model values by



the subscript  $m$  and letting the scale ratio  $\frac{\text{Model}}{\text{Full scale}} = \frac{l_m}{l}$ , the drag relations at a given angle of attack are

$$\text{Model drag} \quad D_m = (C_D)_m \frac{\rho}{2} (V_m l_m)^2 \quad (27)$$

$$\text{Full-scale drag} \quad D = C_D \frac{\rho}{2} (Vl)^2 \quad (28)$$

Since the air forces are assumed to vary as  $(Vl)^2$ , the drag coefficients will be equal  $C_D = (C_D)_m$ . Hence

$$\frac{D}{(Vl)^2} = \frac{D_m}{(V_m l_m)^2}$$

$$\text{or } D = D_m \left(\frac{l}{l_m}\right)^2 \left(\frac{V}{V_m}\right)^2 \quad (29)$$

$$\text{Similarly} \quad L = L_m \left(\frac{l}{l_m}\right)^2 \left(\frac{V}{V_m}\right)^2 \quad (30)$$

In steady horizontal flight  $L = W$ . Hence

$$V = V_m \times \left(\frac{l_m}{l}\right) \sqrt{\frac{W}{L_m}} \quad (31)$$

The minimum, or stalling, speed is given by substituting the maximum model lift  $L_{m \max}$  in equation (31)

$$V_S = V_m \times \left(\frac{l_m}{l}\right) \sqrt{\frac{W}{L_{m \max}}} \quad (32)$$

It is often more convenient to work with speed ratios, rather than to calculate each speed separately,

$$\frac{V}{V_S} = \sqrt{\frac{L_{m \max}}{L}} \quad \text{or} \quad V = V_S \sqrt{\frac{L_{m \max}}{L}} \quad (33)$$

but equation (31) can be simplified for any given model, to

$$V = \sqrt{\frac{K}{L_m}} = \sqrt{\frac{W}{K}} \quad K_L = \frac{W}{L_m} \quad (34)$$

where  $K = W(V_m)^2 \times \left(\frac{l_m}{l}\right)^2$ . Equation (34) can be solved very

get equivalent

rapidly on a slide rule by setting the runner to  $K$  on the A scale and moving the slide so that  $L_m$  on the B scale is in line with  $K$ . Then the index on the C scale will rest at the desired value of  $V$  on the D scale.

Full-scale drag values may be calculated by equation (29) but time can be saved by using the values of  $L/D$  from the model test. Where curves of power required are to be calculated for two or more weights the values of  $L/D$  can be plotted against  $\frac{V}{V_s}$ . This curve can then be used in conjunction with equation

$$(33) \text{ since } D = \frac{W}{L/D} \text{ when } L = W.$$

In order to secure accurate results, particularly at the lower speeds corresponding to large angles of attack, it is advisable to correct both lift and drag for the elevator angle necessary to maintain balance. If lift, drag, and pitching moments are measured for several elevator settings, this may be done with sufficient accuracy by simple interpolation. Elevator angles of  $-10^\circ$  (down),  $0^\circ$  (neutral),  $-5^\circ$  (up),  $-10^\circ$ ,  $-15^\circ$ , and  $-20^\circ$  are usually required, but the  $-5^\circ$ ,  $-15^\circ$ , and  $-20^\circ$  settings need be investigated over only a limited angular range at high angles.

The methods employed in a typical performance calculation will now be illustrated by an example. Table 1 contains the wind tunnel data obtained in the Washington Navy Yard tests on a 1:16 scale model of the DH-4B, the lift and drag values,  $L$  and  $D$ , having been corrected for elevator setting.

The maximum lift is 7.571 lbs. (at  $20^\circ$ ) at a speed of 40 mi./hr. The gross weight is 3,876 lbs. Hence, according to equation (32), the stalling speed is

$$V_s = \frac{40}{16} \sqrt{\frac{3,876}{7.571}} = 56.6 \text{ mi./hr.}$$

For any other weight, the stalling speed is

$$V_s = \frac{40}{16} \sqrt{\frac{W}{7.571}} = 0.91 \sqrt{W}$$

also

$$V_{S_2} = V_{S_1} \sqrt{\frac{W_2}{W_1}}$$

or, the speed at any lift is

$$V = \frac{40}{16} \sqrt{\frac{W}{L_m}}$$

Table 2 contains the calculations for power required in horizontal flight at sea-level with two gross weights. The

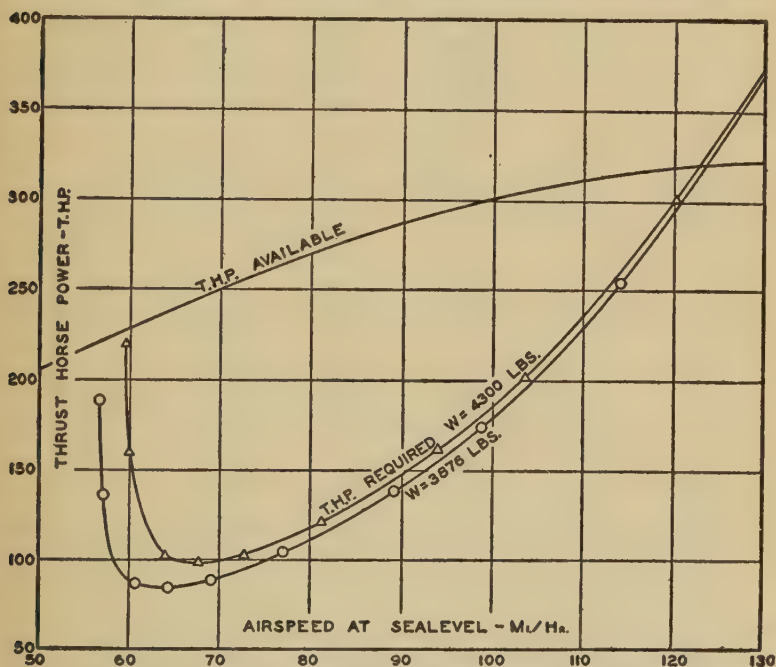


Figure 36. DH-4B Power Curves from Wind Tunnel Tests

corresponding powers required at any desired altitude may be obtained by multiplying each value of  $V$  and  $T.H.p.$ , by the value of  $\sqrt{\frac{\rho_0}{\rho}}$  for the altitude under consideration, as explained in Chapter VII. Figure 36, is a plot of the  $T.H.p.$  available and  $T.H.p.$  required against air speed at sea-level. The performance indicated is in excellent agreement with the flight test results on this airplane in the condition represented by the model.

TABLE 1. WIND TUNNEL TEST DATA ON DH-4B AIRPLANE MODEL  
SCALE RATIO 1/16. TEST SPEED 40 mi./hr. (standard air)

| Angle<br>of<br>Attack<br>$\alpha$ | Elevator<br>Angle<br>for<br>Balance | Lift<br>lbs.<br>$L_m$ | Drag<br>lbs.<br>$D_m$ | $\frac{L}{D}$ | $\frac{V}{V_S}$ |
|-----------------------------------|-------------------------------------|-----------------------|-----------------------|---------------|-----------------|
| 1                                 | .7                                  | 1.214                 | .392                  | 3.10          | 2.500           |
| 2                                 | .4                                  | 1.853                 | .404                  | 4.58          | 2.020           |
| 3                                 | 0                                   | 2.475                 | .420                  | 5.90          | 1.746           |
| 4                                 | — .4                                | 3.040                 | .458                  | 6.63          | 1.576           |
| 6                                 | — 1.2                               | 4.053                 | .527                  | 7.68          | 1.366           |
| 8                                 | 2.0                                 | 5.045                 | .623                  | 8.10          | 1.223           |
| 10                                | 3.2                                 | 5.858                 | .747                  | 7.83          | 1.136           |
| 12                                | 4.7                                 | 6.574                 | .903                  | 7.27          | 1.073           |
| 16                                | 10.0                                | 7.498                 | 1.745                 | 4.29          | 1.004           |
| 20                                | 18.5                                | 7.571                 | 2.641                 | 3.10          | 1.000           |

TABLE 2. CALCULATION FOR T.H.P. REQUIRED IN HORIZONTAL FLIGHT  
AT SEA-LEVEL. DH-4B AIRPLANE.

(Based on Wind Tunnel Data in Table 1)

| Angle<br>of<br>Attack<br>$\alpha$ | Wind Tunnel<br>Data |                                   | $W = 3876$                     |                     |            | $W = 4300$                     |                     |            |
|-----------------------------------|---------------------|-----------------------------------|--------------------------------|---------------------|------------|--------------------------------|---------------------|------------|
|                                   | $\frac{L}{D}$       | Speed<br>Ratio<br>$\frac{V}{V_S}$ | Air<br>Speed<br>$V$<br>mi./hr. | Drag<br>$D$<br>lbs. | $T.H.p._r$ | Air<br>Speed<br>$V$<br>mi./hr. | Drag<br>$D$<br>lbs. | $T.H.p._r$ |
| 1                                 | 3.10                | 2.500                             | 141.4                          | 1,250               | 472.0      | 149.0                          | 1,385               | 550.0      |
| 2                                 | 4.58                | 2.020                             | 114.3                          | 847                 | 258.0      | 120.3                          | 938                 | 301.0      |
| 3                                 | 5.90                | 1.746                             | 98.8                           | 657                 | 173.3      | 104.0                          | 728                 | 202.0      |
| 4                                 | 6.63                | 1.576                             | 89.2                           | 585                 | 139.0      | 93.8                           | 648                 | 162.2      |
| 6                                 | 7.68                | 1.366                             | 77.2                           | 505                 | 104.0      | 81.3                           | 560                 | 121.3      |
| 8                                 | 8.10                | 1.223                             | 69.2                           | 478                 | 88.2       | 72.8                           | 530                 | 103.0      |
| 10                                | 7.83                | 1.136                             | 64.2                           | 495                 | 85.0       | 67.7                           | 548                 | 98.8       |
| 12                                | 7.27                | 1.073                             | 60.7                           | 533                 | 86.3       | 64.0                           | 591                 | 100.7      |
| 16                                | 4.29                | 1.004                             | 56.8                           | 903                 | 136.8      | 59.8                           | 1,002               | 160.0      |
| 20                                | 3.10                | 1.000                             | 56.6                           | 1,250               | 189.0      | 59.6                           | 1,385               | 220.0      |

$$T.H.p._r = \frac{VD}{375}$$

## CHAPTER IV

### PARASITE DRAG DATA

The engineering drag coefficient  $K$  is used frequently in this chapter.  $K$  is defined by the drag equation

$$\text{Drag} = K \left( \frac{\rho}{\rho_0} \right) A V^2$$

where  $A$  is the cross-sectional area in sq. ft., and  $V$  the air speed in mi./hr.  $K$  is therefore the drag per sq. ft. at 1 mi./hr., and is found from

$$K = C_D \frac{\rho_0}{2} (F_V)^2$$

where  $\rho_0 = .002378$  and  $F_V$  is the conversion factor from mi./hr. to ft./sec.  $F_V = \frac{88}{60}$ ,  $(F_V)^2 = 2.151$ . Therefore  $K = .002558 C_D$  and  $C_D = 391 K$ .

**Square flat plates.** Eiffel investigated the drag of square, flat plates over a limited range in Reynolds Number <sup>1</sup> and the author is unable to find any more extensive series of tests. Eiffel's results were originally reported as the effect of area on drag square plates as shown on Figure 37. These data have been converted and for a test speed of 10 m./sec. are as follows:

| Length of Side<br>Cm. | $C_D$ | $K$    | Reynolds<br>Number |
|-----------------------|-------|--------|--------------------|
| 10.0                  | 1.040 | .00266 | 68,000             |
| 15.0                  | 1.055 | .00270 | 102,000            |
| 25.0                  | 1.072 | .00274 | 170,000            |
| 37.5                  | 1.140 | .00292 | 255,000            |
| 50.0                  | 1.193 | .00305 | 340,000            |
| 70.7                  | 1.234 | .00316 | 482,000            |
| 100.0                 | 1.263 | .00323 | 680,000            |

<sup>1</sup> "The Resistance of the Air and Aviation," Chapter II.

Eiffel states that for a very large plate  $C_D$  approaches the value of 1.28, or  $K = .00328$ . This value is generally used in calculating the drag at very high Reynolds Numbers, and in calculating the equivalent flat plate area of parasite drag.

**Rectangular flat plates normal to wind.** The drag coefficient of rectangular flat plates increases as the aspect ratio is increased. Eiffel's tests<sup>2</sup> are well known: Foppl made some similar tests

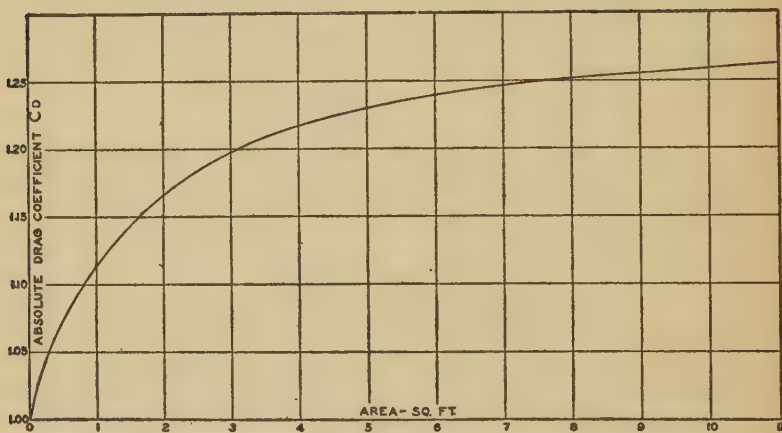


Figure 37. Effect of Area on Drag Coefficient for Square Flat Plates

at Göttingen.<sup>3</sup> Eiffel's and Foppl's data do not agree very well, probably on account of the differences in Reynolds Numbers. Wieselsberger<sup>4</sup> has recently made some tests which appear more consistent; his data are plotted on Figure 38. He also tested a number of annular discs, the data for which are plotted on the same figure. It is interesting to note that the drag coefficient for infinite aspect ratio is  $C_D = 2.00$ , or  $K = .00512$ .

**Circular discs.** The resistance of circular discs has been measured in various wind tunnels, and the results are in excellent agreement. Eiffel found  $C_D = 1.055$  and 1.08 for 15 cm.

<sup>2</sup> Chapter II, "Resistance of the Air and Aviation."

<sup>3</sup> Reported in the "Jahrbuch der Motorluftschiff-Studiengesellschaft," 1910-11, Julius Springer, Berlin.

<sup>4</sup> Göttingen Ergebnisse II (1923), R. Oldenbourg, München.



and 30 cm. discs respectively, when tested at a speed of 10 m./sec. Wieselsberger<sup>5</sup> tested four discs 3, 6, 15.1, and 40 cm. in diameter over a wide range in Reynolds Number obtaining values of  $C_D$  varying from 1.07 to 1.13, with an average value of about 1.11.

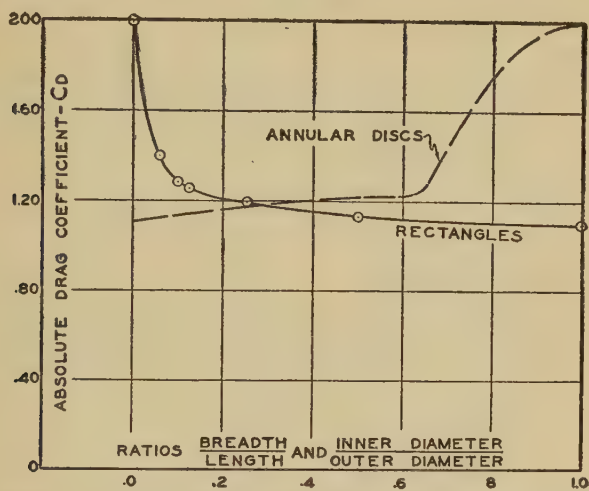


Figure 38. Drag Coefficients for Rectangles and Annular Discs

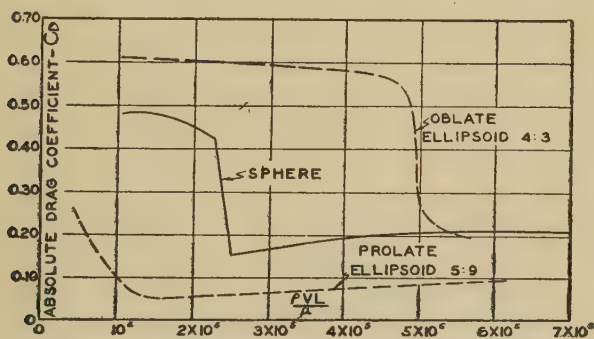


Figure 39. Drag Coefficients for Spheroids and Ellipsoids

A 15 cm. disc tested in the N. A. C. A. variable density wind tunnel<sup>6</sup> at Reynolds Numbers ranging from 210,000 to 4,440,000

<sup>5</sup> See "Ergebnisse der Aerodynamischen Versuch-Anstalt Zu Göttingen" II (1923), pp. 28-32, R. Oldenbourg, München.

<sup>6</sup> James M. Shoemaker, "Resistance of a Fifteen-Centimeter Disc," N. A. C. A. Technical Note No. 252 (1926).

gave values of  $C_D$  between 1.077 to 1.139 with an average value of about 1.11.

In both the Göttingen and the N. A. C. A. tests there was no definite change of  $C_D$  with Reynolds Number, although there is a little evidence of the existence of the critical Reynolds

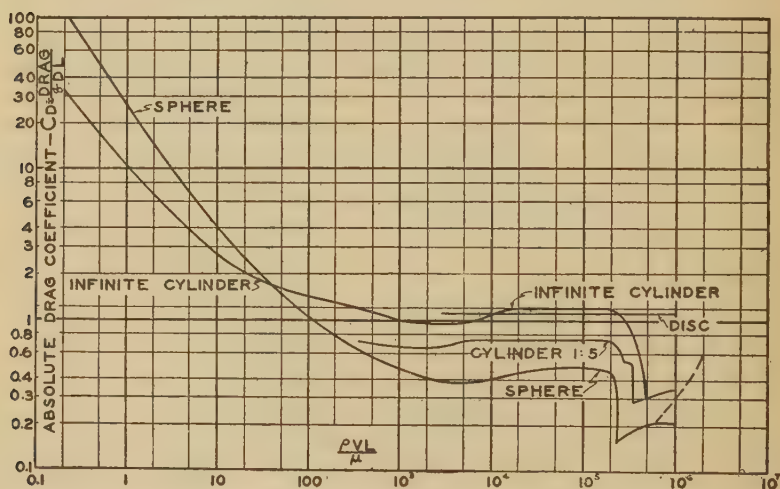


Figure 40. Variation of  $C_D$  with Reynolds Number—Cylinders and Spheres

Number which is found for cylinders and spheres. For any ordinary size of disc or wind speed the average value  $C_D = 1.11$  may be used. For Reynolds Numbers above 6,000,000 the coefficient is unknown.

**Spheres.** The drag of spheres has been measured over an extremely wide range in Reynolds Number. The variation of  $C_D$  with Reynolds Number given on Figure 40 is based on a compilation of test data from various sources made by Dr. A. F. Zahm<sup>7</sup> and Mr. F. A. Loudon of the Washington Navy Yard.

**Spheroids and ellipsoids.** Drag coefficients of two ellipsoids are compared with those for a sphere<sup>8</sup> in Figure 39.

<sup>7</sup>A. F. Zahm, "Flow and Drag Formulas for Simple Quadrics," N. A. C. A. Technical Report No. 253 (1926).

<sup>8</sup>Based on data in "Göttingen Ergebnisse" II.

**Hemispheres—parachutes.** Eiffel tested a hemispherical cup and found  $C_D = 0.33$  when the convex side was presented to the wind, and  $C_D = 1.33$  when the concave side was presented to the wind. The latter value is used for parachutes, on the basis of projected area when inflated. The form of an inflated parachute is approximately a part of the surface of an ellipsoid and the projected area is about 55% of the surface area. The limiting velocity of a parachute is therefore given by

$$V_L = \sqrt{\frac{2W}{1.33\rho(0.55S)}} \quad (35)$$

where  $W$  is the total weight (lbs.) of parachute plus its load,  $S$  the total surface area (sq. ft.), and  $\rho$  the air density. For standard density the limiting velocity in ft./sec. is

$$V_L = 34 \sqrt{\frac{W}{S}} \quad 33.9 \quad (36)$$

**Cylinder with axis normal to wind.** The drag of cylinders normal to the wind has been thoroughly investigated. Figure 40 gives the variation of  $C_D$  with Reynolds Number both for a section of an infinite cylinder and for a section 5 diameters long. These curves are based on test data obtained at the Washington Navy Yard and other laboratories, as compiled by Dr. A. F. Zahm<sup>9</sup> and Mr. F. A. Loudon.

**Cylinder with axis parallel to wind.** Eiffel tested two series of cylinders with the axes parallel to the wind. In one series the diameter was 30 cm. and in the other 15 cm. Lengths from a thin disc up to 7.0 diameters were tested at a wind speed of 10 m./sec. The ratio of the drag of the cylinders to the drag of the discs are plotted against length/diameter ratio in Figure 41.

Subsequent tests on the cylinder with length/diameter = 7.0 showed the value of  $C_D$  to be reduced from 0.94 to 0.19 by fitting hemispherical ends.

<sup>9</sup> See N. A. C. A. Technical Report No. 253.

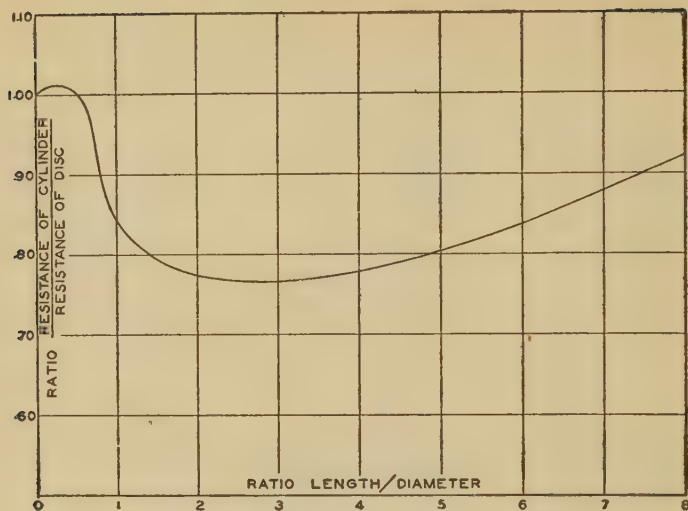


Figure 41. Drag Coefficients for Cylinders Parallel to Wind

**Skin friction.** The skin friction resistance of a very smooth surface such as glass, varnished wood, or doped fabric, is given by

$$D_F = C_{DF} \cdot \frac{\rho}{2} S V^2 \quad (37)$$

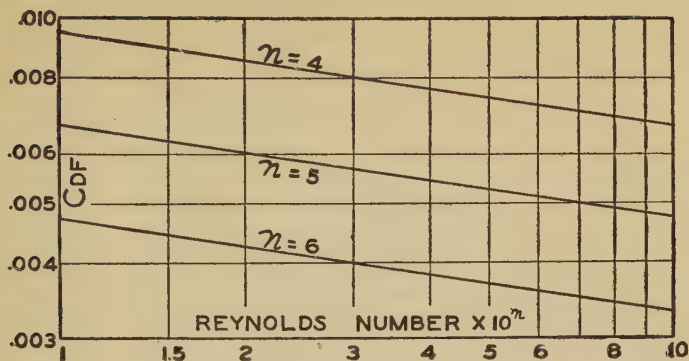
where  $\rho$  is the air density,  $S$  the *total* area or “wetted surface,”  $V$  the air speed, and  $C_{DF}$  the frictional drag coefficient given on Figure 42.  $C_{DF}$  is also given by the relation

$$\begin{aligned} C_{DF} &= .0375 \left( \frac{\rho V L}{\mu} \right)^{-0.15} \\ &= .0375 (R N)^{-0.15} \end{aligned} \quad (38)$$

hence

$$D_F = .0375 (R N)^{-0.15} \cdot \frac{\rho}{2} S V^2 \quad (39)$$

The frictional drag coefficient of a surface which is not *very smooth*, such as unvarnished wood, undoped cloth, etc., varies with the roughness of the surface between .005 and .010

Figure 42. Skin Friction Drag Coefficient  $C_{DF}$ 

and is substantially constant for all Reynolds Numbers over a wide range including the common full-scale values.

Some doubt exists as to the correct form of equation (39) on the basis that breadth should enter directly, so that the frictional coefficient varies as a function of length and Reynolds Number. There is also some doubt as to the validity of the equation for short lengths, but this doubt probably arises from the difficulty which some experimenters have had in separating form resistance from frictional resistance. The total "profile drag" of a thin section may be less than the skin friction. Equation (39) is certainly sufficiently accurate for most engineering purposes.

**Wing section drag.** Wing section, or profile drag is largely a function of camber and thickness. If the *full-scale* values of minimum drag coefficients obtained in the N. A. C. A. variable density wind tunnel be plotted against thickness, as in Figure 43, it is found that the variation is linear over the range investigated and the straight line extended to zero thickness intercepts the drag axis at  $C_{D_0} = .00375$ , which is just half of the skin friction coefficient (for the double surface). Actually the minimum section drag coefficients may be less than the skin friction coefficients for moderate cambers, but it is unlikely that any observed value would ever go below, say  $C_D = .0060$ .

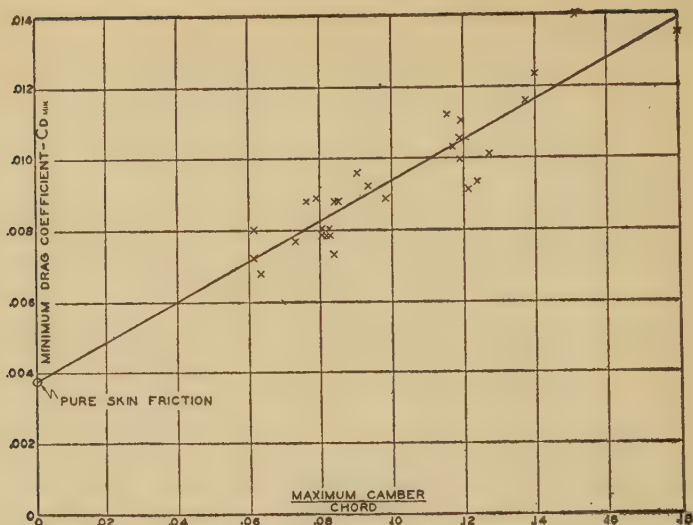


Figure 43. Minimum Section Drag Coefficient as a Function of Camber Ratio

**Struts.** Figure 44 shows the drag coefficients obtained at various speeds, on a series of four geometrically similar Navy No. 1 struts of fineness ratio 3.0, tested at the Washington Navy Yard. This strut<sup>10</sup> has the lowest drag coefficients yet recorded for a strut section. The offsets are given in Table 3.

Drag, in lbs./ft. at 100 mi./hr. in standard air for a strut of fineness ratio 3, is plotted against strut thickness in Figure 45. These values should probably be increased about 20% for the commercial shapes. Rounding the trailing edge does not appear likely to give large increase in drag, providing the radius is small. Cutting off about 15% from the rear of a strut does not affect the drag at all, according to Munk's tests reported in the *Technische Berichte*.

The change in drag with fineness ratio is shown on Figure 46. This is a composite curve based on British, German and American tests, which differ very little.

<sup>10</sup> A. F. Zahm, R. H. Smith, and G. C. Hill, "Point Drag and Total Drag of Navy Struts No. 1 Modified," N. A. C. A. Technical Report No. 137 (1922).



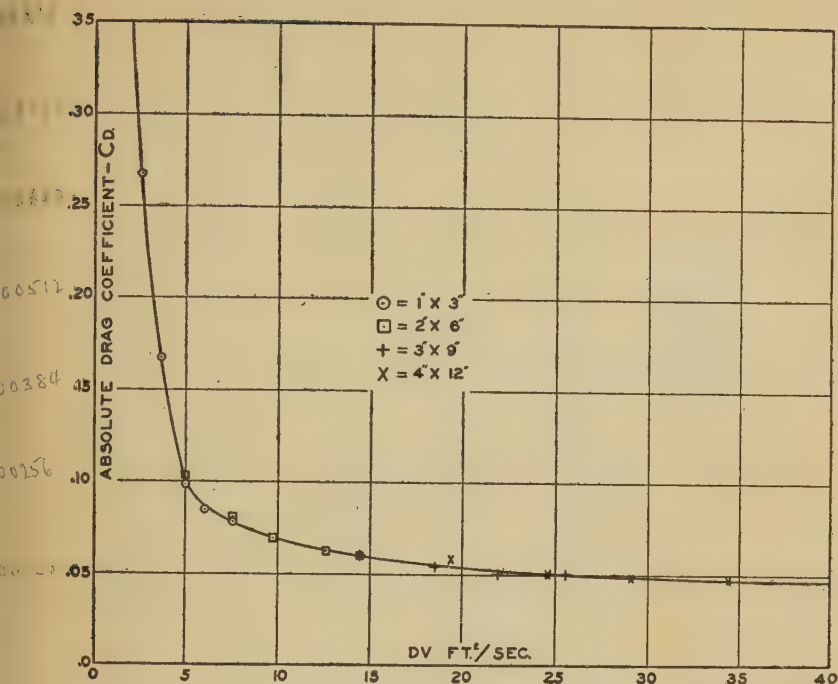
Figure 44. Drag Coefficient  $C_D$  as a Function of  $V_d$  for Navy Struts

TABLE 3. OFFSETS FOR NAVY STRUT AND C-CLASS AIRSHIP

| % L   | % D                    |                         | % L    | % D                    |                         |
|-------|------------------------|-------------------------|--------|------------------------|-------------------------|
|       | Navy<br>No. 1<br>Strut | C-Class<br>Air-<br>ship |        | Navy<br>No. 1<br>Strut | C-Class<br>Air-<br>ship |
| 0     | 0                      | 0                       |        |                        |                         |
| 1.25  | 26.0                   | 20.0                    | 35.00  | 100.0                  | 99.9                    |
| 2.50  | 37.1                   | 33.5                    | 40.00  | 99.5                   | 99.0                    |
| 5.00  | 52.5                   | 52.6                    | 50.00  | 95.0                   | 95.0                    |
| 7.50  | 63.6                   | 65.8                    | 60.00  | 86.1                   | 88.5                    |
| 10.00 | 72.0                   | 75.8                    | 70.00  | 73.2                   | 79.0                    |
| 12.50 | 78.5                   | 83.5                    | 80.00  | 56.2                   | 66.5                    |
| 15.00 | 83.6                   | 88.7                    | 90.00  | 33.8                   | 49.3                    |
| 20.00 | 91.1                   | 94.7                    | 95.00  | 19.0                   | 36.2                    |
| 25.00 | 95.9                   | 98.2                    | 98.00  | 7.8                    | 22.5                    |
| 30.00 | 98.8                   | 99.8                    | 100.00 | 0                      | 0                       |

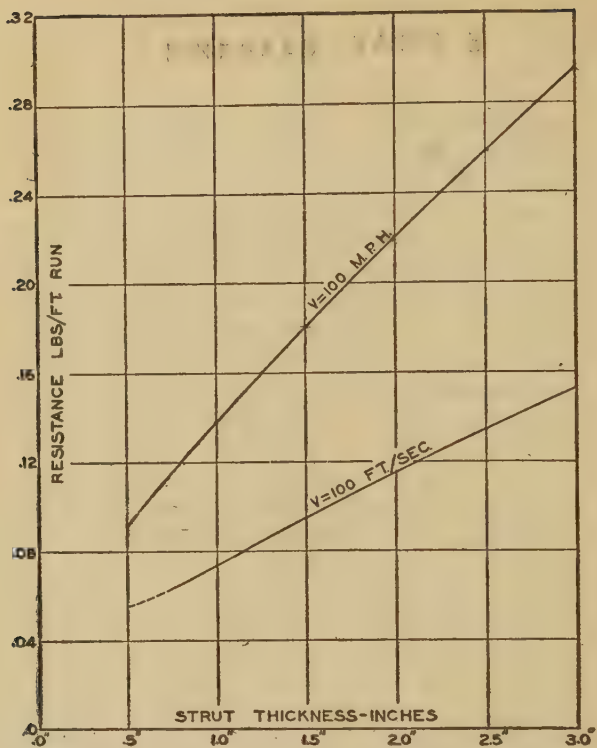


Figure 45. Drag per Foot at 100 mi./hr. Navy Struts

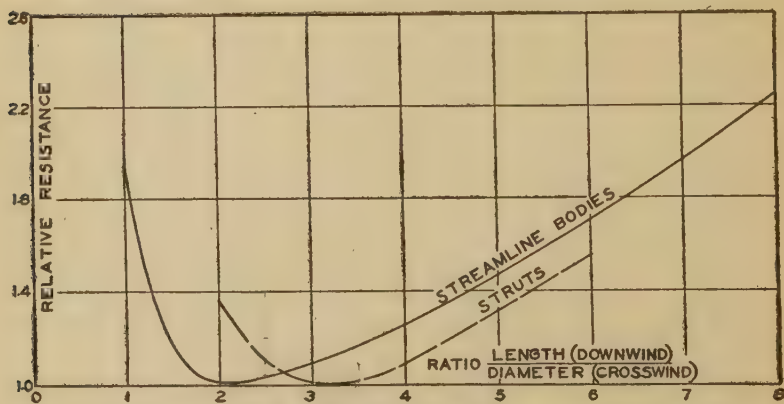


Figure 46. Variation of Drag Coefficient with Fineness Ratio for Struts and Streamline Bodies

**Aircraft cable.** The variation of  $C_D$  with Reynolds Number for round wire is given on Figure 47. The range in  $C_D$  actually encountered is not very large and it is customary to take  $C_D = 1.25$  for all normal sizes and speeds. This applies only to full scale, and gives

$$D \text{ lbs./ft.} = 2.56 d \left( \frac{V}{100} \right)^2 \quad (40)$$

where  $d$  is the diameter in inches and  $V$  the air speed in mi./hr. at standard density. The drag of various sizes of wires and turnbuckles may be found with other data in Table 4.

TABLE 4. DATA ON AIRCRAFT CABLE

| Diameter       |      | Approximate Wt.<br>per 100<br>ft.<br>lbs. | Breaking<br>Strength<br>min.<br>lbs. | Resistance at 100 mi./hr.       |                                           |                                          |
|----------------|------|-------------------------------------------|--------------------------------------|---------------------------------|-------------------------------------------|------------------------------------------|
| Nominal<br>in. | in.  |                                           |                                      | Wire<br>Alone<br>per ft.<br>run | Turn-<br>buckles<br>Plus Eyes<br>Complete | Turn-<br>buckles<br>Plus One<br>Eye Only |
| $\frac{1}{16}$ | .062 | .81                                       | 480                                  | .16                             | .42                                       | .25                                      |
| $\frac{5}{64}$ | .078 | .95                                       | 550                                  | .20                             | .48                                       | .29                                      |
| $\frac{3}{32}$ | .094 | 1.45                                      | 920                                  | .24                             | .52                                       | .33                                      |
| $\frac{1}{8}$  | .125 | 2.45                                      | 1,350                                | .32                             | .65                                       | .43                                      |
| $\frac{5}{32}$ | .156 | 4.67                                      | 2,600                                | .40                             | .82                                       | .53                                      |
| $\frac{3}{16}$ | .187 | 5.80                                      | 3,200                                | .48                             | 1.01                                      | .63                                      |
| $\frac{7}{32}$ | .218 | 8.30                                      | 4,600                                | .56                             | 1.24                                      | .74                                      |
| $\frac{1}{4}$  | .250 | 10.50                                     | 5,800                                | .64                             | 1.50                                      | .86                                      |
| $\frac{5}{16}$ | .312 | 16.70                                     | 9,200                                | .80                             | 2.05                                      | 1.08                                     |
| $\frac{3}{8}$  | .375 | ....                                      | ....                                 | .96                             | 2.60                                      | 1.33                                     |

Figure 48 is a plot of total drag divided by total length, that is, drag per foot, at 100 mi./hr. for various sizes and lengths of cable, complete with terminals.

The effect of fore-and-aft spacing is shown on Figure 49.

The effect of inclination is given on Figure 50. The curves marked A and B are two sets of test data for which the drag ratio

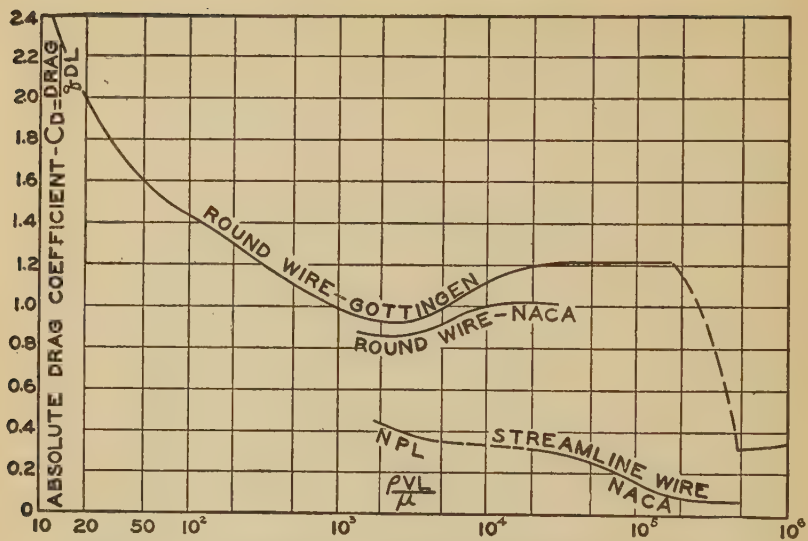
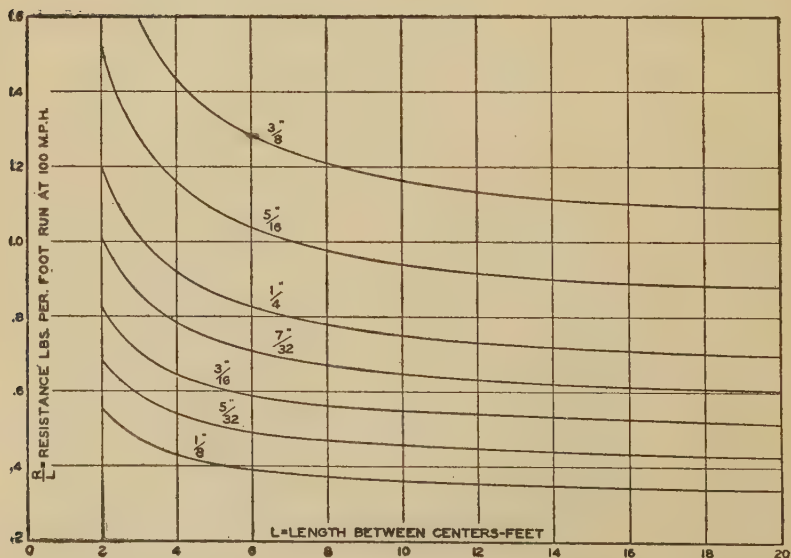
Figure 47. Variation of  $C_D$  with Reynolds Number for Wires

Figure 48. Drag per Foot at 100 mi./hr. for Cables with Turnbuckles and Eyes

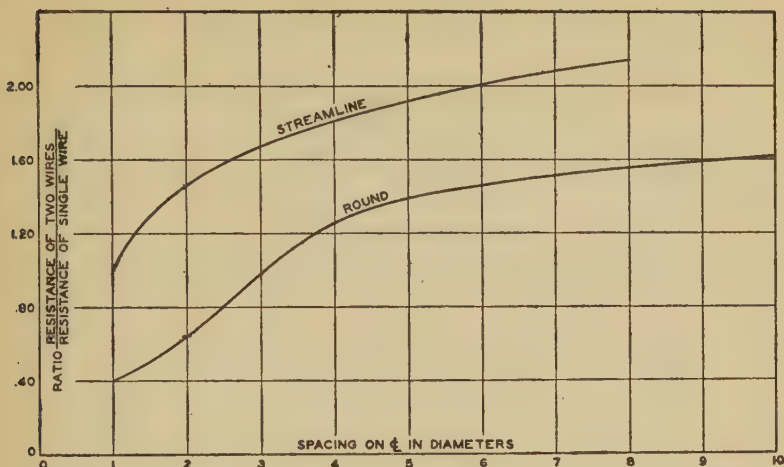


Figure 49. Effect of Fore-and-aft Spacing on the Drag of a Pair of Wires

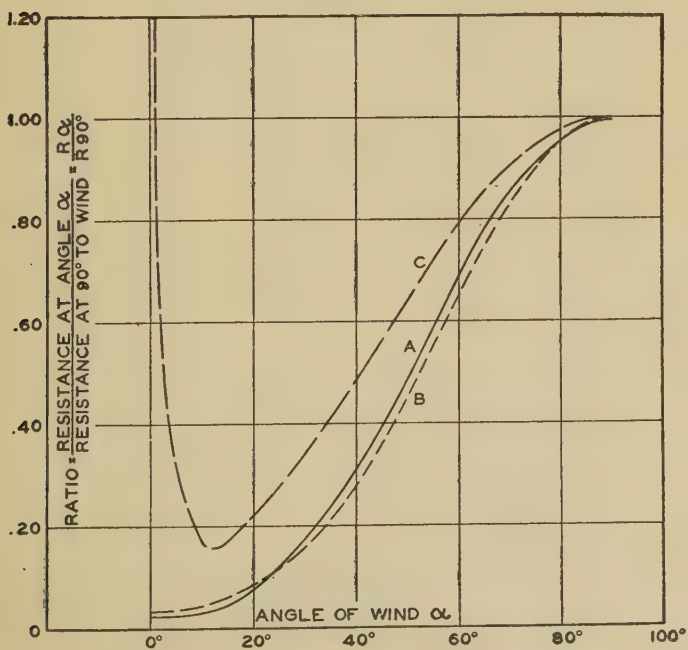


Figure 50. Effect of Inclination on the Drag of Wire and Cable. Curves A and B based on Unit Length, Curve C on Projected Length

refers to unit length. The curve marked C is the average drag ratio per unit of *projected* length.

Lift and drag coefficients as a function of angle of attack are given for both smooth wire and cable on Figure 51. These are the usual absolute coefficients, for example,

$$\text{Lift} = C_L \frac{\rho}{2} S V^2$$

$S$  being the product of the wire length in feet by its diameter in feet.

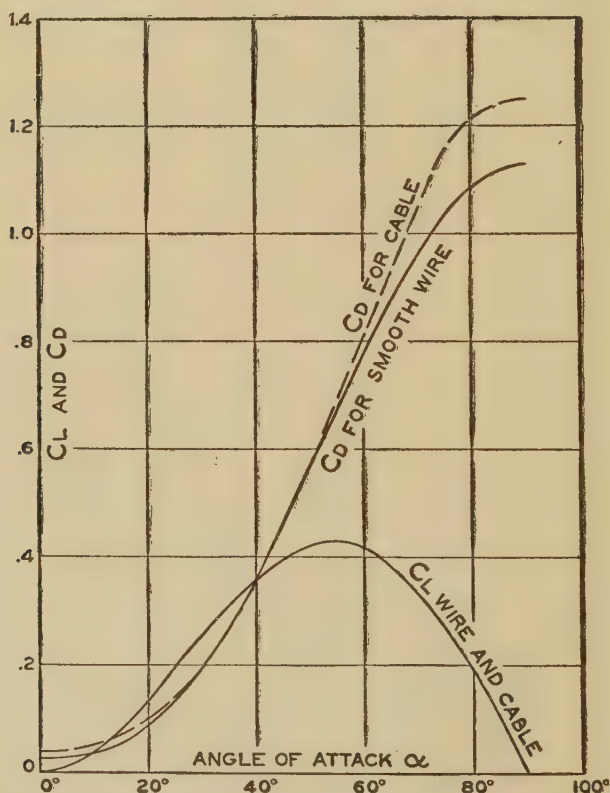


Figure 51. Lift and Drag Coefficients for Round Wire and Cable



**Streamline wire.** The drag of streamline wire has been measured by N. P. L.<sup>11</sup> and N. A. C. A. and the variation of  $C_D$  with Reynolds Number is given on Figure 47. The two sets of data fall on a single curve.

Figure 52 gives the drag in lbs./ft. at 100 mi./hr. and also the drag of two standard terminals at the same speed. It is customary to use the projected length of inclined streamline wire in

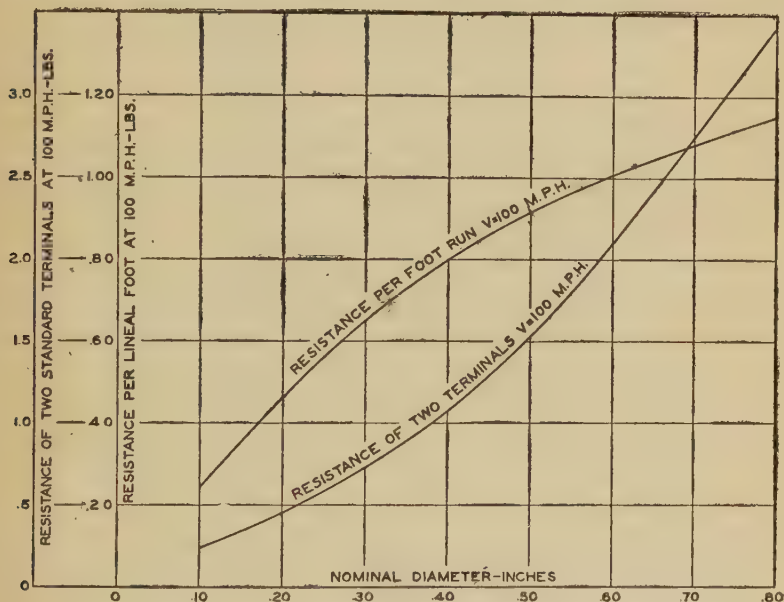


Figure 52. Drag of Streamline Wire and Standard Terminals

calculating the drag. This is an approximation which is sufficiently exact if the angle is not less than about  $40^\circ$ .

Figure 53 gives the total drag divided by total projected length for various sizes and lengths of streamline wires with standard terminals.

The effect of fore-and-aft spacing on double streamline wires is shown on Figure 49, and the effect of yaw on the drag of a single wire is shown on Figure 54.

Table 5 contains data on the standard sizes of streamline wire.

<sup>11</sup> R. & M. 256. N. P. L. is abbreviation for National Physical Laboratory (British).

TABLE 5. STREAMLINE WIRE DATA

| No. | Breaking<br>Strength<br>lbs. | Dia.<br>at<br>Ends<br>in. | Threads | Streamline<br>Section<br>in. | Resistance<br>per ft.<br>Projected<br>Length at<br>100 mi./hr.<br>lbs. | Resistance<br>of 2 End<br>Fittings at<br>100 mi./hr.<br>lbs. | Weight<br>per ft.<br>lbs. |
|-----|------------------------------|---------------------------|---------|------------------------------|------------------------------------------------------------------------|--------------------------------------------------------------|---------------------------|
| 10  | 1,000                        | .1380                     | #6-40   | .048 X .192                  | .033                                                                   | 0.30                                                         | .026                      |
| 21  | 2,100                        | .1900                     | #10-32  | .064 X .256                  | .044                                                                   | 0.43                                                         | .046                      |
| 34  | 3,400                        | .2500                     | 1/4"-28 | .087 X .348                  | .056                                                                   | 0.58                                                         | .085                      |
| 61  | 6,100                        | .3125                     | 1/8"-24 | .110 X .440                  | .067                                                                   | 0.76                                                         | .137                      |
| 80  | 8,000                        | .3750                     | 3/8"-24 | .135 X .540                  | .077                                                                   | 0.97                                                         | .205                      |
| 115 | 11,500                       | .4375                     | 1/2"-20 | .159 X .636                  | .085                                                                   | 1.23                                                         | .286                      |
| 155 | 15,500                       | .5000                     | 1/2"-20 | .183 X .732                  | .092                                                                   | 1.53                                                         | .375                      |

9/16" 269 X .836  
5/8" 341 X .924

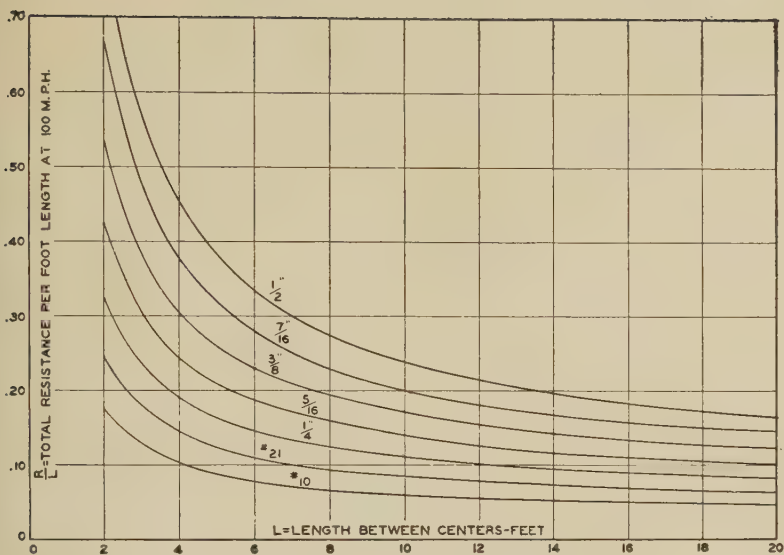


Figure 53. Drag per Foot at 100 mi./hr. for Streamline Wire with Standard Terminals

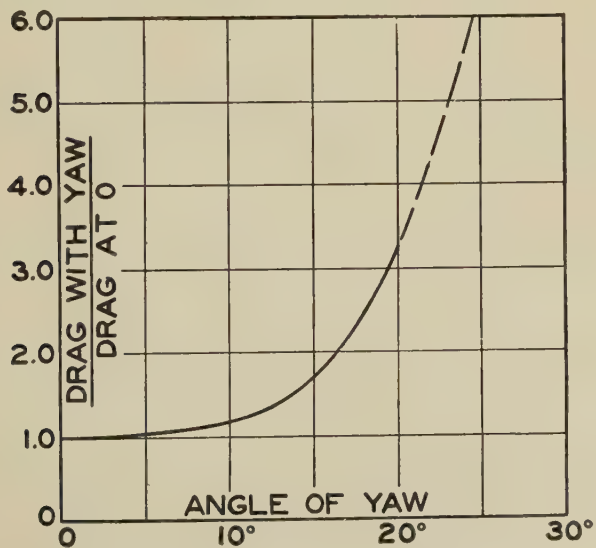
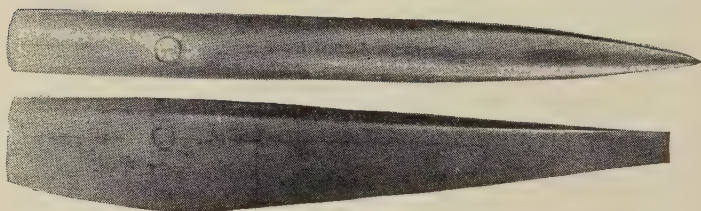


Figure 54. Effect of Yaw on Drag of Streamline Wire

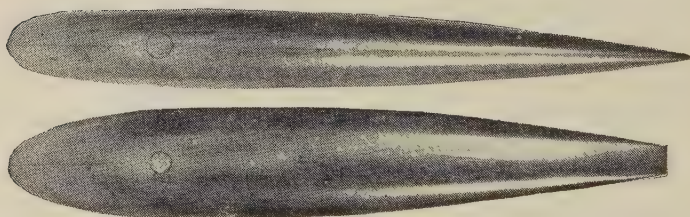
1



2



3



4

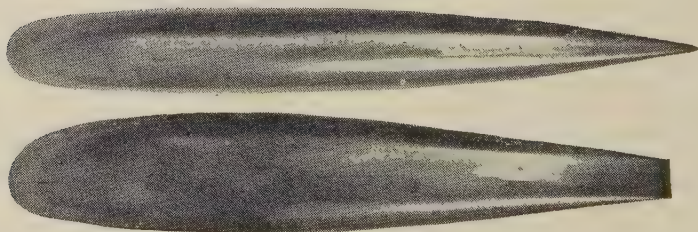
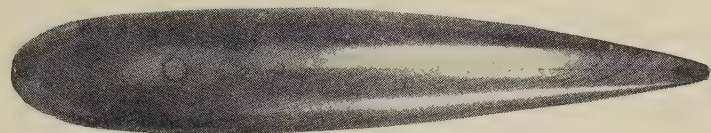
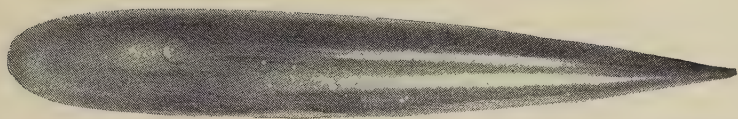
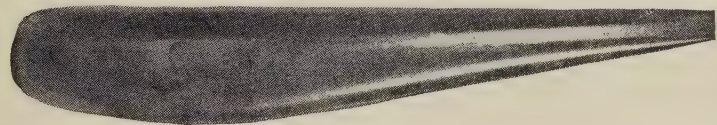
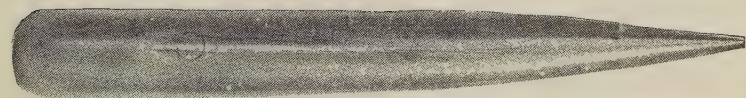


Figure 55a. Fuselage Models

5



6



7

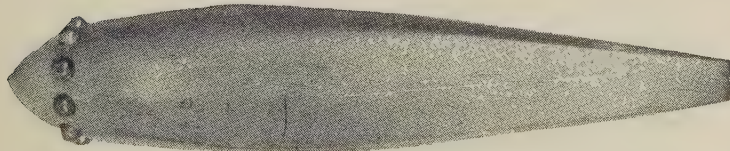
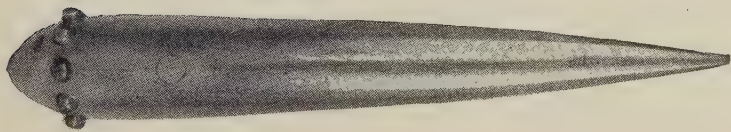
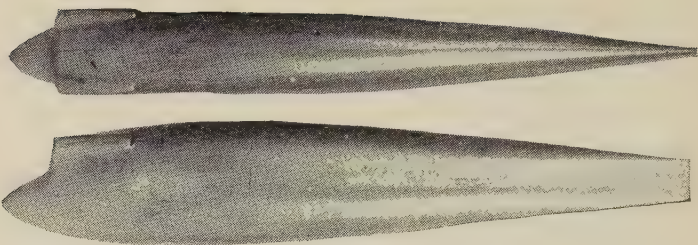


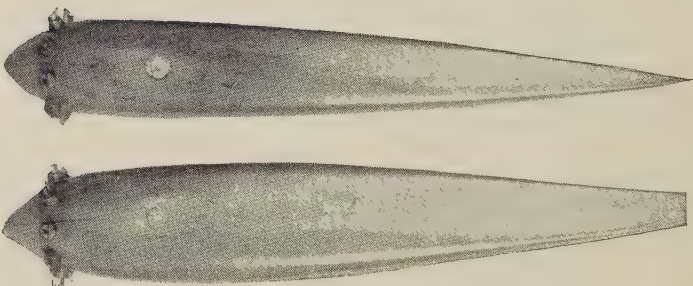
Figure 55b. Fuselage Models



8



9



10

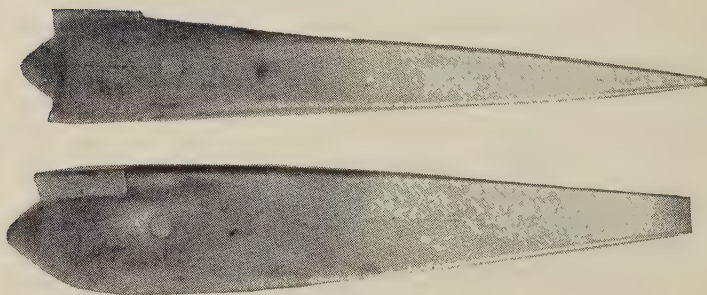
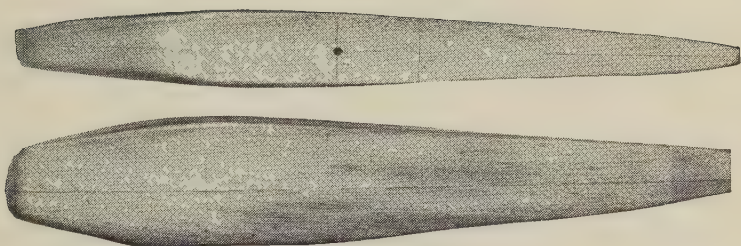


Figure 55c. Fuselage Models



11



12

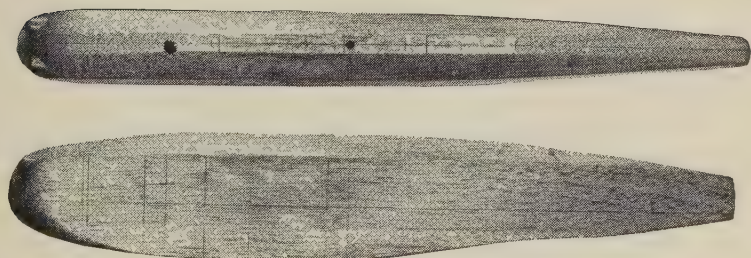
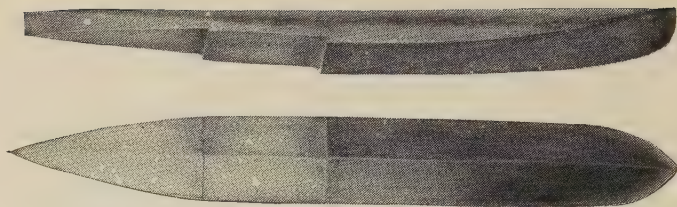
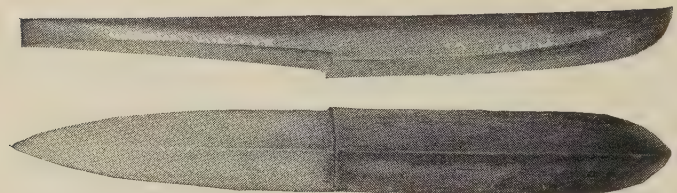


Figure 55d. Fuselage Models

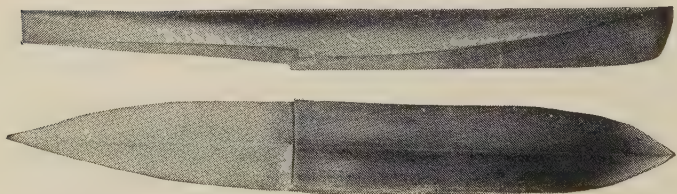
1



2



3



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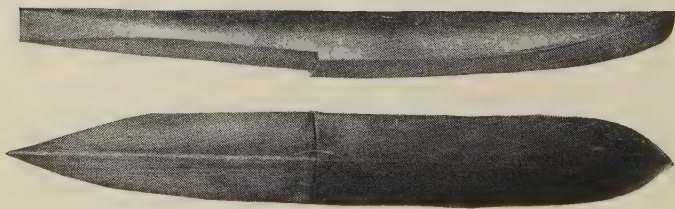


Figure 56a. Float Models

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6



7



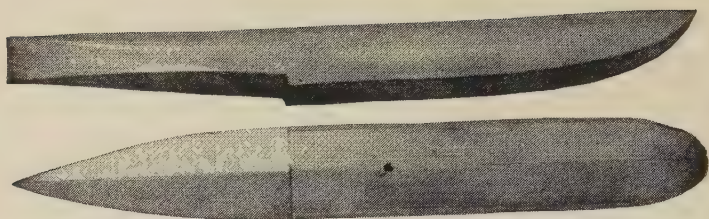
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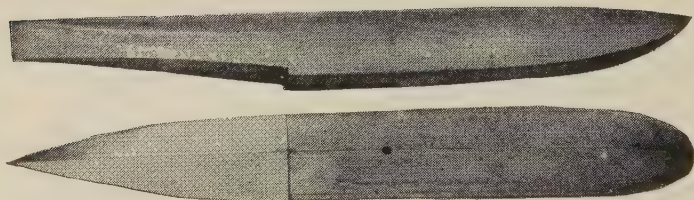
Figure 56b. Float Models



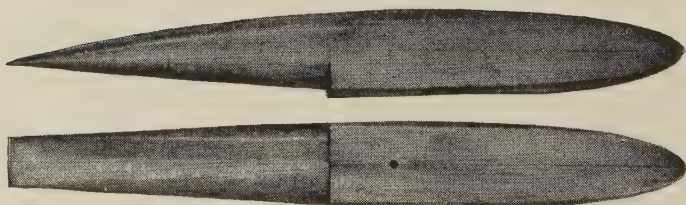
9



10



11



12

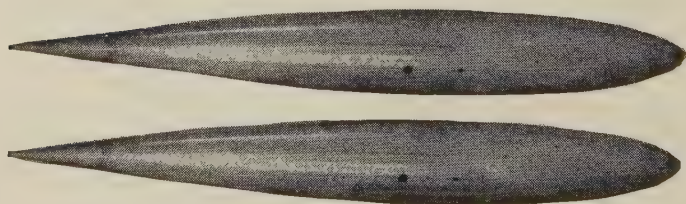
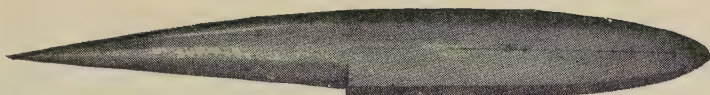


Figure 56c. Float Models

13



14



15

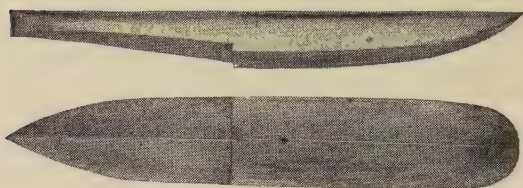


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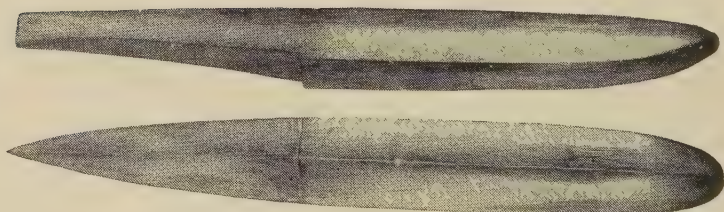


Figure 56d. Float Models

17



18



19

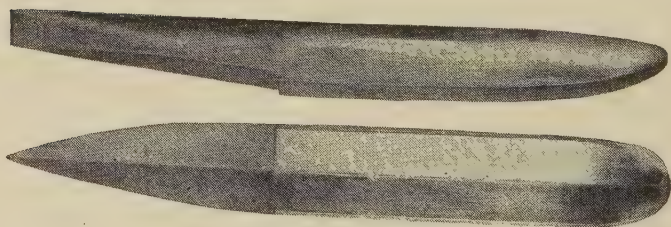


Figure 56e. Float Models



**Fittings.** The average drag of a fitting mounted on a wing, fuselage, float, or tail surface is given approximately by considering the fitting as a flat plate of *double* its projected area. This allowance is necessary in order to account for the "interference" drag due to the fitting. The drag of a fitting is surprisingly high. The drag of an average strut end fitting with wire attachments on a 4,000-lb. airplane will probably be about 3.0 lbs. at 100 mi./hr. Most of this is in the wire terminals and can be included in the wire drag where it belongs.

Miscellaneous projecting parts, not streamlined, should be treated as fittings.

**Control horns.** The value of  $K$  for a well-streamlined control horn is about .00080, but if bolts and nuts are exposed  $K = .0015$  is probably a fair average.

**Fuselage drag.** Drag data on fuselages and similar airplane parts have been compiled by the Bureau of Aeronautics, Navy Department, and published as N. A. C. A. Technical Report No. 236.<sup>12</sup> Since the publication of this report additional tests have been made at the Washington Navy Yard on the models illustrated in Figure 55. The data on these models are given in Table 6.

Models Nos. 3, 4, and 5 are practically pure streamlines, which were tested in order to supply a basis for comparison. Models 7 and 9 were tested in order to find some form of simple model construction that would have the same drag as an air-cooled engine; No. 9 gives a close approximation. Several additional models not shown were tested in this investigation; the conclusion was reached that the model cylinders should be solid to the outer edge of the fins and up to the top of the cylinder. The remaining models closely represent well-known airplane fuselages.

If all the engine, radiators, windshields, fittings, and other details are considered separately, the average value of  $K$  is

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<sup>12</sup> W. S. Diehl, "Tests on Airplane Fuselages, Floats, and Hulls," N. A. C. A. Technical Report No. 236 (1926).

approximately .00040. If all of the minor details are not considered, the average value of  $K$  will be about .00050. The latter figure takes care of the minor items only, such as fittings, and irregularities of construction.

The drag coefficients for nacelles are not appreciably different from those for fuselages.

TABLE 6. DATA ON AIRPLANE FUSELAGE MODELS SHOWN IN FIGURE 55

| Model No. | Dimensions |                              | Measured Drag at 40 mi./hr. lb. | $K$    | Absolute Drag Coefficient |
|-----------|------------|------------------------------|---------------------------------|--------|---------------------------|
|           | Length ft. | Cross Section Area A sq. ft. |                                 |        |                           |
| 1         | 1.670      | .0457                        | .066                            | .00091 | .353                      |
| 2         | 1.668      | .0459                        | .050                            | .00068 | .266                      |
| 3         | 1.669      | .0472                        | .012                            | .00016 | .062                      |
| 4         | 1.668      | .0548                        | .016                            | .00018 | .071                      |
| 5         | 1.673      | .0621                        | .016                            | .00016 | .063                      |
| 6         | 1.673      | .0463                        | .022                            | .00030 | .116                      |
| 7         | 1.673      | .0708                        | .058                            | .00051 | .200                      |
| 8         | 1.663      | .0550                        | .021                            | .00024 | .094                      |
| 9         | 1.662      | .0662                        | .068                            | .00064 | .251                      |
| 10        | 1.667      | .0862                        | .036                            | .00026 | .102                      |
| 11        | 1.667      | .0481                        | .026                            | .00034 | .132                      |
| 12        | 1.667      | .0397                        | .027                            | .00042 | .166                      |

**Effect of pitch on fuselage drag.** The drag of a fuselage with well-rounded section does not change greatly with angle of pitch while the drag of a fuselage with square or rectangular sections increases rapidly with increasing pitch. Consider models Nos. 2 and 3 in Figure 55.

The measured model drag in lbs. at 40 mi./hr. varies with pitch as follows:

| Angle of Pitch.... | 0°   | 5°   | 10°  | 15°  | 20°  |
|--------------------|------|------|------|------|------|
| Model No. 2.....   | .050 | .057 | .070 | .083 | .129 |
| Model No. 3.....   | .012 | .012 | .010 | .014 | .022 |

Rounding the sections of a fuselage reduces the drag at 0°, and gives an even greater improvement at cruising and climbing speeds.

**Floats.** Table 7 gives data for a number of seaplane floats shown on Figure 56. These models, representing about an equal number of actual floats and of design studies, were tested at the Washington Navy Yard. The actual floats represented by models Nos. 1-10, and No. 19, show fairly uniform drag coefficients. For the average float  $K = .00055$ .

Float No. 12 is practically a pure streamline form tested for comparison. Models Nos. 11, 12, 13, and 14 are included for comparison purposes only. They cannot be used as actual floats on account of undesirable water performance.

The drag of a float may be calculated from the "shape coefficient"  $C_s$ , the gross weight of the seaplane, and the excess buoyancy. Since the weight of sea water is 64 lbs./cu. ft., the cubic displacement for a weight  $W$  is  $\left(\frac{W}{64}\right)$ . Letting the ratio of submerged displacement to load displacement equal  $(1 + e)$ , the total volume of the float is  $\frac{W(1 + e)}{64}$ , and the drag

$$D = C_s \frac{\rho}{2} V^2 \left[ \frac{W(1 + e)}{64} \right]^{\frac{2}{3}} \quad (41)$$

The factor  $e$  is the decimal value of the usual percentage excess displacement. For example, with 80% excess, the value of  $e$  is 0.80.

The use of a pronounced hollow bottom of the Short-Macchi type increases the drag 10% to 15% over that for the conventional "Vee" bottom, according to wind tunnel test data now available. However, a moderate amount of hollowness does not appear to affect the drag appreciably.

Wind tunnel tests have been made to determine the effect

of a step on float drag. In general there is no measurable change in drag when a fairing is fitted behind the step. Test data indicate that a good streamline body fitted with hydrovanes has more drag than a conventional float even when due allowance is made for scale effect.

The drag of an average float is comparatively low, and the drag of two floats may be slightly less than the drag of the two wheels in the corresponding land-type landing gear. The float bracing has more drag, however, than the land-type bracing. For an airplane of 3,000 lbs. gross weight, the parasite area for the seaplane arrangement is normally 1.0 to 1.5 sq. ft. greater than for the landplane arrangement.

TABLE 7. DATA ON SEAPLANE FLOAT MODELS SHOWN IN FIGURE 56

| Model No. | Dimensions |                                | Drag at 40 mi./hr. lb. | Model $K$ | Absolute Coefficients |       |
|-----------|------------|--------------------------------|------------------------|-----------|-----------------------|-------|
|           | Length ft. | Cross Section Area $A$ sq. ft. |                        |           | $C_D$                 | $C_S$ |
| 1         | 1.670      | .0272                          | .0252                  | .00058    | .226                  | .060  |
| 2         | 1.664      | .0258                          | .0253                  | .00061    | .239                  | .063  |
| 3         | 1.667      | .0276                          | .0236                  | .00053    | .209                  | .058  |
| 4         | 1.671      | .0304                          | .0268                  | .00055    | .215                  | .061  |
| 5         | 1.670      | .0274                          | .0239                  | .00055    | .213                  | .059  |
| 6         | 1.667      | .0269                          | .0228                  | .00053    | .207                  | .056  |
| 7         | 1.667      | .0340                          | .0286                  | .00053    | .205                  | .063  |
| 8         | 1.667      | .0300                          | .0339                  | .00071    | .276                  | .076  |
| 9         | 1.659      | .0292                          | .0243                  | .00052    | .203                  | .054  |
| 10        | 1.655      | .0305                          | .0178                  | .00036    | .143                  | .041  |
| 11        | 1.665      | .0365                          | .0139                  | .00024    | .093                  | .029  |
| 12        | 1.667      | .0369                          | .0085                  | .00014    | .056                  | .020  |
| 13        | 1.666      | .0368                          | .0118                  | .00020    | .079                  | .026  |
| 14        | 1.662      | .0370                          | .0105                  | .00018    | .070                  | .025  |
| 15        | 1.665      | .0339                          | .0184                  | .00034    | .132                  | .041  |
| 16        | 1.665      | .0253                          | .0247                  | .00061    | .238                  | .062  |
| 17        | 1.654      | .0405                          | .0313                  | .00048    | .189                  | .058  |
| 18        | 1.733      | .0319                          | .0163                  | .00032    | .124                  | .034  |
| 19        | 1.605      | .0305                          | .0165                  | .00034    | .132                  | .038  |

**Wing tip floats.** The drag of wing tip floats varies between  $.00040 AV^2$  and  $.00080 AV^2$ . In the absence of specific data, the average value of  $K = .00060$  may be used.

**Flying boat hulls.** The drag of the F5L hull is approximately  $.00052 AV^2$  and the drag of the NC hull approximately  $.00040 AV^2$ . The NC hull is exceptionally clean, and the value of  $K$  is probably about  $.00050$  for the average flying boat hull.

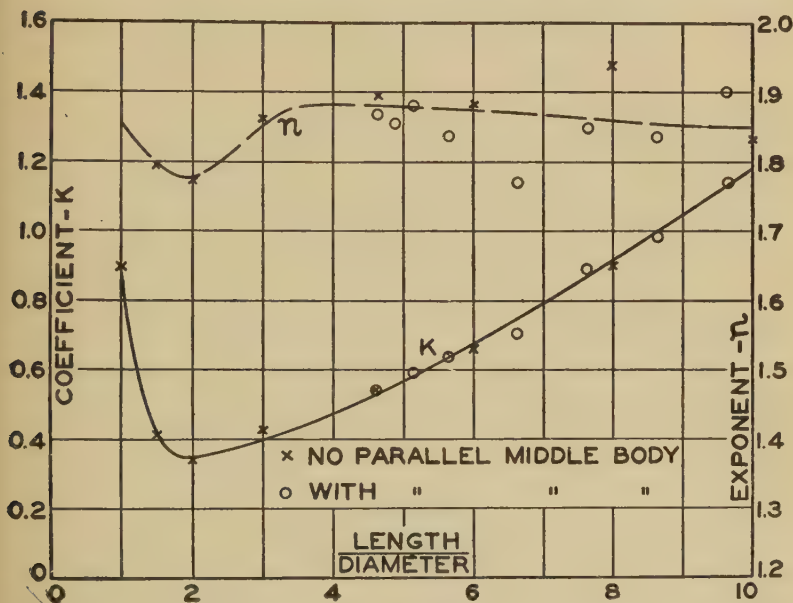


Figure 57. Effect of Fineness Ratio on  $K$  and  $n$  in Equation for Drag of C-Class Airship

**Streamline bodies.** The drag of streamline bodies at  $0^\circ$  pitch and yaw is not only very difficult to measure accurately but it is also greatly affected by surface irregularities, disturbance, or turbulence in the air flow, and by change in Reynolds Number. The form having the least drag per unit of cross-section area does not necessarily have the least drag per unit of volume, and the form which shows up best at one Reynolds



Number may be comparatively poor at some other Reynolds Number.

The C-Class airship is a form which has a very low drag under all conditions. This shape has been thoroughly tested at the Washington Navy Yard for the effect of inserting parallel middle body<sup>13</sup> and for the effect of varying the spacing between stations. The drag may be written in the form

$$D = K \left( \frac{\rho}{\rho_0} \right) \left( \frac{Vd}{100} \right)^n \quad (42)$$

where  $D$  is the drag in lbs.,  $V$  the air speed in ft./sec., and  $d$  the diameter in ft. Both  $K$  and  $n$  vary with fineness ratio as shown on Figure 57. Allowing for the fact that the value of  $n$  is very difficult to determine, the test data are quite consistent.

Offsets for the C-Class are given in Table 3.

**Cellular radiators.** Tests at the Washington Navy Yard<sup>14</sup> on cellular radiators gave values of  $C_D$  between 0.57 and 0.78, with an average value of about 0.70. This coefficient is for the core alone and it decreases slightly as the core thickness is increased. The average  $C_D$  corresponds to about 18.0 lbs./sq. ft. of core at 100 mi./hr.

When a cellular radiator is covered by closed shutters, the drag coefficient is substantially that of a flat plate, i.e.,  $C_D = 1.10$  approximately. Allowance must be made for the additional resistance of the internal flow over engine, etc., when calculating the drag of nose radiators. The drag of headers on free-air radiators should be considered as that of a flat plate of the same area.

**Lamblin radiators.** Tests at Issy-les-Moulineaux<sup>15</sup> on a Lamblin radiator 10" in diameter by 23.6" in length, gave  $D = .0038V^2$  in kg.-m.-sec. units which is equivalent to

<sup>13</sup> A. F. Zahm, R. H. Smith, and G. C. Hill, "The Drag of the C-Class Airship Hull with Varying Lengths of Cylindric Midships," N. A. C. A. Technical Report No. 138 (1922).

<sup>14</sup> R. H. Smith, "Resistance and Cooling Power of Various Radiators," N. A. C. A. Technical Report No. 261 (1927).

<sup>15</sup> "Bulletin Technique" No. 28, August, 1925.



$$D = .001675 V^2 \text{ lbs.}$$

for  $V$  in mi./hr. at standard density. Tests at the Washington Navy Yard<sup>16</sup> on a larger size Lamblin radiator gave

$$D = .0023 V^2 \text{ lbs.}$$

**Drag of Sperry Messenger airplane parts.** The drag of various parts of a full-scale Sperry Messenger airplane has recently been measured in the N. A. C. A. 20-ft. wind tunnel at Langley Field.<sup>17</sup> The results were as follows:

| Part                                | lbs. Drag<br>at 80 mi./hr. |
|-------------------------------------|----------------------------|
| Bare closed fuselage.....           | 15.95                      |
| Engine (3-cylinder air-cooled)..... | 11.30                      |
| Cockpit.....                        | 2.00                       |
| Windshield and upholstery.....      | 2.00                       |
| Tail surfaces.....                  | 8.00                       |
| Landing gear.....                   | 26.00                      |

The drag coefficient of the bare fuselage with faired-over cockpit is approximately  $K = .0042$ . The addition of the windshield and upholstery and the opening of the cockpit raises the value of  $K$  to approximately .0048.

The drag of the 3-cylinder air-cooled engine is equivalent to  $D = .00176 V^2$  for  $V$  in mi./hr. On this basis the drag of a similar 9-cylinder air-cooled engine, without additional interference allowance, would be  $D = .0053 V^2$ . This is comparable with the average value  $D = .0060 V^2$  obtained from flight test data on the Wright "Whirlwind."

The total area of the tail surfaces is about 30 sq. ft. The measured drag indicates a value of  $C_D = .011$  ( $K = .000042$ ). This is equivalent to about 0.40 lb. per sq. ft. at 100 mi./hr.

**Air-cooled engines.** The drag of an air-cooled engine depends greatly on the cowling. With average cowling the drag of the Wright "Whirlwind" is about  $.0060 V^2$  lbs.,  $V$  being in mi./hr.

<sup>16</sup> See N. A. C. A. Technical Report No. 261.

<sup>17</sup> Fred E. Weick, "Full-Scale Drag Tests on Various Parts of Sperry Messenger Airplane," N. A. C. A. Technical Note No. 271 (1928).

Flight test data indicate values for this engine from approximately  $.0050V^2$  to  $.010V^2$ . The drag of the Pratt and Whitney "Wasp" has been calculated for normal cowling as  $D = .0080V^2$ . The drag of the Pratt and Whitney "Hornet" or the Wright "Cyclone," on the same basis, would be  $D = .0090V^2$ .

Data now available show a consistent indication that the drag of a 9-cylinder air-cooled engine is equivalent to an increase between 1 and 2 sq. ft. of parasite flat-plate area over the value for the corresponding water-cooled installation with a good free-air or tunnel type radiator. Most of this increase is unnecessary and may be avoided by proper cowling.

**Wheels—landing gear.** British tests<sup>18</sup> gave the drag of wheels as follows:

- (a) Wheel without fairing,  $C_D = 0.70$   
( $D = 18.0$  lbs./sq. ft. at 100 mi./hr.)
- (b) With the usual fairing from hub to rim  $C_D = 0.43$   
( $D = 11.0$  lbs./sq. ft. at 100 mi./hr.)
- (c) With complete fairing from hub to edge of tread,  
 $C_D = 0.23$   
( $D = 6.0$  lbs./sq. ft. at 100 mi./hr.)

French tests (Bulletin Technique No. 28, August, 1925) on the complete landing gears for the Spad 33 and Spad 40, indicated a total drag of 55 and 61 lbs. respectively at 100 mi./hr. In both cases about 31 lbs. of this was due to the two wheels and the remainder to the four struts and faired axle. The wheels were 800 x 150 mm. or about 32" x 6". The drag of one 32" x 6" wheel is, therefore, about 15.5 lbs. at 100 mi./hr.

Tests on an 800 x 150 mm. Breguet shock-absorbing wheel fitted with side plates were reported in the same bulletin. This wheel had a drag of 16 lbs. at 100 mi./hr. These values agree very closely with flight test data.

The drag in lbs. per wheel at 100 mi./hr. in standard air is approximately:

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<sup>18</sup> Br. A. C. A. R. & M. No. 207.

| Size of Wheel | lbs. Drag<br>at 100 mi./hr. |
|---------------|-----------------------------|
| 26 x 3        | 7.5                         |
| 28 x 4        | 10.0                        |
| 30 x 5        | 12.5                        |
| 32 x 6        | 16.0                        |

**Machine guns.** Flight tests indicate that the drag of a machine gun, lined fore-and-aft, with a scarff ring mount, is about  $.0020V^2$ . This value is increased about 50% by putting the gun broadside.

**Searchlights.** A searchlight 21 cm. (8.27") maximum diameter by 42 cm. (16.55") in length, complete with wing-mounting brackets was tested at St. Cyr<sup>19</sup> and the drag found to be  $2.9V^2$  in gr-m-sec. units which is equivalent to

$$D = .00132 V^2 \text{ lbs.}$$

for  $V$  in mi./hr. at standard density.

**Interference.** The "interference" effect between two objects varies greatly, sometimes decreasing the total resistance and sometimes increasing it. The reduction of drag when two wires are lined up fore-and-aft has been given. When two wires are side by side the total resistance, for spacings less than about 6 diameters, is greater than twice that of a single wire, but very little actual test data are available. It is usually assumed that no interference exists for lateral spacings of 10 diameters or greater. This has been verified at the Washington Navy Yard, but no record was made of the tests.

The "upstream wake" of a cylinder, which is very aptly designated as an "air prow" by Dr. Zahm, extends to a distance of about 5 diameters and the lateral disturbance extends to a similar distance before the velocity is normal. The theoretical velocity distributions about a number of simple shapes are

<sup>19</sup> "Bulletin Technique" No. 28, August, 1925.

shown in N. A. C. A. Technical Report No. 253.<sup>20</sup> From these one may secure an idea of the extent of the disturbance created by a moving object.

The drag of two adjacent struts was partially investigated by Nayler and Jones.<sup>21</sup> Two 1" x 3" streamline struts were tested at several spacings and the average drags found to be as follows:

|                  |          |       |       |       |
|------------------|----------|-------|-------|-------|
| Spacing          | $\infty$ | 4.90  | 3.970 | 2.890 |
| Thickness        |          |       |       |       |
| Total Drag, lbs. | 0.048    | 0.050 | 0.052 | 0.053 |

These data indicate that two struts spaced laterally 3 "diameters" apart on their center lines have about 10% more drag than with normal spacing.

The most important form of interference is that which occurs when anything is attached to an airplane wing, particularly on the upper surface. This may be illustrated by the results of some tests on a twin engine airplane model at the Washington Navy Yard. The drag in the first test appeared high and the nacelles were removed and tested alone. At 40 mi./hr. the nacelle drag was found to be .078 lb. in free air, and 0.120 lb. on the model. A plasticine fairing with generous fillets between the nacelle and the wing reduced the interference drag from .042 lb. to .002 lb. This model was a biplane with the nacelles mounted directly on the upper surface of the lower wing. A still greater effect probably would be found if the nacelles were located on the upper surface of the upper wing of a biplane or on the upper surface of a monoplane wing. When it is necessary to mount nacelles above a wing, wind tunnel tests with and without nacelles are advisable in order to determine whether or not the fairing is adequate, or to find the fairing which gives a minimum interference drag.

Any projection from the upper surface of an airplane wing such as strut fittings, bolts, brackets, etc., will show interference

<sup>20</sup> A. F. Zahm, "Flow and Drag Formulas for Simple Quadrics" (1926).

<sup>21</sup> "The Determination of the Forces on Two Struts in Close Proximity to one Another," Br. A. C. A. R.&M. No. 204 (1915).

drags ranging from 50% to 200% of the free air drag of the part in question. Any form of fairing between wings and fuselage that interferes with the natural flow of air over the wing may be expected to show a pronounced interference effect. Horizontal tail surfaces fitted just above the fuselage may also be expected to show interference drag effects if the gap is not filled.

## CHAPTER V

### CONTROL SURFACE DESIGN

**Axes, forces, and moments.** The three major axes of an airplane are the longitudinal or  $X$ , lateral or  $Y$ , and normal or  $Z$ . Motion about the  $X$ ,  $Y$ , and  $Z$  axes is designated as roll, pitch, and yaw, respectively. Control about these axes is secured by the ailerons, elevators, and rudder, respectively. By universal agreement in this country the positive direction along the axes is to the rear for  $X$ , to the left (as seen from the rear) for  $Y$ , and up for  $Z$ . The positive direction for angles or moments is from  $+Y$  toward  $+Z$  (or right wing down) in roll, from  $+Z$  to  $+X$  (or nose up) in pitch, and from  $+X$  to  $+Y$  (or nose swinging to right) in yaw.

**Stability.** An airplane is *statically stable* if any displacement from a given attitude sets up forces and moments tending to restore the original attitude. It is *dynamically stable* if the resulting motion is stable, that is, if any oscillations due to static stability are quickly damped. Static stability may be easily measured by wind tunnel tests on an airplane model; it is directly proportional to the slope of the moment curve. Dynamic stability, on the other hand, must be laboriously calculated from rather extensive wind tunnel tests, using assumptions which are questionable. It is not surprising, therefore, to find that practically all aeronautical engineers ignore dynamic stability as a design factor.

In pitch, a fair degree of static stability with normal design proportions is usually accompanied by dynamic stability. Stability in pitch is called "longitudinal stability." Stability in roll and stability in yaw are not easily separated. They are always treated in combination as "lateral stability." Static



stability in both roll and yaw does not insure dynamic lateral stability and in fact lateral instability may easily result from too much "directional stability." In this connection, the use of moderate dihedral is probably desirable in all seaplanes and in most land planes. The prejudice of many pilots against the use of dihedral in pursuit type airplanes is not substantiated by full-scale rolling and turning data.

The study of dynamic stability is of greater value to the aeronautical engineer than is generally realized. It is an excellent method of analyzing the effects of changes in design proportions and will probably be used more extensively for this purpose in the future when the demands for more refinement in design methods make it necessary.

The literature on stability calculations is extensive. The most important papers are:

- L. Bairstow, "Investigation into the Stability of an Airplane, Etc." Br. A. C. A. R.&M. No. 77 (1913).
- L. Bairstow, "The Experimental Determination of Rotary Coefficients." Br. A. C. A. R.&M. No. 78.
- L. Bairstow, "An Examination into the Longitudinal Stability of a Monoplane." Br. A. C. A. R. & M. No. 79.
- J. C. Hunsaker, "Experimental Analysis of Inherent Longitudinal Stability for a Typical Biplane." N. A. C. A. Technical Report No. 1.

Excellent treatment of stability is also given in the following books:

- L. Bairstow, "Applied Aerodynamics." Longmans, Green & Co. (1920).
- E. B. Wilson, "Aeronautics." John Wiley & Son. (1920).

**Control.** One of the most important problems in airplane design is to determine the sizes and proportions of the control surfaces so as to obtain a satisfactory degree of control and stability under all conditions of flight. The remainder of this chapter is devoted to the general question of control surface design.

**Horizontal tail area.** *First Method:* The horizontal tail area required to give static longitudinal stability may be obtained by writing the equation for pitching moments about the center of gravity. If the resultant air force be assumed normal to the wing chord and equal to the lift (which is a close approximation except at very low lift coefficients), the slope of the wing moment curve against angle of attack is

$$\frac{dM_w}{d\alpha} = qS_w C \frac{dC_L}{d\alpha} \left[ \frac{a}{\bar{C}} - \left( C_p + \alpha_a \frac{dC_p}{d\alpha} \right) \right] \quad (43)$$

where  $S_w$  is the total wing area,  $C$  the mean wing chord,  $a$  the fore-and-aft center of gravity location on the mean chord,  $C_p$  the center of pressure,  $\alpha_a$  the absolute angle of attack measured from zero lift, and  $q$  the dynamic pressure. The slope of the tail moment curve is

$$\frac{dM_t}{d\alpha} = \frac{dM_t}{d\alpha_t} \cdot \frac{d\alpha_t}{d\alpha} = - \frac{dC_{Lt}}{d\alpha_t} \cdot \frac{d\alpha_t}{d\alpha} q S_t l \quad (44)$$

where  $\alpha_t$  is the angle of attack,  $C_{Lt}$  the lift coefficient, and  $S_t$  the total area, of the horizontal tail surfaces.  $l$  is the distance from the c.g. of the airplane to the center of pressure of the tail surfaces, and may be taken for convenience as the distance from the c.g. to the hinge axis.

The slope of the total resultant moment curve for the airplane may be written

$$\frac{dM}{d\alpha} = KqWC \quad (45)$$

where  $W$  is the gross weight,  $C$  the mean wing chord, and  $K$  a constant depending on the stability.

Assuming that  $\frac{dM}{d\alpha} = \frac{dM_w}{d\alpha} + \frac{dM_t}{d\alpha}$  and combining equations (43), (44), and (45) gives

$$KqWC = qS_w C \frac{dC_L}{d\alpha} \left[ \frac{a}{\bar{C}} - \left( C_p + \alpha_a \frac{dC_p}{d\alpha} \right) \right] - \frac{dC_{Lt}}{d\alpha_t} \cdot \frac{d\alpha_t}{d\alpha} q S_t l$$

Dividing by  $\left(qS_w C \frac{dC_{L_t}}{d\alpha_t} \cdot \frac{d\alpha_t}{d\alpha}\right)$  and arranging terms

$$\frac{S_t}{S_w} \cdot \frac{l}{C} = \frac{1}{\frac{dC_{L_t}}{d\alpha_t} \cdot \frac{d\alpha_t}{d\alpha}} \left\{ -K \left( \frac{W}{S_w} \right) + \left[ \frac{a}{C} - \left( C_p + \alpha_a \frac{dC_p}{d\alpha} \right) \right] \frac{dC_L}{d\alpha} \right\} \quad (46)$$

letting  $\frac{dC_{L_t}}{d\alpha_t} = F_1$ ,  $\frac{d\alpha_t}{d\alpha} = F_2$ ,  $\left( C_p + \alpha_a \frac{dC_p}{d\alpha} \right) = F_3$ , and  $\frac{dC_L}{d\alpha} = F_4$ , then equation (46) becomes

$$\frac{S_t}{S_w} \cdot \frac{l}{C} = \frac{1}{F_1 \cdot F_2} \left[ -K \left( \frac{W}{S_w} \right) + \left( \frac{a}{C} - F_3 \right) F_4 \right] \quad (47)$$

$F_1$  and  $F_4$  are the slopes of the lift curves for tail and wings respectively and depend on the effective aspect ratio and section.

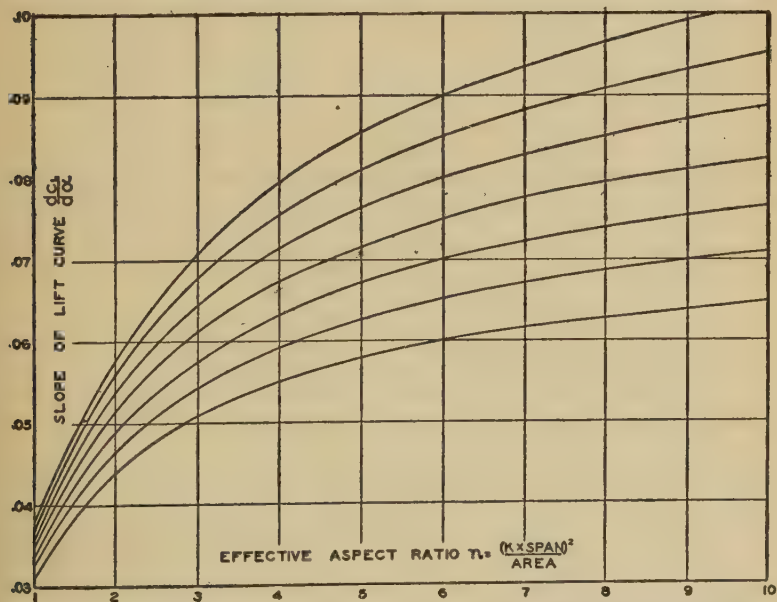


Figure 58. Variation of Slope of Lift Curve with Effective Aspect Ratio. Stability Factors  $F_1$  and  $F_4$

Values of  $F_1$  and  $F_4$  may be read from Figure 58. For example, if  $\frac{dC_L}{d\alpha} = .072$  at aspect ratio 6, then  $\frac{dC_L}{d\alpha} = .065$  at aspect ratio 4, or  $.076$  at aspect ratio 8. The variation with aspect

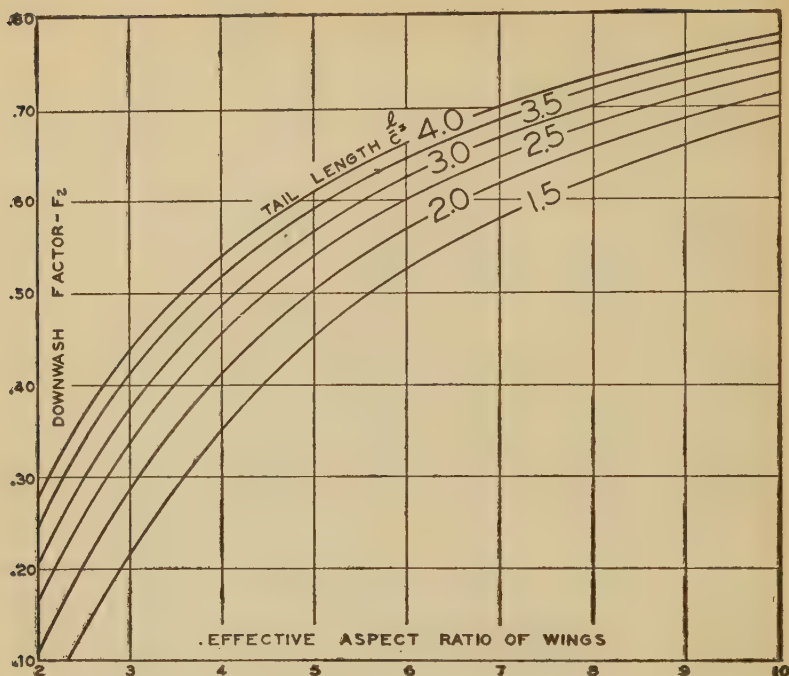


Figure 59. Variation of the (Downwash) Stability Factor  $F_2$  with Effective Aspect Ratio and Tail Length

ratio is along a curve similar to those shown and if  $\frac{dC_L}{d\alpha}$  is known for any aspect ratio, the value is found at any other aspect ratio by following along or in between the curves given. The average value of  $\frac{dC_L}{d\alpha}$  is about .072 at aspect ratio 6. Note that this is the effective aspect ratio  $n = \left(\frac{kb}{S}\right)^2$ . (See Chapter II for values of  $k$ .)

$F_3$  is a wing section stability factor depending on the camber and trailing edge curvature. Values of  $\frac{dC_L}{d\alpha}$  and  $F_3$  for many of the wing sections in common use are given in Table 8.

$F_2$  is a downwash factor depending on the effective aspect ratio of the wings and the tail length  $l$ , as shown on Figure 59.

The static stability constant  $K$  has been obtained from wind tunnel tests on a great number of satisfactory airplanes and varies between  $-.0005$  and  $-.0010$ . The former value is about the minimum for static stability at all speeds, while values greater than  $-.0010$  give excessive stability or "stiffness."

Equation (47) has given uniformly excellent results for design purposes. It is of interest to note that the left-hand side is the familiar "th." This is not a constant as many designers have assumed; it varies with design proportions. The author has found values varying from 0.25 to 1.0 and the extreme limits were certainly not attained.

The effect of each variable on the horizontal area required for constant static stability has been calculated. Figure 60 shows the effect of wing aspect ratio, Figure 61 the effect of tail aspect ratio, Figure 62 the effect of tail length, and Figure 63 the effect of fore-and-aft c.g. location.

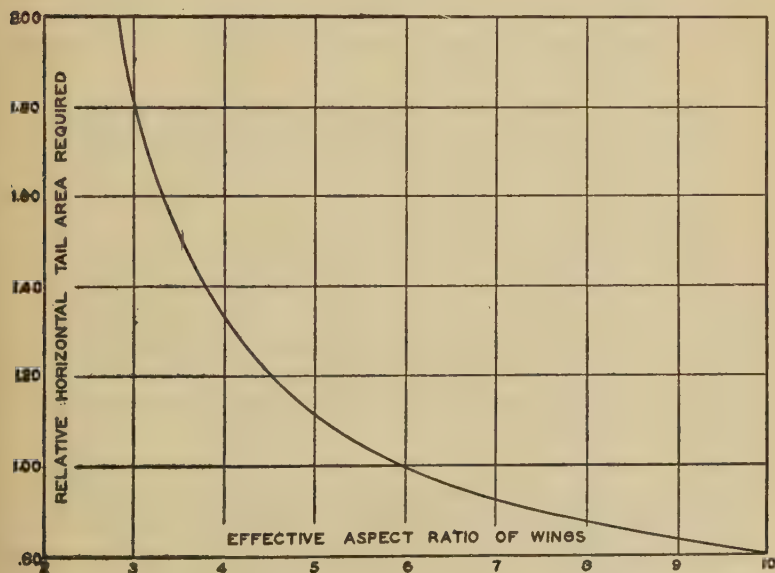


Figure 60. Effect of Wing Aspect Ratio on Horizontal Tail Area Required for Constant Static Stability

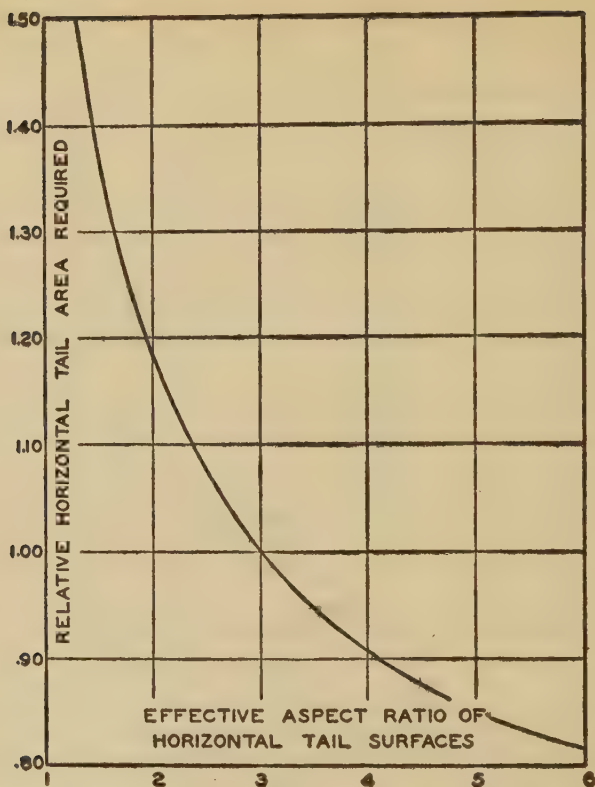


Figure 61. Effect of Tail Aspect Ratio on Tail Area Required for Constant Static Stability

For constant static stability the area of the horizontal tail surfaces continues to decrease as the c.g. is moved forward, but in order to provide control, the decrease cannot be utilized. Experience indicates that control and stability combine to give a minimum area for a c.g. location at about 30% of the mean chord.

*Second Method:* The horizontal tail area may also be calculated by writing moments about the c.g. at zero lift, when the downwash is zero. The wing moment is

$$M_w = C_{M_0} q S_w C \quad (48)$$



where  $C_{M_0}$  is the moment about the leading edge (or any point) at zero lift,  $S_w$  the total wing area, and  $C$  the mean chord, as before. The tail moment is

$$M_t = C_{Lt} q S_t l = \alpha_v \cdot \frac{d C_{Lt}}{d \alpha_i} \cdot q S_t l \quad (49)$$

where  $\alpha_v$  is the "effective longitudinal dihedral," measured between the no-lift lines of the wings and tail, and the other terms as in equation (44). The resultant moment on the airplane at zero lift should be given by

$$M_0 = K_0 q W C \quad (50)$$

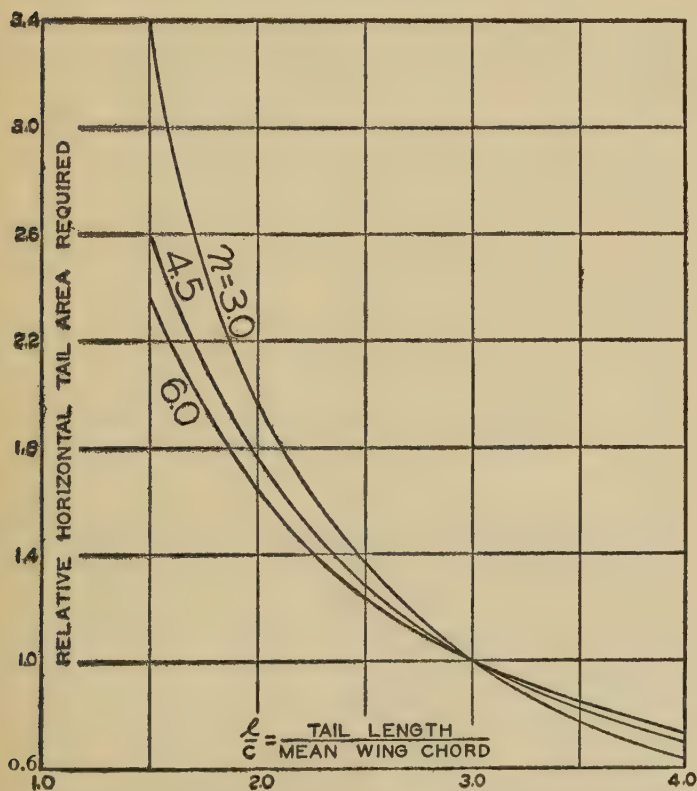


Figure 62. Effect of Tail Length on Tail Area Required for Constant Static Stability.  $n$  = Effective Aspect Ratio

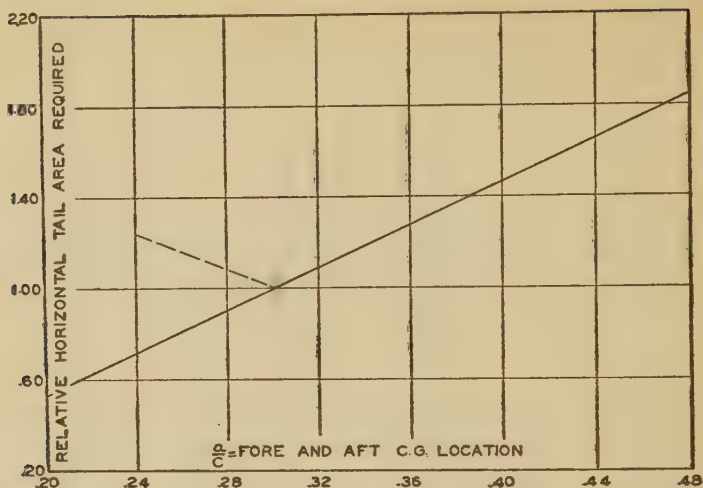


Figure 63. Effect of c.g. Location on Tail Area Required for Constant Static Stability

from equation (45). Equating these moments gives

$$\alpha_v \cdot \frac{dC_{Lt}}{d\alpha_t} \cdot q S_t l + C_{M_0} q S_w C = K_0 q W C$$

from which

$$\frac{S_t}{S_w} \cdot \frac{l}{C} = \frac{1}{\alpha_v F_1} \left[ K_0 \left( \frac{W}{S_w} \right) - C_{M_0} \right] \quad (51)$$

Analysis of wind tunnel test data shows that

$$K_0 = K \alpha_a'$$

where  $\alpha_a'$  is the absolute angle of attack for balance of moments and  $K$  a constant varying from about .0005 to .0010 according to the stability desired.  $K$  has the same value as in equation (45), but it is now positive, because  $\alpha_a'$  is considered positive. The analysis also shows a definite relation between the "stabilizer setting"  $\alpha_v$  and the absolute angle of attack for balance. The average value is

$$\alpha_v = (3.0^\circ + 0.25 \alpha_a')$$

Equation (51) may therefore be written

$$\frac{S_t}{S_w} \cdot \frac{l}{C} = \frac{1}{F_1 (3.0 + 0.25\alpha_a')} \left[ K_0 \alpha_a' \left( \frac{W}{S_w} \right) - C_{M_0} \right] \quad (52)$$

The moment coefficient at zero lift,  $C_{M_0}$ , is normally negative, and may be found in Table 8 for many of the wing sections in common use.

As an example of the calculation for horizontal tail area, consider an airplane having the following characteristics:

$$\left( \frac{W}{S_w} \right) = 11.20 \text{ lbs./sq. ft. Clark Y section}$$

$$\text{Effective wing aspect ratio} = 4.55$$

$$\text{Effective tail aspect ratio} = 3.35$$

$$\text{Tail length } \frac{l}{C} = 2.85$$

$$\text{Balance at } \alpha = 0^\circ \quad \alpha_a = 5.4^\circ$$

$$\text{C. G. location at } 32\% \text{ of mean chord.}$$

TABLE 8. WING SECTION STABILITY DATA

| Wing Section         | Slope of<br>Lift Curve<br>at Aspect<br>Ratio 6<br>$\frac{dC_L}{d\alpha}$ | $\left( C_p - \alpha_a \frac{dC_p}{d\alpha} \right)$<br>$F_3$ | Moment<br>Coefficient<br>at<br>Zero Lift<br>$C_{M_0}$ |
|----------------------|--------------------------------------------------------------------------|---------------------------------------------------------------|-------------------------------------------------------|
| R. A. F.-15.....     | .077                                                                     | .25                                                           | -.052                                                 |
| Sloane.....          | .080                                                                     | .24                                                           | -.060                                                 |
| Clark Y.....         | .071                                                                     | .23                                                           | -.080                                                 |
| Göttingen 387.....   | .072                                                                     | .21                                                           | -.095                                                 |
| Göttingen 398.....   | .072                                                                     | .23                                                           | -.079                                                 |
| Göttingen 420.....   | .075                                                                     | .21                                                           | -.090                                                 |
| Göttingen 436.....   | .072                                                                     | .23                                                           | -.078                                                 |
| U. S. A.-27.....     | .071                                                                     | .23                                                           | -.086                                                 |
| U. S. A.-35A.....    | .073                                                                     | .20                                                           | -.120                                                 |
| Navy N-9.....        | .072                                                                     | .24                                                           | -.060                                                 |
| Navy N-10.....       | .080                                                                     | .23                                                           | -.075                                                 |
| Navy N-22.....       | .074                                                                     | .22                                                           | -.083                                                 |
| N. A. C. A. M-6..... | .072                                                                     | .27                                                           | +.010                                                 |
| N. A. C. A. M-12.... | .074                                                                     | .26                                                           | -.005                                                 |

From these data

$$F_1 = 0.0613$$

$$F_2 = 0.520$$

$$F_3 = 0.23$$

$$F_4 = 0.0662$$

from which by equation (47)

$$\begin{aligned} \frac{S_t}{S_w} \cdot \frac{l}{C} &= \frac{1}{.0613 \times 0.520} [0.0006 \times 11.20 + (0.32 - 0.23) \times 0.662] \\ &= 0.397 \end{aligned}$$

and by equation (52)

$$\begin{aligned} \frac{S_t}{S_w} \cdot \frac{l}{C} &= \frac{1}{.0613 (3.0 + .25 \times 5.4)} [0.0006 \times 5.4 \times 11.20 + .080] \\ &= 0.434 \end{aligned}$$

a coefficient of about 0.42 is probably a good compromise.

**Elevator and stabilizer areas.** The best distribution of area between the stabilizer and elevators obviously depends on several design factors, but in general the elevator area should lie between 35% and 50% of the total. The recommended range is between 40% and 45%. In a seaplane, values below 40% usually lead to deficient control in take-off.

**Tail-surface sections.** If tail surfaces are to give maximum stabilizing and control effects, the section must be as thin as practicable. The chief disadvantages of thick surfaces are: (1) low value of  $\frac{dC_L}{d\alpha}$ ; (2) low full-scale maximum  $C_L$  (or  $C_N$ ); (3) unsteady air flow leading to irregularities in moment curves and control effect; and (4) sensitiveness to slight irregularities in section due to method of construction or to handling in service.

It is particularly desirable to avoid rapid changes in section. The sections should preferably be geometrically similar, but slight deviations for structural reasons are allowable. Overhanging balances must have reasonably thin sections.

Wind tunnel tests indicate that aerodynamically there is no one section that is notably superior to the others. In the matter of minimum drag, the Göttingen sections Nos. 443, 444,

and 445 show up very well, but these are not entirely satisfactory in regard to beam depths. A section similar to that shown in Figure 64 is apparently the best compromise between aerodynamic and structural requirements. This section is shown

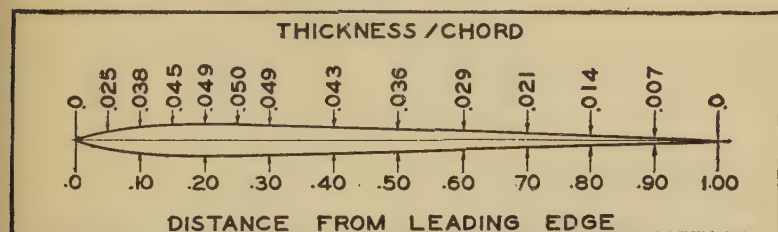


Figure 64. Airfoil Section for Tail Surfaces

with sharp leading and trailing edges, but any reasonable radius may be used on either, or both, leading and trailing edges. The sharp edge is not recommended. The maximum thickness should not be greater than about 8% of the chord.

**Plan-form of horizontal tail surfaces.** Figure 65 gives the most frequently used plan-forms for horizontal tail surfaces. Form A is approximately rectangular with a modified elliptical tip. Form B has a circular or elliptical tip with raked, but straight, leading, and trailing edges. Form C is elliptical, and Form D is approximately parabolical or triangular in shape.

**Vertical tail surface required.** The only certain method of designing vertical tail surfaces is by wind tunnel tests, but a close approximation may be secured by a simple formula. Let the full-scale yawing moment at, say,  $10^\circ$  yaw, be  $N_{10}$  lbs.-ft., the total vertical tail area  $S_V$  sq. ft., and the tail length (from C.G. to rudder hinge axis)  $l$  ft. Then it may be expected that

$$\frac{N_{10}}{10} = \left( \frac{dN}{d\psi} \right)_e = k_n q S_V l \quad (53)$$

If  $k_n$ , as obtained from wind tunnel tests be plotted against  $l$ , the points are found to fall into three fairly definite groups representing landplanes, seaplanes, and flying boats.

$$q = \frac{1}{2} \times 1.225 \times (5280)^2 \times 0.00255$$

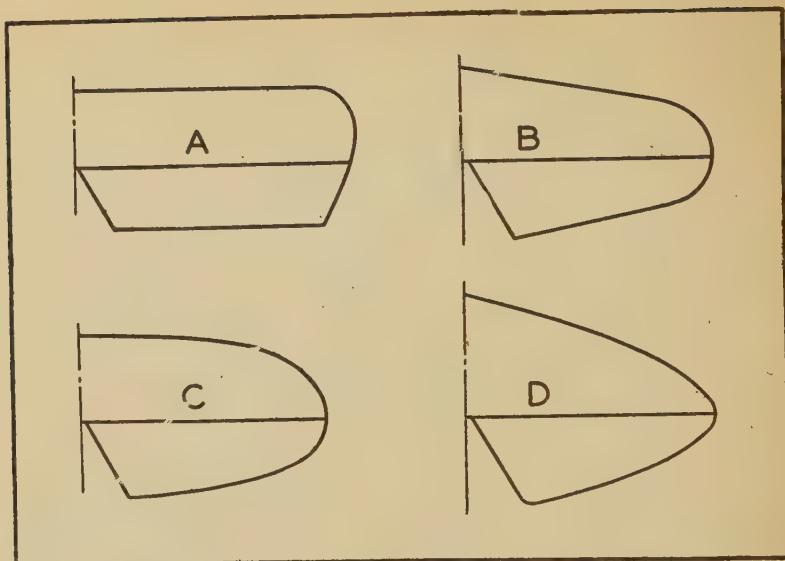


Figure 65. Tail Surface Plan-Forms

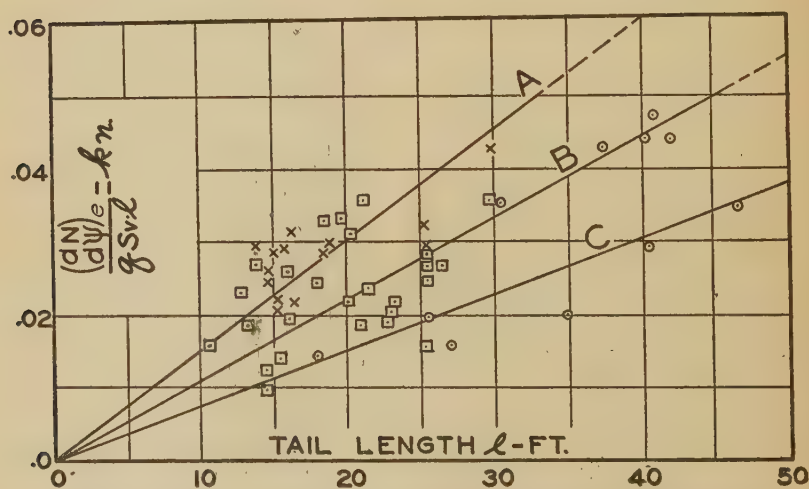


Figure 66. Effect of Tail Length on Yawing Moment Slope Coefficient



Figure 66 is a plot of  $k_n$  against  $l$  for about fifty airplanes for which wind tunnel data are available.  $k_n$  appears to vary directly as  $l$  and has average values of .0015 for landplanes, .0011 for seaplanes, and .00075 for flying boats. From which  $k_n = K_l l$  and for

$$\text{Landplanes} \quad \left(\frac{dN}{d\psi}\right)_e = .0015 q S_V l^2 \quad (54)$$

$$\text{Seaplanes} \quad \left(\frac{dN}{d\psi}\right)_e = .0011 q S_V l^2 \quad (55)$$

$$\text{Flying Boats} \quad \left(\frac{dN}{d\psi}\right)_e = .00075 q S_V l^2 \quad (56)$$

It will be noted that the points for part of the flying boats are on line B in Figure 66. These have a comparatively short length of hull forward of the wings, as compared with the older types.  $\left(\frac{dN}{d\psi}\right)_e$  is the effective slope of the yawing moment curve for an appreciable angle of yaw, and is used in preference to the actual slope at  $0^\circ$  yaw on account of the difficulty in measuring the latter.

It is logical to assume that the desirable slope of the moment curve should be proportional to the type of airplane, its gross weight, and maximum span. That is

$$\frac{dN}{d\psi} = K_N q W b \quad (57)$$

Analysis of wind tunnel test data on about fifty airplanes, in the light of flight test reports on the same airplanes, shows that the best values of  $K_N$  are between  $4 \times 10^{-5}$  and  $8 \times 10^{-5}$ . The desirable limits for various types of airplanes appear to be

Pursuit,  $K_N$  between  $4 \times 10^{-5}$  and  $6 \times 10^{-5}$

Observation,  $K_N$  between  $5 \times 10^{-5}$  and  $7 \times 10^{-5}$

Bombing, training, commercial, etc.,  $K_N$  between  $6 \times 10^{-5}$  and  $10 \times 10^{-5}$

$\left(\frac{dN}{d\psi}\right)_e$  from equation (53) and  $\left(\frac{dN}{d\psi}\right)$  from equation (57) may now be equated, giving

$$K_{1q}S_V l^2 = K_{Nq}Wb$$

from which

$$S_V = \frac{K_N}{K_1} \frac{Wb}{l^2} \quad (58)$$

The values of  $K_N$  have just been given, while  $K_1$  is .0015 for landplanes, .0011 for seaplanes, and .00075 for flying boats. For equal directional stability, a seaplane requires about 35% more, and a flying boat 100% more, vertical tail surface than a landplane of the same gross weight, span, and tail length.

As an example of calculation for vertical tail area, consider a pursuit type landplane for which  $W = 2,800$  lbs.,  $b = 32$  ft., and  $l = 15.0$  ft.  $K_1 = .0015$  and  $K_N$  should be about  $5 \times 10^{-5}$ .

$$\therefore S_V = \frac{5 \times 10^{-5}}{.0015} \times \frac{2,800 \times 32}{225} = 13.3 \text{ sq. ft.}$$

As a seaplane, this design would probably weigh about 10% more, or  $W = 3,080$  lbs. For equal directional stability, since  $K_1 = .0011$  for a seaplane:

$$S_V = \frac{5 \times 10^{-5}}{.0011} \times \frac{3,080 \times 32}{225} = 19.9 \text{ sq. ft.}$$

If the same vertical surface is to be used on both landplane and seaplane, the area should be about 15 or 16 sq. ft.

**Rudder and fin area.** The correct division of the vertical area between the rudder and fin depends upon the type of airplane; landplane, seaplane, or flying boat. It also depends upon the length and shape of the fuselage, hull, and floats, and upon the wing arrangement. The only satisfactory method is to make wind tunnel tests, but analysis of available wind tunnel test data shows that a majority of airplane designs will be satisfactory with the following proportions:

| Type              | Rudder Area         |
|-------------------|---------------------|
|                   | Total Vertical Area |
| Landplanes.....   | .55 to .60          |
| Seaplanes.....    | .50 to .55          |
| Flying Boats..... | .45 to .50          |

**Aileron area required.** The effectiveness of an aileron, as measured by the rolling moment due to a given setting, varies with the aileron span, and chord. It may be shown<sup>1</sup> from test data that the variation of efficiency with chord is given by

$$\frac{\eta_1}{\eta_0} = 1.50 - 2.00 \left( \frac{t}{c} \right) \quad (59)$$

where  $t$  is the aileron chord and  $c$  the total wing chord, including

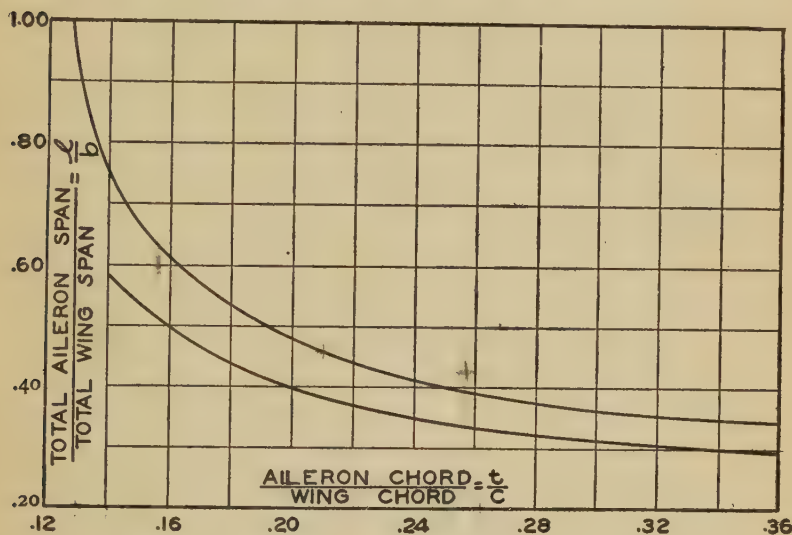


Figure 67. Aileron Proportions. Relative Span and Chord

the aileron chord. In a similar manner the variation of efficiency with span is

$$\frac{\eta_2}{\eta_0} = 1.20 - 0.60 \left( \frac{l}{b} \right) \quad (60)$$

where  $l$  is the aileron span and  $b$  the wing span.

Assuming that satisfactory control is given by an aileron of the proportions  $\frac{t}{c} = 0.25$  and  $\frac{l}{b} = 0.40$ , it is possible to calculate

<sup>1</sup> W. S. Diehl, "Notes on the Design of Ailerons," N. A. C. A. Technical Note No. 144 (1923).

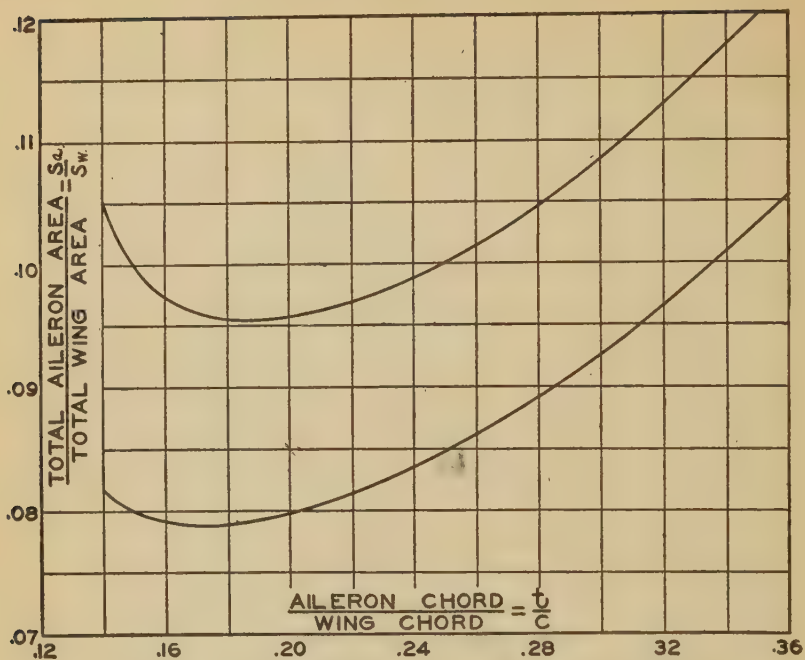


Figure 68. Aileron Proportions. Relative Area and Chord

the proportions of all other ailerons having the same effectiveness in producing rolling moments. The assumed aileron has  $\frac{S_a}{S_w} = 0.10$ , where  $S_a$  is the aileron area and  $S_w$  the wing area. The relative efficiencies are  $\frac{\eta_1}{\eta_0} = 1.00$  and  $\frac{\eta_2}{\eta_0} = 0.96$ , and the "effective area" is the product of these efficiencies by the actual area, or

$$S_e = 0.10 \times 1.00 \times 0.96 = .096$$

The proportions of all ailerons having this effective area are given by

$$t \left[ 1.50 - 2.00 \left( \frac{t}{c} \right) \right] \cdot l \left[ 1.20 - 0.60 \left( \frac{l}{b} \right) \right] = .096 bc$$

If the satisfactory aileron proportions be assumed as  $\frac{t}{c} = 0.20$  and  $\frac{l}{b} = 0.40$ , the proportions of equivalent ailerons are given by

$$t \left[ 1.50 - 2.00 \left( \frac{t}{c} \right) \right] \cdot l \left[ 1.20 - 0.60 \left( \frac{l}{b} \right) \right] = .0843 bc$$

Figure 67 is a plot of  $\frac{l}{b}$  against  $\frac{t}{c}$  and Figure 68 is a plot of  $\frac{S_a}{S_w}$  against  $\frac{t}{c}$  as calculated from the two equations. These are not the extreme limits for satisfactory ailerons, but the best results have been obtained from approximately the proportions given.

The aileron when displaced from neutral affects the lift distribution over the whole wing, but chiefly over the part of which it forms the trailing edge. If the area thus affected is  $S_e$ , with the center of this area at distance  $d$  from the fore-and-aft axis, then a convenient measure of control effectiveness is given by

$$K_a = \frac{d}{b} \cdot \frac{S_e}{S_w} \quad (61)$$

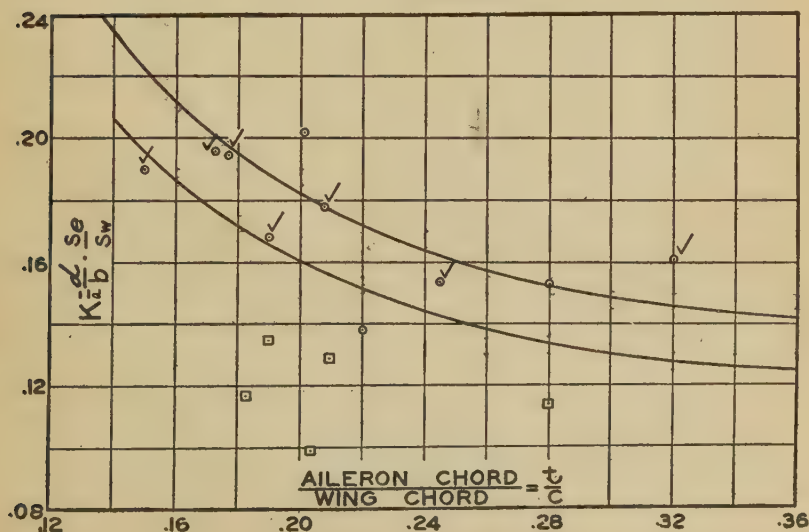


Figure 69. Aileron Coefficient  $K_a$

Figure 69 contains a plot of  $K_a$  against  $\frac{t}{c}$  for the two effective areas previously considered. The points on this figure represent recent designs. Airplanes which were reported to have excellent lateral control are indicated by circles with check marks ( $\checkmark$ ). A plain circle indicates no particular comment, while a square indicates unfavorable comment. An aileron design based on  $K_a$  between the limits shown on Figure 69 will certainly be satisfactory, in so far as area is concerned.

The aileron chord should lie between the extreme limits of  $\frac{t}{c} = 0.16$  and  $\frac{t}{c} = 0.28$ . The most desirable chord, aerodynamically, is between  $\frac{t}{c} = 0.20$  and  $\frac{t}{c} = 0.25$ .

**Aileron types.** Figure 70 illustrates eight types of wing tips and ailerons. Types A and C are simple ailerons on elliptical tips. These are very satisfactory, and should be used whenever practicable on all high-speed airplanes. Type B is a simple aileron on a modified elliptical tip. This aileron is slightly improved if the outer tip is cut away as shown by the dotted line. Type D is an inset aileron on a modified elliptical tip. This type does not extend to the wing tip. It is desirable on wings of great span and unusual flexibility.

Types E, F, G, and H are undesirable for various reasons. All of these tend to be "heavy" on control owing to the peak in loading at the tip. Types G and H should never be used on very high-speed designs owing to probability of flutter.

Care must be taken in the design of hinges for long, narrow ailerons in order to avoid binding when the wings deflect under load.

**Aileron moments.** The best data available at this time on rolling moments and aileron hinge moments are given in R. & M. Nos. 550 and 615.<sup>2</sup> These data were obtained at a test speed of

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<sup>2</sup> "An Investigation of the Aerodynamic Properties of Wing Ailerons," Part I by H. B. Irving, E. Ower, and G. A. Hankins (1918), and Part II by Irving and Ower. (1919)



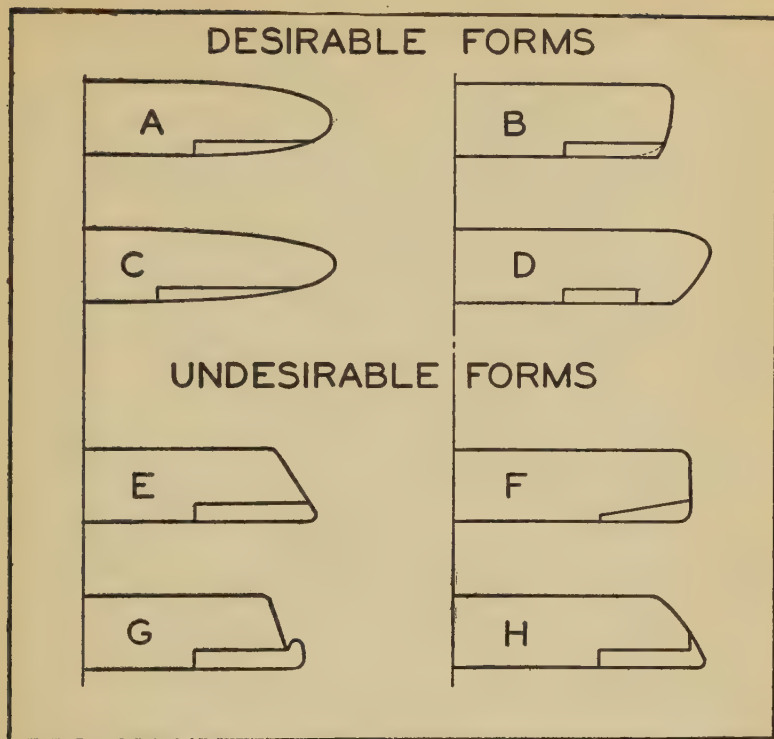


Figure 70. Aileron Types

40 ft./sec. on a biplane, gap = chord, no stagger and both wings 6" x 36", R. A. F.-15. The wing tips were square (with a small radius at each corner, however) and ailerons were fitted on both wings. From these tests it appears that the hinge moment does not change very much with angle of attack. Figure 71 is a plot of the hinge moment coefficient  $K_H$  against the ratio  $\frac{t}{c}$  for various aileron angles at an angle of attack  $\alpha_w = 8^\circ$ . The hinge moment for any condition is

$$H = K_H \cdot \frac{\rho}{2} \cdot C S_a V^2 \quad (62)$$

where  $C$  is the wing chord,  $S_a$  the total aileron area, and  $V$  the

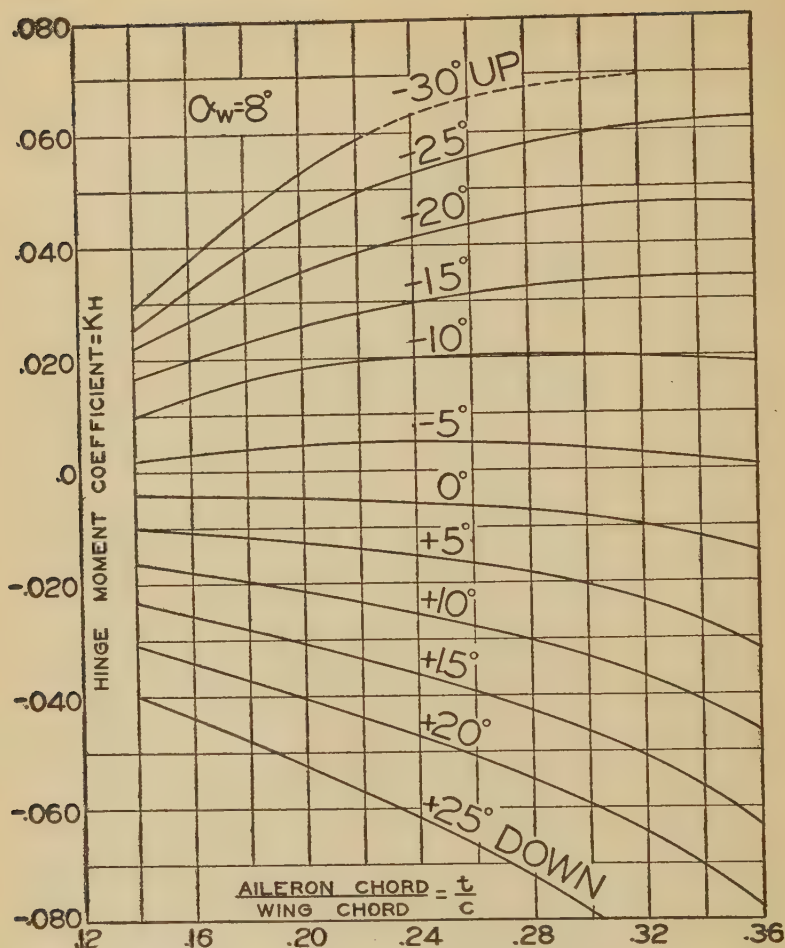


Figure 71. Aileron Hinge Moment Coefficients

air speed.  $K_H$  must be taken as the average for the "up" and "down" aileron angles. This method of plotting allows differential movements to be studied.

In R. & M. No. 515 the coefficient for rolling moment per unit aileron area is defined by the equation

$$L = K_L \frac{\rho}{2} b S_a V^2 \quad (63)$$

where  $b$  is the span and  $S_a$  the total aileron area. This form is about as satisfactory as any that can be written and it has the advantage of showing that the decrease in efficiency is almost exactly equal to the decrease in moment arm from the center line of the airplane to the center of the area affected by the aileron. That is, if instead of the span  $b$ , the moment arm  $d$  were used,  $K_L$  would show no appreciable variation with aileron

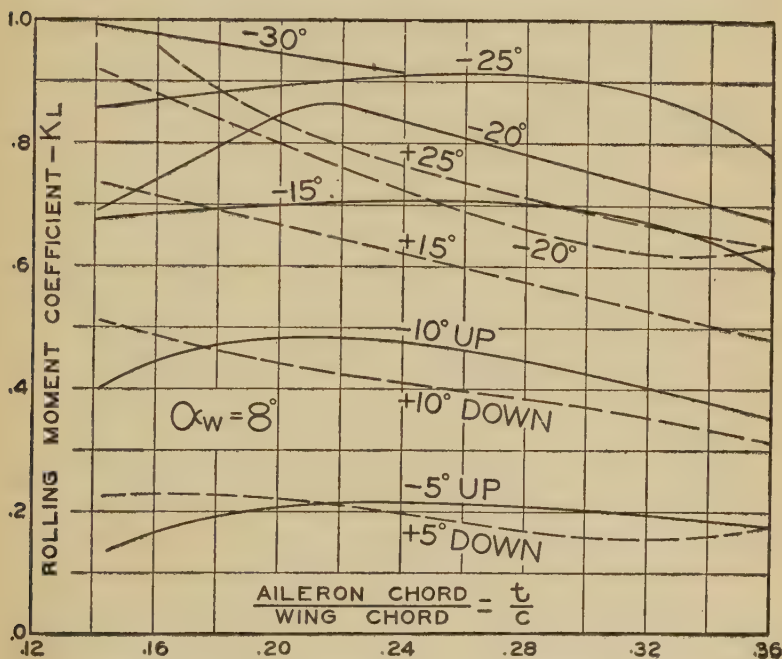
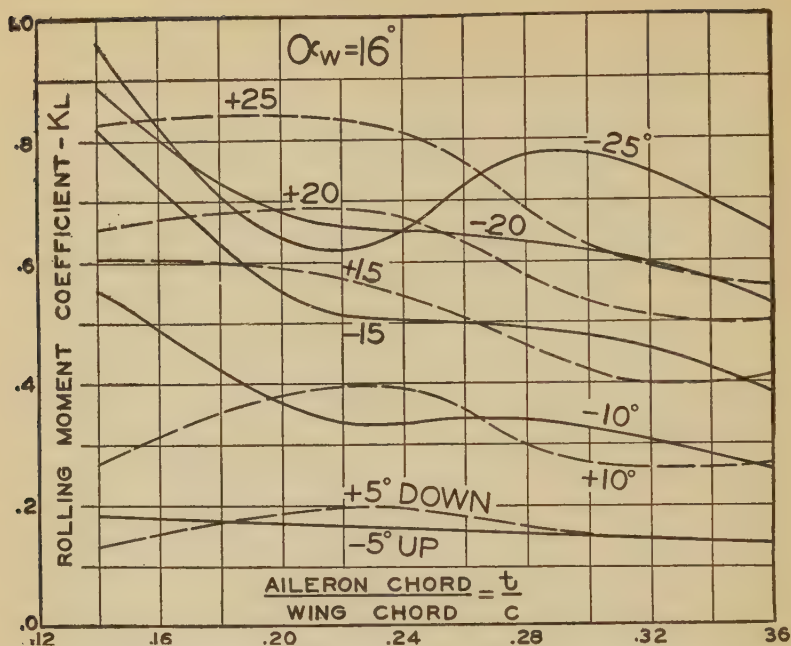


Figure 72. Aileron Rolling Moment Coefficients  $\alpha = 8^\circ$

span. This may be done by using instead of  $b$ , the factor  $d = \left( \frac{2b - b_a}{2} \right)$ , where  $b_a$  is the total span of the ailerons.

Values of  $K_L$  for various aileron angles are plotted against  $\frac{t}{c}$  in Figures 72 and 73 for angles of attack of  $8^\circ$  and  $16^\circ$  respectively. These values of  $K_L$  are based on ailerons of 6" span on a wing of 36" span. Consequently, the actual effective arm was

Figure 73. Aileron Rolling Moment Coefficients  $\alpha = 16^\circ$ 

$\frac{5}{8}b$  instead of  $b$  and for exact results with any aileron span, the rolling moments should be obtained from

$$L = K_L \frac{\rho}{2} d S_a V^2 \quad (64)$$

where  $d = \left( \frac{2b - b_a}{2} \right) \frac{6}{5} = (1.2b - 0.6b_a)$ . Note the relation between  $d$  and the variation of the efficiency with span, equation (60).

**Balanced controls.** Elevators, rudders, and ailerons are often "balanced" in order to reduce control forces. For small or moderate-sized airplanes the chief reason for balancing is to improve maneuverability, but for large airplanes it is necessary in order that the control forces do not exceed the pilot's strength. The most desirable type of balance varies with the use and size of the control surface. For example, what is probably the most

satisfactory type of balance for a rudder is unsuited for an aileron on a high-speed airplane.

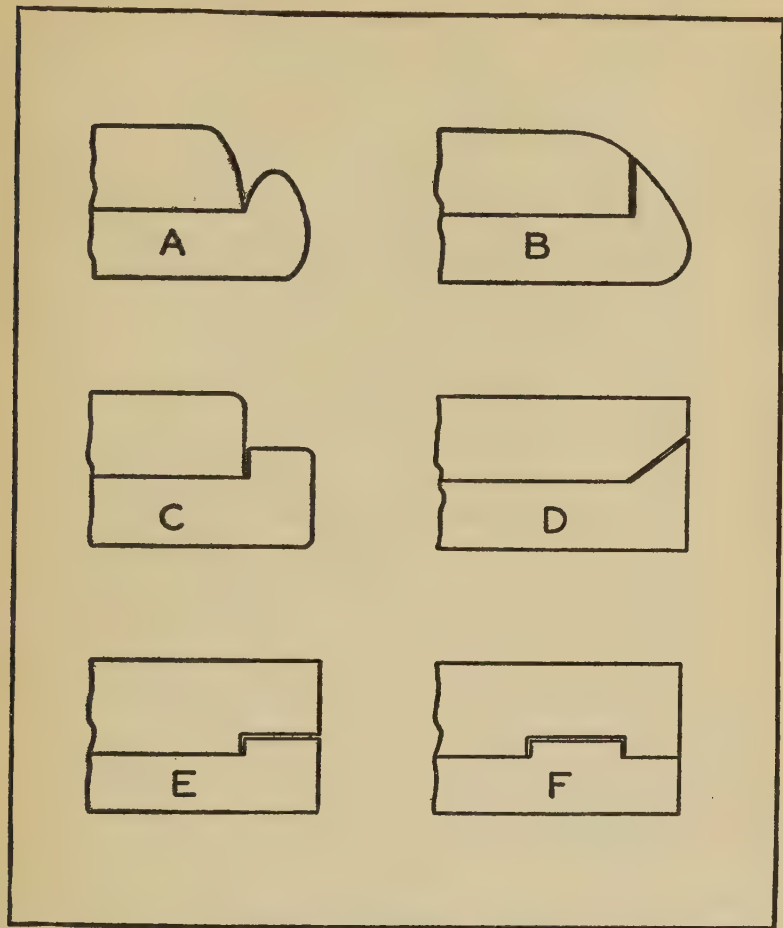


Figure 74. Balanced Controls

In general, no type of aileron balance should be considered satisfactory unless it allows the use of an efficient wing tip, and tends to bring the aileron c.g. near to the hinge axis. Likewise, no type of rudder balance can be considered satisfactory if it reduces the maximum rudder control, or in any way reduces the

rudder control. Elevator balance requirements are similar to those for rudders.

**Types of balanced controls.** Figure 74 shows six types of balances. Types A and B were formerly used on all controls, but they are now used only on rudders and elevators. There is little choice between them, but type B is the more common at present. Neither A nor B should be used on ailerons owing to the poor wing tip and to the high peak loadings on the balanced portion. Type C is liable to overbalance and flutter. Type D has no advantages and is rarely used. Types E and F have been used to a limited extent, but have no outstanding advantage.

Figure 75 shows three widely used forms of balances. Type I is usually called the "Handley Page balance." It is satisfactory for control surfaces that need not be moved through large angles. With a normal airfoil section, the leading edge of the control emerges from the wake of the fixed surface at a control angle of about  $15^\circ$ . Above this critical angle no appreciable increase in control is obtained. For best results, the hinge axis should be located at approximately 25% of control chord from its leading edge. Type II is the familiar "paddle" balance. Full balance cannot easily be secured with this type but it has been fairly satisfactory on all surfaces. The auxiliary surface should be of large aspect ratio, and as far forward as practicable. The area should be generous for good balance. A symmetrical section is generally used, and on ailerons it is set at an angle of  $+3^\circ$  to  $+5^\circ$  to the wing chord. On rudders and elevators, the auxiliary surface would be set between  $0^\circ$  and  $1^\circ$  to the main surface axis. The amount of balance is readily calculated.

Type III is the Frise balance. This is the most satisfactory form of balance now available for ailerons. When the aileron is up, the leading edge projects below the wing and thus adds a drag which tends to counteract the yawing moment due to the drag of the down aileron. The hinge axis is preferably located below the aileron center-line and at about 22% to 25% of the



aileron chord. The hinge axis should not be further aft than 25% of the aileron chord.

The effective area of a balanced control surface of the Handley Page or the Frise type is measured between the hinge

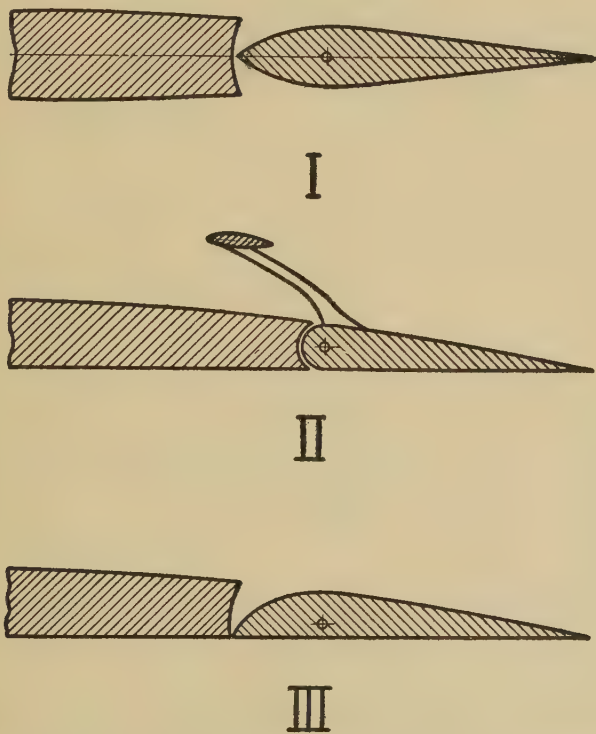


Figure 75. Balanced Controls

axis and the trailing edge. That part of the surface forward of the hinge axis does not contribute to the control effect.

**Calculation for simple balances.** For a long time naval architects have balanced rudders by assuming: (1) uniform loading,

(2) that the center of pressure on a flat plate is at  $0.20 C$ , (3) that the center of pressure on a movable surface of chord  $C_1$  which trails a fixed surface is at  $0.33 C_1$ . With these assumptions the average center of pressure may be calculated and the axis placed

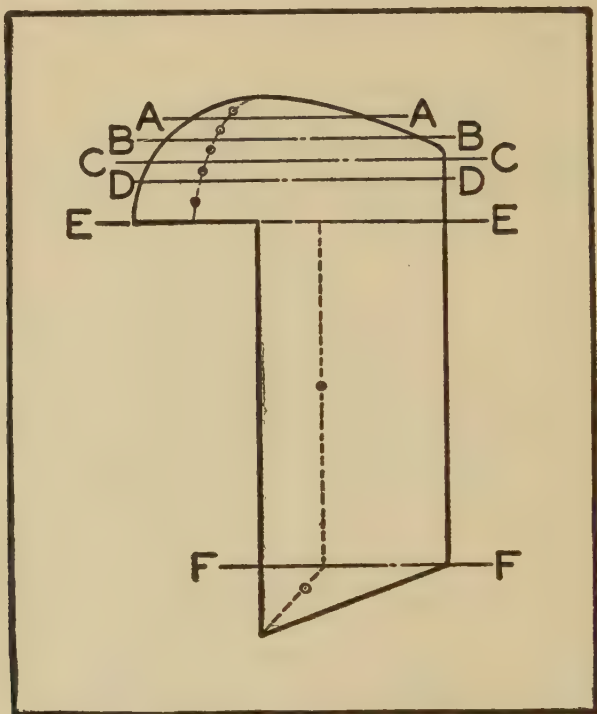


Figure 76. Illustrating Calculation for Rudder Balance

forward of this point at a distance sufficient to avoid overbalance as determined by accumulated experience. This method is applicable to air as well as water, and it gives excellent results.

As applied to a rudder balance, for example, consider Figure 76. The rudder is divided into a number of strips  $A, B, C, D$ , etc., spaced as necessary in order to secure accuracy. The center of pressure of each strip is located as indicated by the circles, assuming  $C_p$  at  $0.20 C$  for all of the overhanging strips and at  $0.33 C_1$  for all of the trailing strips. The area of each strip is

then multiplied by the distance of its  $C_p$  from the axis, considering distances forward as negative. The algebraic sum of these moments divided by the total area is the average distance of the  $C_p$  from the axis. This distance should be between 4 in. and 6 in. for satisfactory balance, the distance decreasing slightly with the size of the airplane. Four and one-half inches to five inches is probably the best location for any airplane having a gross weight between 2,500 and 10,000 lbs. If the first calculation shows the center of pressure outside of the desired range, balance is added or subtracted as required and additional calculations made until the desired location is obtained.

## CHAPTER VI

### ENGINE AND PROPELLER CONSIDERATIONS

**General power curves.** If the ratio  $\frac{\text{B.Hp. at given r.p.m.}}{\text{B.Hp. at 1,800 r.p.m.}}$  be plotted against r.p.m. for various modern airplane engines, it is found that a single curve may be drawn, from which no point differs appreciably more than the variation between individual engines on test. Such a curve, based on the average power curves for nine engine types, is given in Figure 77. This curve may be expanded as in Figure 78 to take care of the maximum r.p.m. actually used. Figure 78 is of great value in performance estimation when specific engine data are not obtainable.

It should be noted that most of the engines designed prior to about 1920 delivered their maximum power at what is now a normal operating r.p.m. For example, the Liberty "peaked" at less than 2,000 r.p.m. Curve three on Figure 78 should be used in this case, and the value of  $N$  taken as about 1,750 r.p.m. instead of the 2,400 r.p.m. there given.

**Variation of B.Hp. with altitude.** The variation of B.Hp. with altitude may be obtained either from altitude chamber tests or free flight tests. Using altitude chamber test data it can be shown<sup>1</sup> that at constant r.p.m.

$$\begin{aligned}\frac{B.Hp.}{B.Hp._0} &= \left(\frac{P}{P_0}\right)^{1.15} \text{ Temperature constant} \\ \text{and } \frac{B.Hp.}{B.Hp._0} &= \left(\frac{T}{T_0}\right)^{-0.5} \text{ Pressure constant} \\ \text{or } \frac{B.Hp.}{B.Hp._0} &= \left(\frac{P}{P_0}\right)^{1.15} \left(\frac{T}{T_0}\right)^{-0.5} \quad (65)\end{aligned}$$

<sup>1</sup> W. S. Diehl, "Engine Performance and The Determination of Absolute Ceiling," N. A. C. A. Technical Report No. 171.

In the standard atmosphere, pressure and temperature are connected by the relation

$$\left(\frac{T}{T_0}\right) = \left(\frac{P}{P_0}\right)^{0.19} \quad (66)$$

and pressure and density by the relation

$$\left(\frac{P}{P_0}\right) = \left(\frac{\rho}{\rho_0}\right)^{1.235} \quad (67)$$

Therefore, in the standard atmosphere

$$\frac{B.Hp.}{B.Hp._0} = \left(\frac{P}{P_0}\right)^{1.055} = \left(\frac{\rho}{\rho_0}\right)^{1.305} \quad (68)$$

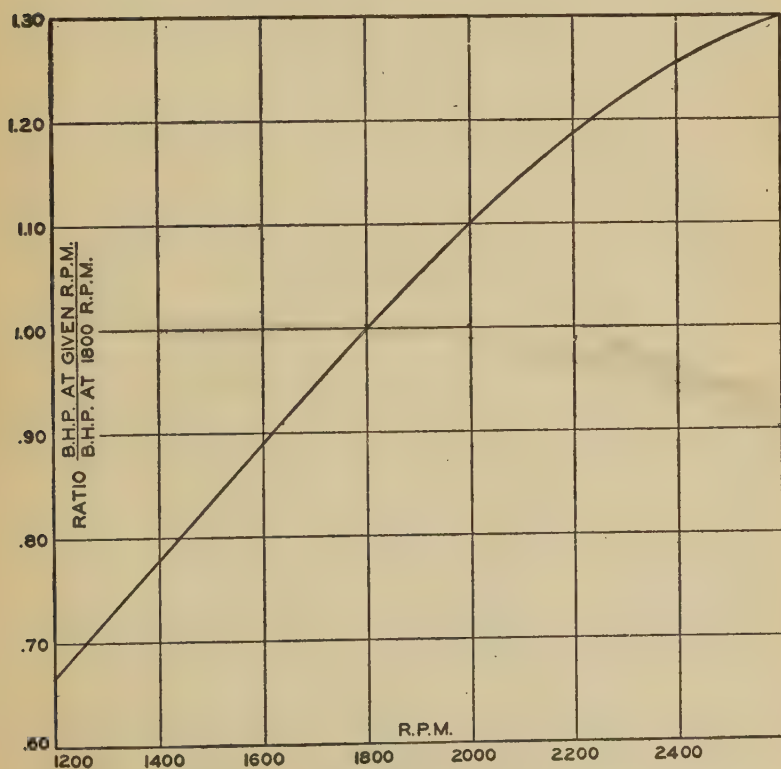


Figure 77. General Brake Horsepower Curve

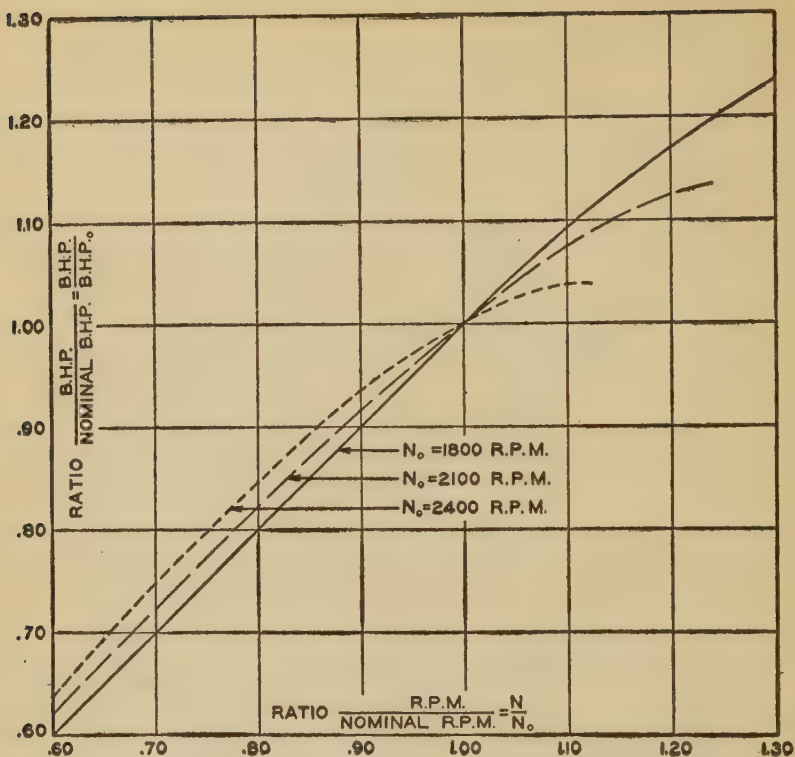


Figure 78. General Brake Horsepower Curves

As a result of the N. A. C. A. flight tests on a DH-4 airplane fitted with a Bendemann dynamometer hub on the Liberty engine<sup>2</sup>, Gove and Green recommend the relation

$$\frac{B.H\phi.}{B.H\phi._0} = 1.088 \frac{\rho}{\rho_0} - .088 \quad (69)$$

which is given in the "Handbook of Instructions for Airplane Designers."<sup>3</sup>

<sup>2</sup> W. D. Gove and M. W. Green, "The Direct Measurement of Engine Power on an Airplane in Flight with a Hub Dynamometer," N. A. C. A. Technical Report No. 252 (1926).

<sup>3</sup> Engineering Division, Air Service, McCook Field (4th edition, 1925).



A replot of the N. A. C. A. data on logarithmic paper shows that it would probably be better to write

$$\frac{B.H.p.}{B.H.p._0} = \left( \frac{\rho}{\rho_0} \right)^{1.20} \quad (70)$$

and in fact  $(1.088 \frac{\rho}{\rho_0} - .088)$  is almost exactly equivalent to  $\left( \frac{\rho}{\rho_0} \right)^{1.15}$ . In view of the fact that these data do not extend to altitudes above 13,000 ft., while pressures and temperatures were not recorded in the report, there is some question as to what was actually measured, or rather to what altitudes or densities the measured data correspond.

The actual difference between equations (68) and (69) is not great, as the following comparison will show:

| Altitude<br>ft. | $\frac{\rho}{\rho_0}$ | $\left( \frac{\rho}{\rho_0} \right)^{1.20}$ | $\left( \frac{\rho}{\rho_0} \right)^{1.305}$ | $\Delta h$ |
|-----------------|-----------------------|---------------------------------------------|----------------------------------------------|------------|
| 0               | 1.000                 | 1.000                                       | 1.000                                        | 0          |
| 5,000           | .862                  | .836                                        | .824                                         | -400       |
| 10,000          | .738                  | .695                                        | .673                                         | -800       |
| 15,000          | .629                  | .573                                        | .547                                         | -1,100     |
| 20,000          | .533                  | .471                                        | .442                                         | -1,600     |
| 25,000          | .448                  | .373                                        | .351                                         | -1,300     |

$\Delta h$  is the difference in altitude represented by the exponent. A difference of about 8% in altitude or 6% in density is indicated.

Equations (65) and (68) are believed to give the variation of B.Hp. with altitude (at constant r.p.m.) as accurately as required for any engineering purpose. The variation of T.Hp. with altitude is given later.

**Specific fuel consumption.** The specific fuel consumption (lbs./B.Hp./hr.) varies with the condition of an engine and the carburetor adjustment, but the effects of throttling, compression ratio, and mixture control are fairly definite.

The effects of throttling with and without mixture control are given on Figure 79, which is based on the average of a great number of ground tests. The variation of specific fuel con-

sumption with compression ratio is given on Figure 80 for the two conditions of normal mixture and lean mixture. The latter is about the minimum that can be obtained in flight operation,

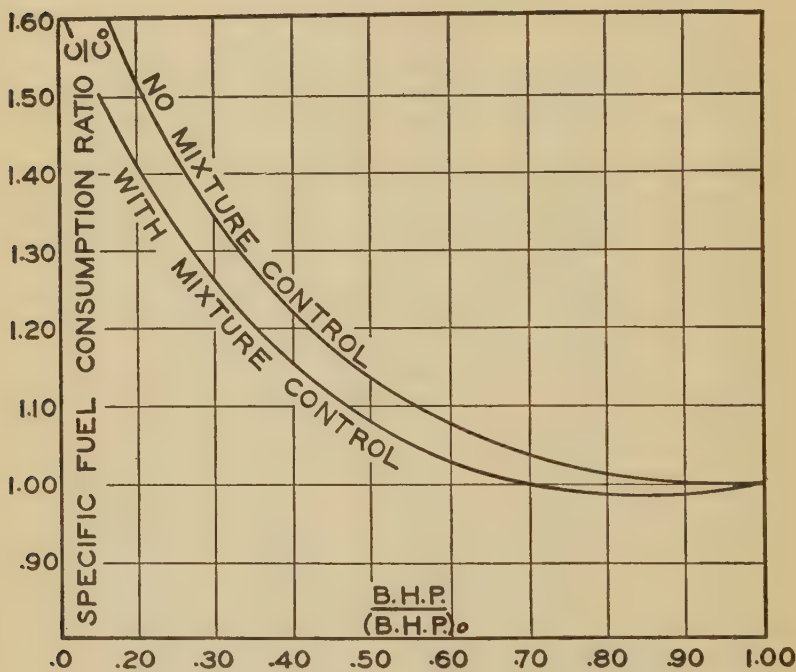


Figure 79. Specific Fuel Consumption at Part Throttle

although it is possible to secure slightly lower values on the test stand.

The variation in specific fuel consumption with altitude is given on Figure 81. This curve is based on the Bureau of Standards altitude chamber tests.<sup>4</sup>

**Correcting test-stand data to standard conditions.** Since the brake horsepower of an engine varies with pressure and temperature, it is necessary, in general, to correct the observed data for deviations from the standard  $P = 29.92''$  Hg.,  $t = 15^{\circ}\text{C}$ . While

<sup>4</sup> N. A. C. A. Technical Reports Nos. 102 and 103.

altitude chamber test data indicate that the  $B.H.p. \propto P^{1.15} T^{-0.5}$ , the deviations from standard pressure during a test are normally quite small and by general agreement the correction is made

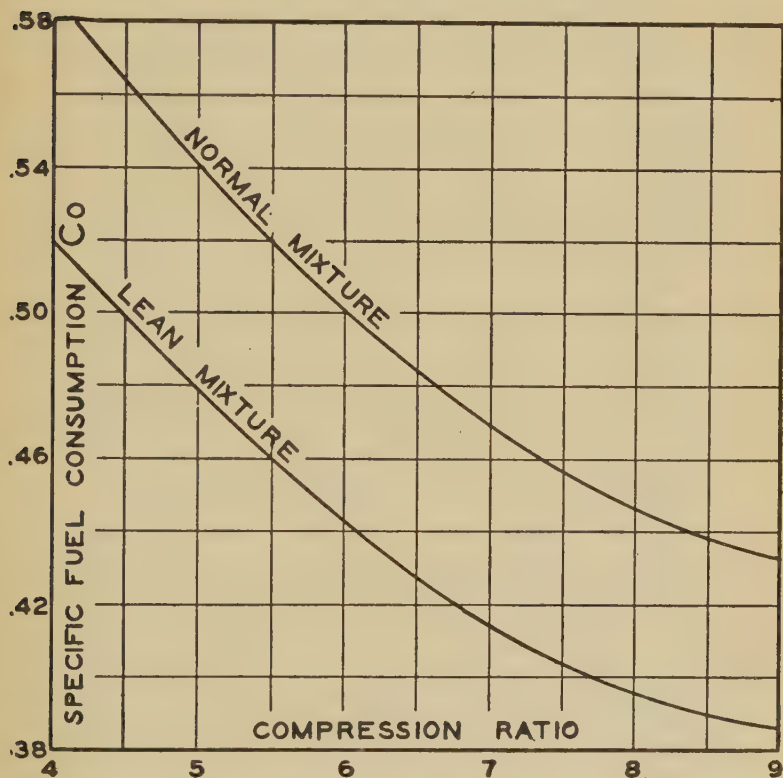


Figure 80. Variation of Specific Fuel Consumption with Compression Ratio and Mixture Control

on the basis of  $B.H.p. \propto PT^{-0.5}$ . If the tests are made with a dynamometer, this correction may be applied directly, but if the tests are made with a propeller or a test "club," allowance must be made for the effect of air density on the power absorbed by the club.

At constant r.p.m., the power absorbed by the club varies directly as the air density, or

$$B.Hp._a = B.Hp._m \cdot \frac{P}{P_o} \cdot \frac{T_o}{T} \quad (71)$$

where  $B.Hp._m$  is the B.Hp. absorbed by the club at  $P = P_o$ , and  $T = T_o$ . That is,  $B.Hp._m$  is the value read from the power

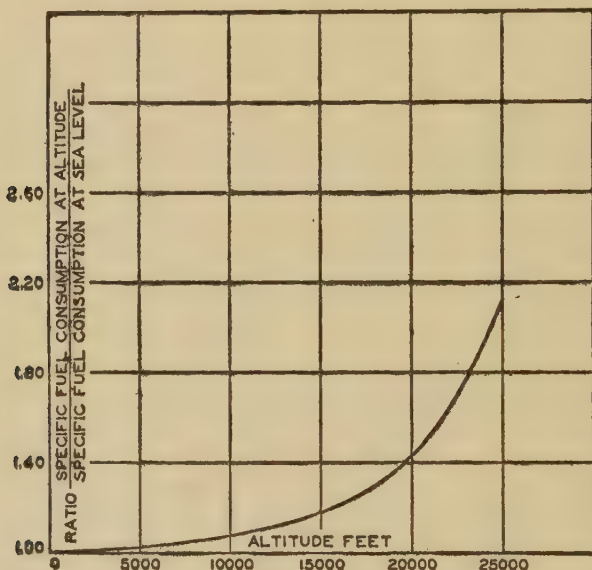


Figure 81. Variation of Specific Fuel Consumption with Altitude

curve for the club. The power delivered by the engine is the same as that absorbed by the club but the engine power is

$$B.Hp._d = B.Hp._s \cdot \frac{P}{P_o} \sqrt{\frac{T_o}{T}} \quad (72)$$

equating  $B.Hp._d$  and  $B.Hp._a$  and solving for  $B.Hp._s$ ,

$$B.Hp._s = \frac{B.Hp._m \cdot \frac{P}{P_o} \cdot \frac{T_o}{T}}{\frac{P}{P_o} \sqrt{\frac{T_o}{T}}} = B.Hp._m \cdot \sqrt{\frac{T_o}{T}} \quad (73)$$

This is on the assumption that the air temperature is the same at the carburetor and propeller. If these temperatures

differ and have the values  $T_c$  and  $T_p$  respectively, equation (73) becomes

$$B.Hp_s = B.Hp_m \frac{T_o}{T_p} \sqrt{\frac{T_c}{T_o}} \quad (74)$$

**Propeller diameter.** The best diameter for a propeller is not sharply defined since it varies with design constants, such as blade area, blade sections, plan-form, etc., which are also variable. However, the propellers commonly in use do not differ greatly in the minor proportions so that it is possible to calculate the diameter by formula and secure results satisfactory for general design purposes and performance calculations.

If the power coefficient  $C_x$  be plotted against  $\frac{V}{nD}$  for a family of geometrically similar propellers, it is found that the points for maximum efficiency lie on straight line passing through the origin. That is

$$C_x = \frac{P}{\rho n^3 D^5} = K' \cdot \frac{V}{nD}$$

from which

$$D = K' \sqrt[4]{\frac{P}{\rho n^2 V}} = K \sqrt[4]{\frac{\rho_o}{\rho}} \sqrt[4]{\frac{B.Hp.}{(r.p.m.)^2 V}} \quad (75)$$

For  $V$  in mi./hr. the values of  $K$  in common use for design purposes are approximately as follows:

#### WOODEN PROPELLERS

| No. of Blades..... |                           | 2   | 3   | 4   |
|--------------------|---------------------------|-----|-----|-----|
| $K$<br>for         | Speed (optimum slip)..... | 285 | 255 | 240 |
|                    | Climb (zero slip).....    | 320 | 290 | 270 |
|                    | Compromise.....           | 305 | 275 | 260 |

#### METAL PROPELLERS

| No. of Blades..... |                           | 2   | 3   |
|--------------------|---------------------------|-----|-----|
| $K$<br>for         | Speed (optimum slip)..... | 280 | 260 |
|                    | Climb (zero slip).....    | 315 | 300 |
|                    | Compromise.....           | 300 | 290 |

The solution when,  $\frac{\rho}{\rho_0} = 1.00$ , is conveniently made if equation (75) is written in the form

$$D = \sqrt[4]{\left(\frac{K^2}{r.p.m.}\right)^2 \cdot \frac{B.H.p.}{V}} \quad (76)$$

This is easily handled on the A, B, C, and D scales of any slide rule; divide  $K^2$  (average value is 90,000) on D scale by r.p.m. on C scale, set runner to index which reads  $\left(\frac{K}{r.p.m.}\right)^2$  on A scale, move slide until  $V$  (mi./hr.) is under hair line, (dividing by  $V$ ), move runner to B.H.p. on B scale (multiplying by B.H.p.). The hair line now indicates  $D^4$  on the A scale and  $D^2$  on the D scale, from which the diameter is determined. This operation may be performed very quickly and is often necessary in making performance estimates. An average value of  $K^2 = 90,000$  is sufficiently accurate for all general purposes, such as performance calculations, preliminary design studies, etc.

A very convenient method<sup>5</sup> of determining diameter and maximum efficiency is to calculate  $F = \frac{V}{n} \sqrt{\frac{\rho V^3}{P}}$  and use Figure 82 to find  $\frac{V}{nD}$  for maximum efficiency. The diameter is

obviously  $D = \frac{\frac{V}{n}}{\frac{V}{nD}}$ . If  $V$  is in mi./hr.,  $n$  in r.p.m., and  $P$  in

$$\text{B.H.p., then } F = 0.325 \frac{V}{n} \sqrt{\frac{\left(\frac{\rho}{\rho_0}\right) V^3}{B.H.p.}}$$

**General efficiency curve.** The curve of efficiency against  $\frac{V}{nD}$  is similar for all conventional propellers,<sup>6</sup> and if the ratios of

<sup>5</sup> For additional data see W. S. Diehl, "The Application of Propeller Test Data to Design and Performance Calculations," N. A. C. A. Technical Report No. 186 (1924).

<sup>6</sup> W. S. Diehl, "The General Efficiency Curve for Air Propellers," N. A. C. A. Technical Report No. 168 (1923).



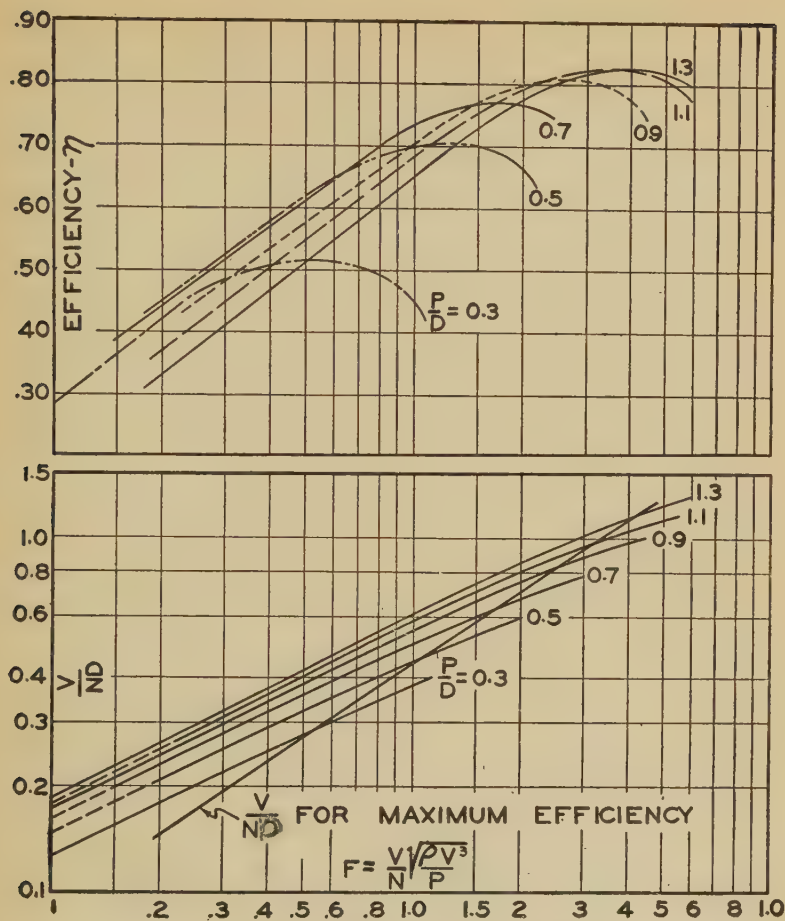


Figure 82. Chart for Determination of Propeller Pitch, Diameter, and Maximum Efficiency

$\frac{\eta}{\eta_m}$  be plotted against  $\left(\frac{V}{nD}\right) / \left(\frac{V}{nD}\right)_o$ , where  $\left(\frac{V}{nD}\right)_o$  corresponds to the maximum efficiency  $\eta_m$ , it is found that the points lie on a single curve. This curve, which may be designated as a "general efficiency curve," is given on Figure 83. From it the efficiency curve for any propeller may be drawn when  $\left(\frac{V}{nD}\right)_o$  and  $\eta_m$  are known.

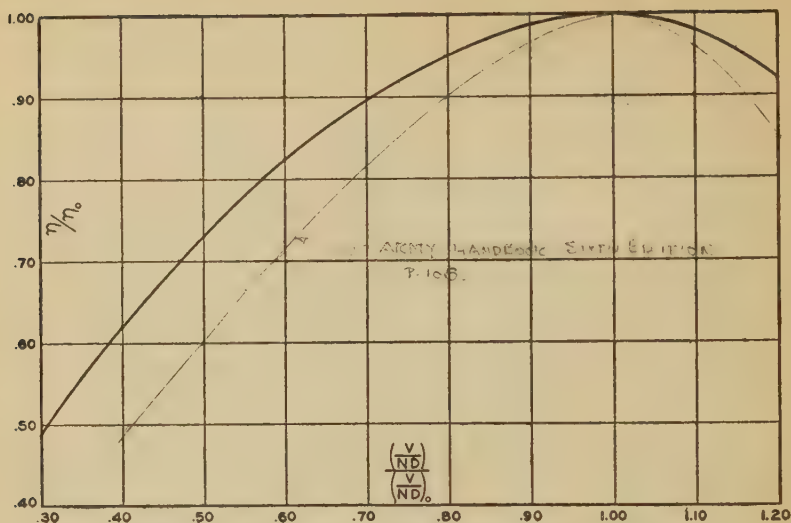


Figure 83. General Efficiency Curve for Propellers

**Maximum efficiency.** The maximum efficiency for any propeller of a geometrically similar series depends on  $\frac{V}{nD}$ , slip

stream obstruction, and tip speed. Figure 84 gives the normal maximum efficiencies for conventional two-bladed wooden and metal propellers with unrestricted flow.

When the slip stream is obstructed, as by a fuselage or nacelle, there is a reduction in efficiency depending on the size and shape of the interfering body. The approximate value of this reduction is given by Figure 85.<sup>7</sup> The average reduction is about 2% or 3%. The body diameter should be taken at a point about one propeller diameter aft of the propeller.

The effect of high tip speed is very noticeable on a propeller having thick blade sections. No appreciable effect is found below about  $0.8a$ , where  $a$  is the velocity of sound. Between  $0.8a$  and  $1.0a$  the efficiency decreases rapidly with a total change which

<sup>7</sup> Derived from data in British A. R. C. R. & M. No. 830, "Experiments on Airscrews with Tractor and Pusher Bodies—Part II," by Fage, Lock, Bateman, and Williams.

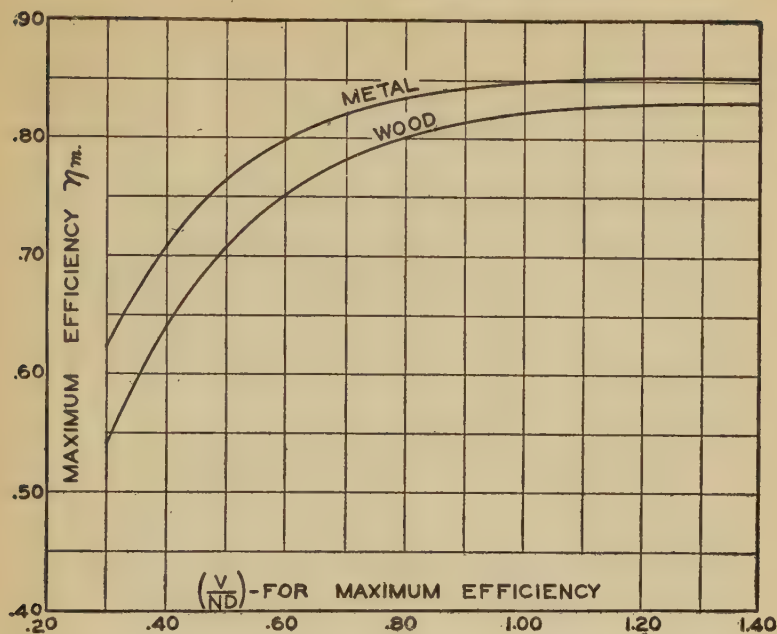


Figure 84. Variation of Maximum Propeller Efficiency with  $\frac{V}{nD}$

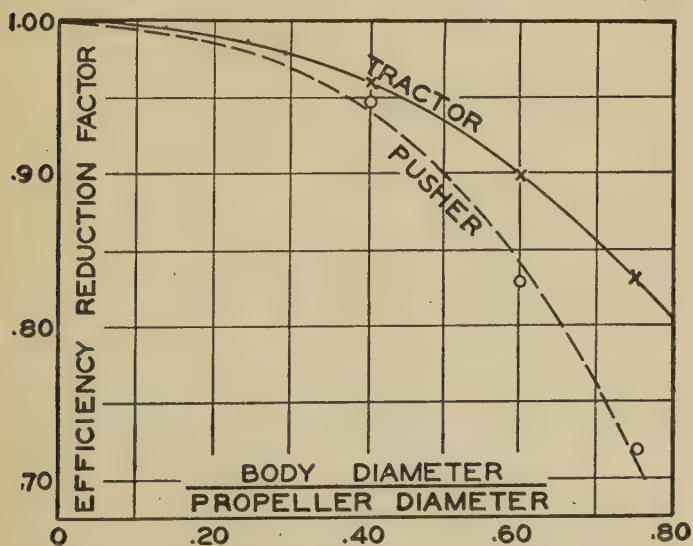


Figure 85. Effect of Body Interference on Propeller Efficiency

may be as much as 30% according to British tests. There is considerable evidence now available to show that the efficiency of a metal propeller with thin blade sections is not seriously affected for tip velocities below 0.9a.

**Relation between  $\frac{p}{D}$  and the value of  $\frac{V}{nD}$  for maximum efficiency.** Definite relations exist between the nominal  $\frac{p}{D}$  ratio and the value of  $\frac{V}{nD}$  for maximum efficiency. Designating this value of  $\frac{V}{nD}$  by the subscript 0, Durand's data on wooden propellers, show that for *general service*

$$\left(\frac{V}{nD}\right)_0 = 0.945 \left(\frac{p}{D}\right) - .046 \left(\frac{p}{D}\right)^2 \quad (77)$$

$\left(\frac{V}{nD}\right)_0$  is sometimes called the "design  $\left(\frac{V}{nD}\right)$ ." This equation may be written in the form

$$\left(\frac{p}{D}\right) = \frac{\left(\frac{V}{nD}\right)_0}{0.945 - .053 \left(\frac{V}{nD}\right)_0} \quad (78)$$

Best climb is obtained when

$$\left(\frac{V}{nD}\right)_0 = \left(\frac{p}{D}\right)$$

and maximum speed when

$$\left(\frac{V}{nD}\right)_0 = 0.905 \left(\frac{p}{D}\right) - .046 \left(\frac{p}{D}\right)^2$$

**Adjustable pitch propellers.** A large number of metal propellers now in use have detachable blades which can be adjusted on the ground to any desired blade angle, or pitch. The usual method of specifying the pitch of these propellers is to give the blade angle at a definite station. Figure 86 gives the pitch in feet corresponding to blade angles at the 42" radius.

The maximum efficiencies obtained from metal propellers

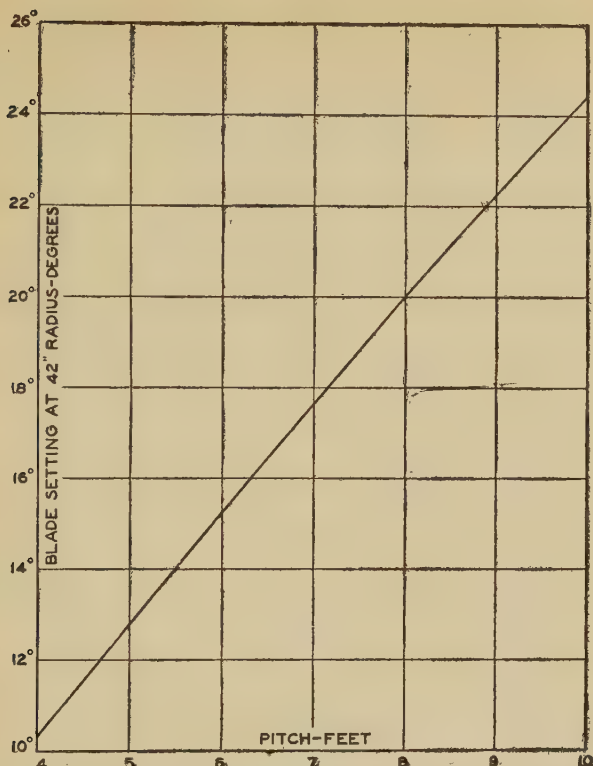


Figure 86. Relation Between Blade Setting and Pitch of Adjustable Blade Propellers

with adjustable blades are, for all practical purposes, identical with those given on Figure 83. Some slight differences are certain to occur when a blade is used at a pitch angle widely different from its designed value, but data now available indicate that the characteristics are still determined by the effective  $\frac{p}{D}$  and the value of  $\frac{V}{nD}$  for maximum efficiency.

**Static thrust.** The static thrust of a propeller is given closely by the empirical equation

$$T_o = 6,000 \left[ 18.7 - 9.5 \left( \frac{p}{D} \right) \right] \frac{B.H.p._o}{(r.p.m.) \cdot D} \text{ lbs.} \quad (79)$$

Where  $p$  is the nominal pitch and  $D$  the diameter.

This equation is based on data<sup>8</sup> for two-bladed wooden propellers. No similar test data for metal propellers is available at this time, but the indications are that little difference exists.

**Negative thrust.** The ratio of  $\frac{V}{nD}$  for zero thrust to  $\frac{V}{nD}$  for maximum efficiency, varies with blade area and  $P/D$  as shown on Figure 87. At values of  $\frac{V}{nD}$  higher than that giving zero thrust the propeller acts as a brake and the thrust is negative.

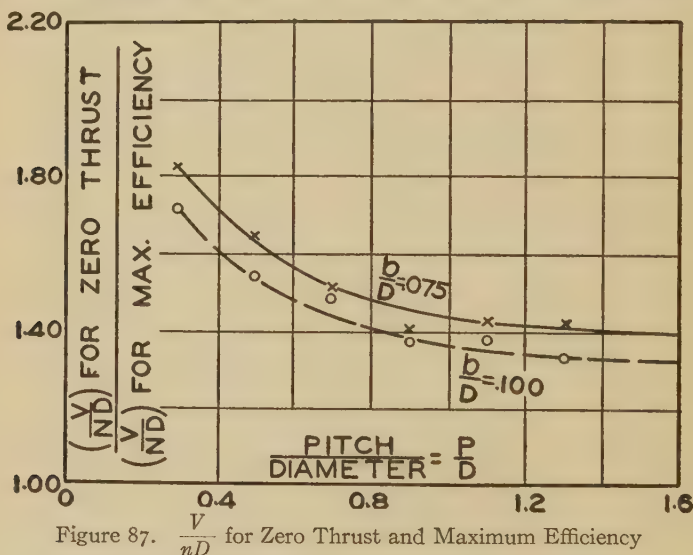


Figure 87.  $\frac{V}{nD}$  for Zero Thrust and Maximum Efficiency

The value of this negative thrust may be expressed as a drag coefficient  $C_D$  defined by the relation

$$\text{Negative thrust} = C_D \frac{\rho}{2} V^2 \left( \frac{b}{D} \right) D^2 \quad (80)$$

where  $V$  is the forward velocity,  $b$  the maximum blade width, and  $D$  the diameter. Values of  $C_D$  for 6 of Durand and Lesley's propellers<sup>9</sup> are plotted against  $\frac{nD}{V}$  in Figure 88. It is of great

<sup>8</sup> From W. F. Durand and E. P. Lesley, "Experimental Research on Air Propellers, II," N. A. C. A. Technical Report No. 30 (1918).

<sup>9</sup> Propellers Nos. 3, 4, 7, 8, 11, and 12. Data from W. F. Durand and E. P. Lesley, "Experimental Research on Air Propellers, II," N. A. C. A. Technical Report No. 30 (1918).



interest to note that the drag of a propeller is, in general, considerably greater when idling than when "dead." The increase in drag due to idling is greater as the pitch decreases.

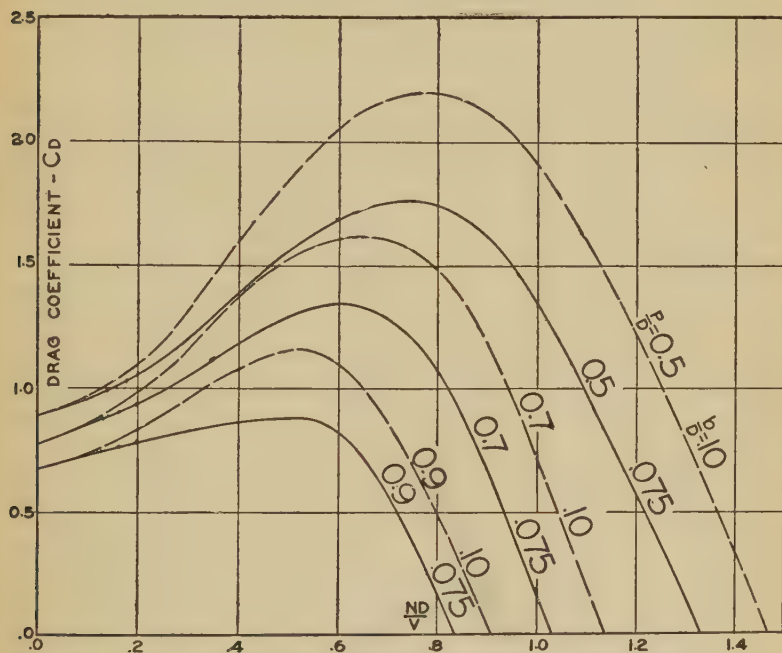


Figure 88. Drag Coefficients for Stationary and Idling Propellers in Terms of Pitch and Blade Widths

The drag coefficient of a dead propeller varies with the pitch/diameter ratio and, from the data on Figure 88 has the value

$$C_D = 1.16 - 0.44 \frac{p}{D} \quad (81)$$

**General power coefficient curves.** A general curve similar to the general efficiency curve may be drawn for the power coefficient  $C_2 = \frac{P}{\rho V^3 D^2}$ . Figure 89 is a plot of the ratio  $\frac{C_2}{(C_2)_0}$  against

$\left(\frac{V}{nD}\right) / \left(\frac{V}{nD}\right)_0$  for a number of Durand's propellers.  $(C_2)_0$  is

the value of the coefficient at  $\frac{V}{nD} = \left(\frac{V}{nD}\right)_0$  and  $\eta = \eta_m$ . Figure 89 is a folded logarithmic plot, the three lines being parts of a single curve, with the limits indicated. This arrangement is necessary if the scale is to be open. Values of  $\frac{C_2}{(C_2)_0}$  read from a large scale plot are given in Table 9.

A similar general curve may be drawn for  $C'_2 = \eta C_2$  as in Figure 90. This curve necessarily follows from Figure 89 and the general efficiency curve. Values of  $\frac{\eta C_2}{(\eta C_2)_0}$  obtained from a large-scale plot are also given in Table 9.

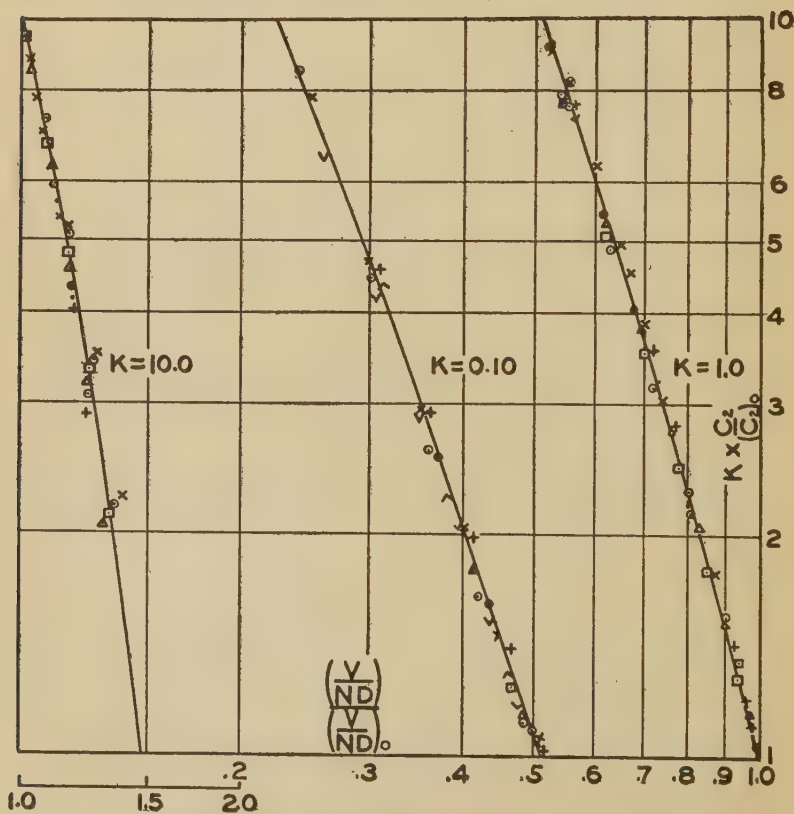


Figure 89. General Power Coefficient Curve

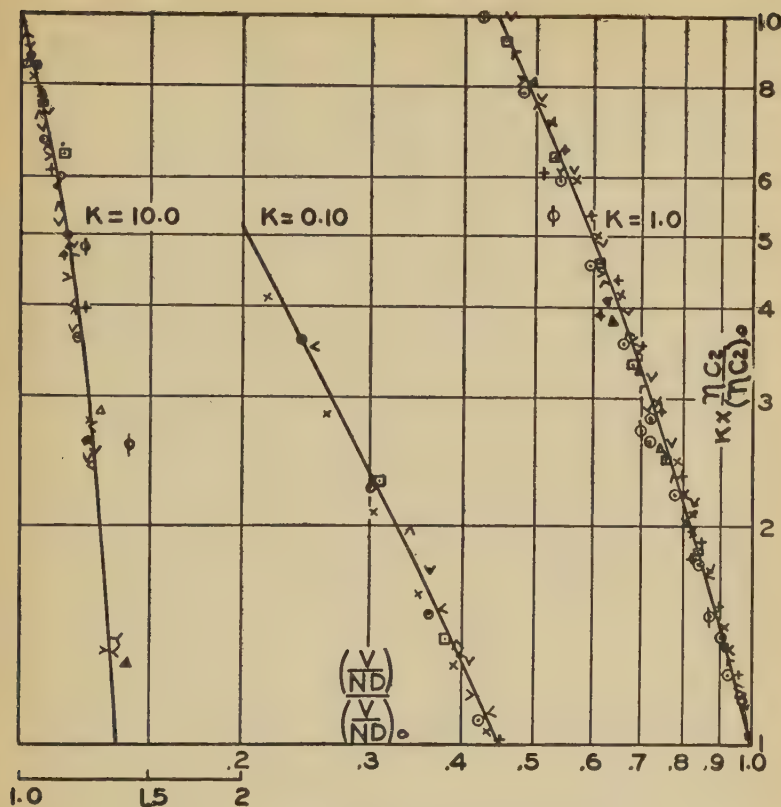


Figure 90. General Thrust Power Coefficient Curve

Figures 89 and 90 lead to the general solution of a number of common problems involving the relations between  $P$ ,  $V$ , and  $n$ ; some of these will be given.

**Variation of thrust power with altitude.** The general power coefficient curves provide a simple method for determining the variation of either B.Hp. or T.Hp. with altitude. It is necessary that a climb be made to ceiling at constant true air speed and the variation of r.p.m. with altitude determined. Then, at any altitude, the ratio  $\left(\frac{V}{nD}\right) / \left(\frac{V}{nD}\right)_0$  is known, consequently

TABLE 9. NON-DIMENSIONAL PROPELLER DATA

| $\frac{V}{nD}$<br>$\left(\frac{V}{nD}\right)_o$ | $C_2$<br>$(C_2)_o$ | $\eta C_2$<br>$(\eta C_2)_o$ | $\eta$<br>$\eta_m$ | $\frac{V}{nD}$<br>$\left(\frac{V}{nD}\right)_o$ | $C_2$<br>$(C_2)_o$ | $\eta C_2$<br>$(\eta C_2)_o$ | $\eta$<br>$\eta_m$ |
|-------------------------------------------------|--------------------|------------------------------|--------------------|-------------------------------------------------|--------------------|------------------------------|--------------------|
| .25                                             | 75.00              | 33.70                        | .450               | .80                                             | 2.300              | 2.200                        | .956               |
| .30                                             | 45.50              | 23.60                        | .522               | .85                                             | 1.850              | 1.800                        | .975               |
| .35                                             | 30.00              | 17.60                        | .586               | .90                                             | 1.490              | 1.470                        | .988               |
| .40                                             | 20.50              | 13.20                        | .645               | .95                                             | 1.210              | 1.210                        | .998               |
| .45                                             | 14.50              | 10.20                        | .700               | 1.00                                            | 1.000              | 1.000                        | 1.000              |
| .50                                             | 10.60              | 7.90                         | .753               | 1.05                                            | .830               | .830                         | .995               |
| .55                                             | 7.90               | 6.30                         | .798               | 1.10                                            | .680               | .670                         | .982               |
| .60                                             | 6.05               | 5.08                         | .840               | 1.15                                            | .523               | .500                         | .956               |
| .65                                             | 4.73               | 4.15                         | .871               | 1.20                                            | .404               | .370                         | .917               |
| .70                                             | 3.70               | 3.36                         | .908               | 1.25                                            | .324               | .280                         | .864               |
| .75                                             | 2.90               | 2.71                         | .934               | 1.30                                            | .250               | .198                         | .792               |

$\frac{C_2}{(C_2)_o}$  or  $\frac{\eta C_2}{(\eta C_2)_o}$  is known. The ratio of B.Hp. at altitude to B.Hp. at sea level is equal to the product of the density ratio by  $\frac{R_h}{R_o}$ , where  $R_h$  is the value of  $\frac{C_2}{(C_2)_o}$  at altitude and  $R_o$  is the value at sea level. The T.Hp. ratio is determined in the same manner, by using the general curve for  $\eta C_2$  instead of  $C_2$ .

Table 10 contains the ratios of  $\frac{T.Hp.}{T.Hp._o}$  at various altitudes as calculated in this manner from flight test data. These data extended up to an average altitude of about 22,000 ft.; above this altitude the values in the table have been obtained by careful extrapolation. The relation between T.Hp. and density in the standard atmosphere is approximately

$$\frac{T.Hp.}{T.Hp._o} = \left(\frac{\rho}{\rho_o}\right)^{1.35} \quad (82)$$

These data have been checked by comparison of calculated performance with observed flight test data for a number of airplanes and excellent agreement found in every case.

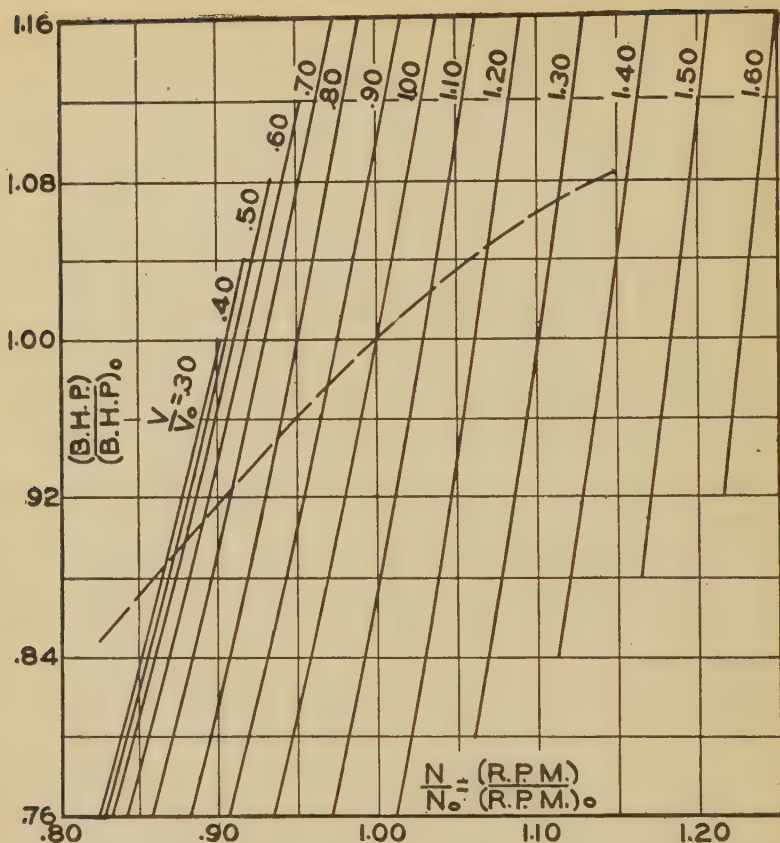
TABLE 10. VARIATION OF THRUST HORSEPOWER WITH ALTITUDE AT CONSTANT AIR SPEED (STANDARD ATMOSPHERE)

| Altitude<br>Ft. | $\frac{T.Hp.}{T.Hp._0}$ | $\sqrt{\frac{\rho_0}{\rho}}$ | Altitude<br>Ft. | $\frac{T.Hp.}{T.Hp._0}$ | $\sqrt{\frac{\rho_0}{\rho}}$ |
|-----------------|-------------------------|------------------------------|-----------------|-------------------------|------------------------------|
| 0               | 1.000                   | 1.000                        | 16,000          | .508                    | 1.282                        |
| 2,000           | .925                    | 1.030                        | 18,000          | .465                    | 1.325                        |
| 4,000           | .854                    | 1.061                        | 20,000          | .425                    | 1.370                        |
| 5,000           | .820                    | 1.077                        | 22,000          | .387                    | 1.418                        |
| 6,000           | .787                    | 1.094                        | 24,000          | .352                    | 1.468                        |
| 8,000           | .723                    | 1.128                        | 25,000          | .336                    | 1.521                        |
| 10,000          | .663                    | 1.164                        | 26,000          | .320                    | 1.577                        |
| 12,000          | .608                    | 1.201                        | 28,000          | .291                    | 1.635                        |
| 14,000          | .556                    | 1.240                        | 30,000          | .261                    | 1.697                        |
| 15,000          | .532                    | 1.261                        | 32,000          | .234                    | 1.763                        |

**B.Hp. and r.p.m. at various speeds.** A general relation between full throttle,  $B.Hp.$ ,  $r.p.m.$ , and  $V$  is readily determined from the general power coefficient curve. Holding  $V$  constant and varying  $n$ , the ratio  $\frac{B.Hp.}{B.Hp._0}$  may be calculated from the known ratios

$$\left(\frac{V}{nD}\right) \text{ and } \frac{C_2}{(C_2)_0}$$

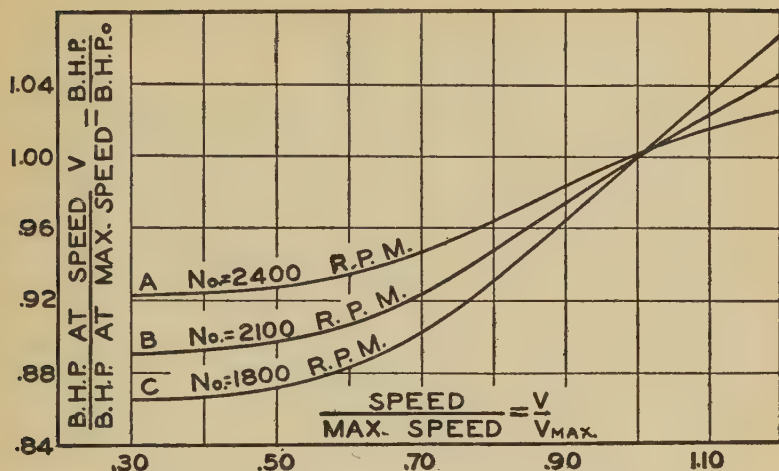
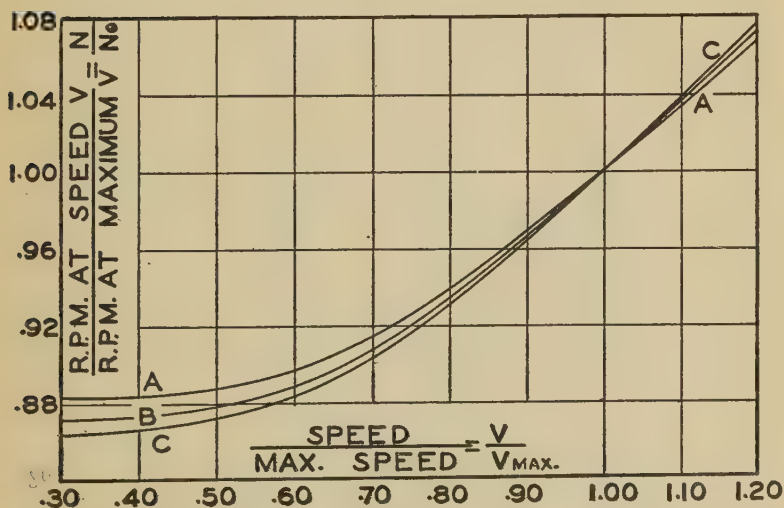
If this procedure is carried out, using ratios instead of definite values, a family of curves may be plotted as in Figure 91. Each of the steeply inclined curves represents the variation in  $\frac{B.Hp.}{B.Hp._0}$  with  $\frac{n}{n_0}$  at a constant ratio  $\frac{V}{V_0}$ , where  $B.Hp._0$  is the  $B.Hp.$  at  $r.p.m. = n_0$  and at the designed speed for maximum efficiency  $V$ . If an engine power curve is plotted on this figure, in the form of  $\frac{B.Hp.}{B.Hp._0}$  against  $\frac{n}{n_0}$ , the intersections with the velocity curves determine the B.Hp. and r.p.m. at any speed. The power curves from Figure 78 may be plotted on Figure 91. These curves represent normal maximum r.p.m. of 1,800, 2,100, and 2,400 on

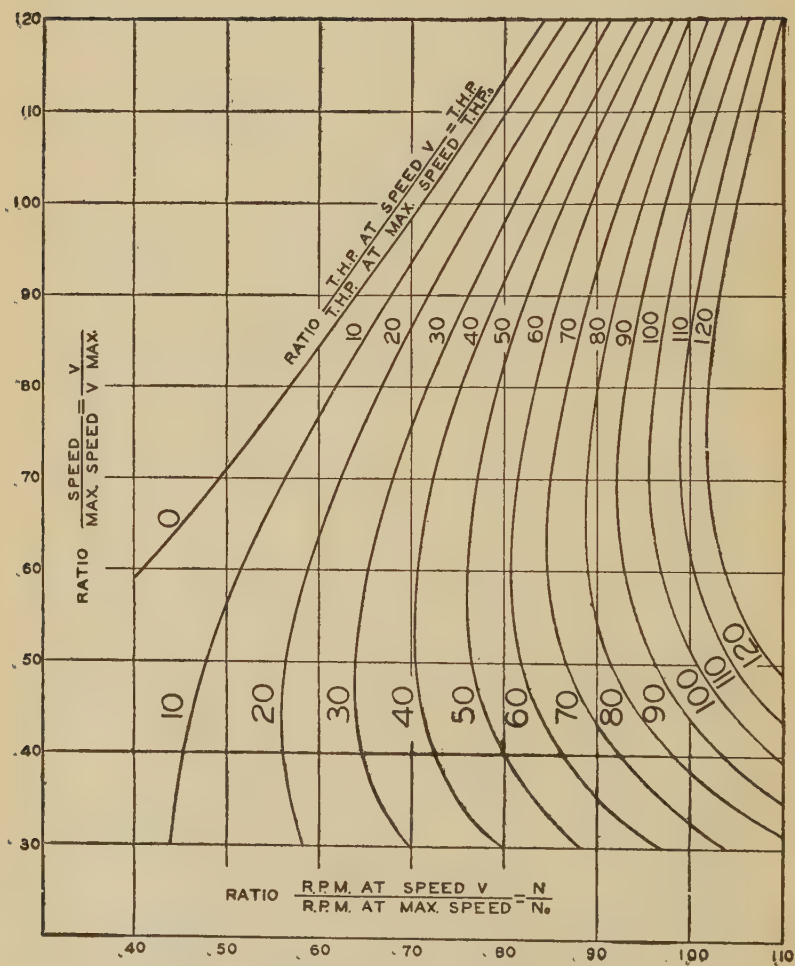
Figure 91. General Curves for  $B.H.p.$ ,  $r.p.m.$  and  $V$ 

engines which deliver maximum  $B.H.p.$  between 2,500 and 2,700 r.p.m. The curves might also be considered to represent approximately 75%, 85%, and 95% of the r.p.m. for maximum  $B.H.p.$  In this manner, allowance may be made for the fact that certain engines are operated nearer to their maximum power than others, or the same engine may be used with widely different r.p.m. in different airplanes.

Figure 92 is a plot of  $\frac{B.H.p.}{B.H.p._0}$  against  $\frac{V}{V_0}$  and Figure 93 is a plot of  $\frac{r.p.m.}{r.p.m._0}$  against  $\frac{V}{V_0}$  as determined by Figure 91.



Figure 92. General Curves for *B.H.p.* and *V*Figure 93. General Curves for *r.p.m.* and *V*.  $N_0 = 2,400$  r.p.m. for Curve A, 2,100 r.p.m. for Curve B, and 1,800 r.p.m. for Curve C

Figure 94. General Curves for  $T.H.p.$ ,  $r.p.m.$ , and  $V$

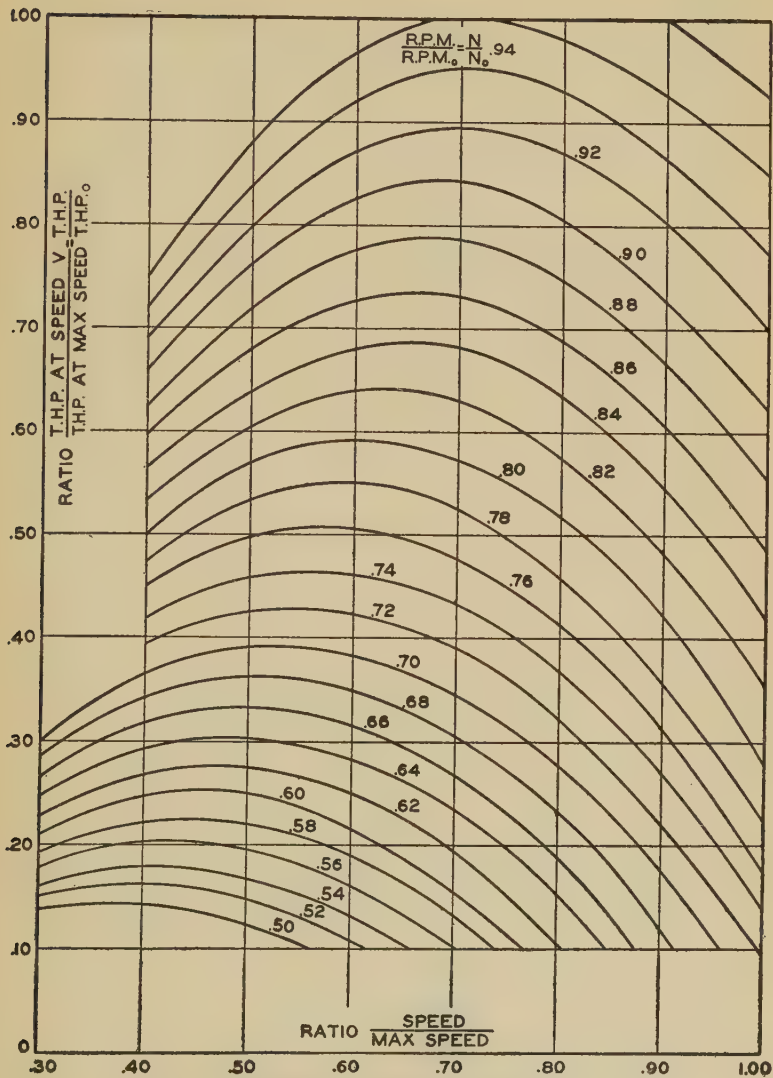
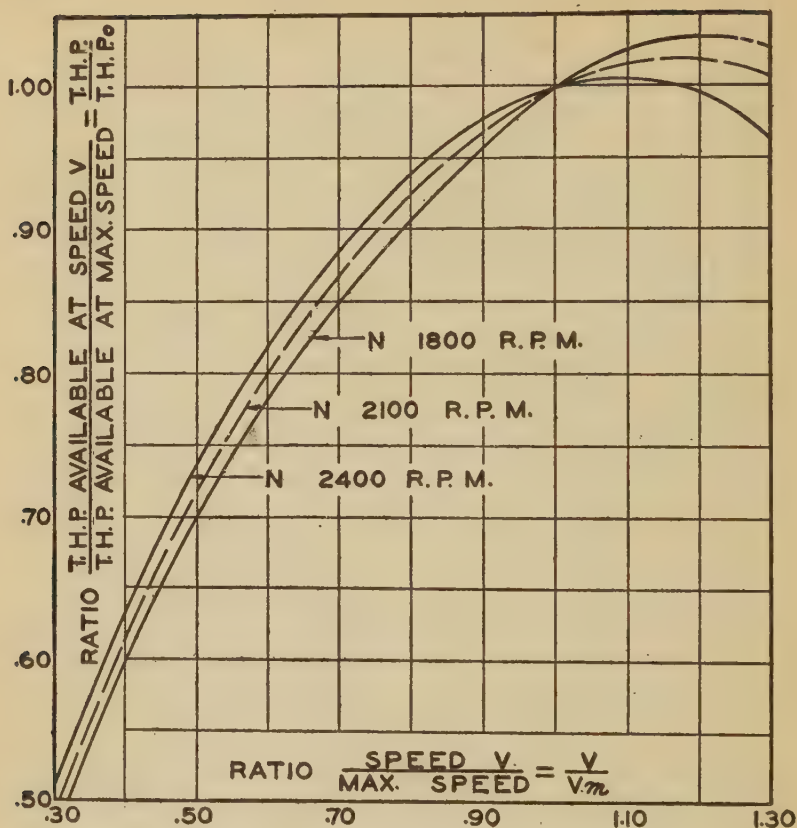


Figure 95. General Curves for  $T.H.p.$ ,  $r.p.m.$ , and  $V$

Figure 96. General Curves for  $T.H.p.$  and  $V$ 

$T.H.p.$ ,  $r.p.m.$ , and  $V$ . General relations between  $T.H.p.$ ,  $r.p.m.$ , and  $V$  in throttled flight may be determined from the general curve for  $\eta C_2$  by holding  $\frac{n}{n_0}$  constant and varying  $\frac{V}{V_0}$ .

$$\left(\frac{V}{nD}\right) \text{ and } \frac{\eta C_2}{\left(\frac{V}{nD}\right)_0}$$

are therefore known and the thrust power ratio is

$$\frac{T.H.p.}{T.H.p._0} = \frac{\eta C_2}{(\eta C_2)_0} \cdot \left(\frac{V}{V_0}\right)^3 \quad (83)$$

Figures 94 and 95 give the relation between  $T.Hp.$ ,  $r.p.m.$ , and  $V$ , enabling the solution of any of the common problems involving these factors.

The relation between  $T.Hp.$  and  $V$  with full throttle is found from the general curves of  $B.Hp.$ ,  $r.p.m.$ , and  $\eta$  against  $V$ . This relation given on Figure 96 is of great value in performance estimation since it gives directly the variation of  $\frac{T.Hp.}{T.Hp._0}$  with  $\frac{V}{V_0}$ .

When the value of  $\frac{T.Hp.}{T.Hp._0}$  is known at the designed speed  $V_0$  for maximum propeller efficiency, the curve of maximum thrust power available is obviously determined by one of the curves on Figure 96.

## CHAPTER VII

### PERFORMANCE CALCULATION

The complete performance of an airplane comprises its maximum and minimum speeds, rate of climb at various altitudes, time to climb to various altitudes, and range and endurance at given speeds with specified fuel loads. While it is possible to calculate performance by purely analytical methods, a combination of the graphical and analytical will be found more simple and direct. The usual method is to calculate, for horizontal flight at various speeds and at a given altitude, the thrust power required for horizontal flight and the maximum thrust power available. If the curves of power required and power available are plotted against speed, their intersection obviously will determine the maximum speed at the altitude under consideration. The difference between power available and power required at any given speed is the excess power available for climb, and the maximum rate of climb occurs at that speed at which the excess power is a maximum.

Before going into a detailed explanation of the general methods for calculating performance, a brief statement of the basic assumptions will be given. In steady horizontal flight the forces acting on an airplane are: lift, drag, gravity, and propeller thrust. Since the motion is steady, the vector sum of the forces or their components in any direction, and the resultant moment about any axis must be zero. The equations of motion are easily obtained. Vertical and horizontal components give respectively

$$L + T \sin \theta - W = \frac{dh}{dt} = 0 \quad (84)$$

$$D - T \cos \theta = \frac{dV}{dt} = 0 \quad (85)$$



where  $\theta$  is the angle of pitch of the thrust axis,  $L$  the lift,  $D$  the drag, and  $T$  the thrust. The thrust correction  $T \sin \theta$  in equation (84) is negligible except at fairly large angles, and it is usually neglected, although the effect of a moderate throttle opening may be a reduction of the order of 5 mi./hr. in landing speed for airplanes of the pursuit type.  $\theta$  is ordinarily less than  $20^\circ$ , so that  $\cos \theta = 1.0$  for all practical purposes. Therefore, unless extreme accuracy is required, it is always assumed that  $L = W$  and  $D = T$ .

With the foregoing assumptions, the equation connecting weight and speed is

$$L = W = C_L \frac{\rho}{2} S V^2 \quad (86)$$

from which

$$V = \sqrt{\frac{W}{C_L \frac{\rho}{2} S}} = \sqrt{\frac{W}{K_y S}} \cdot \sqrt{\frac{\rho_0}{\rho}} \quad (87)$$

since  $K_y = C_L \frac{\rho}{2}$ . It must be noted that when the engineering coefficient  $K_y$  is used, the *density ratio* must be introduced, otherwise the equation holds only for standard density. Since absolute coefficients will be used more and more in the future, the engineer should familiarize himself with them. It is often convenient to use equation (87) in the form

$$V = 29.00 \sqrt{\frac{\left(\frac{W}{S}\right)}{C_L}} \cdot \sqrt{\frac{\rho_0}{\rho}} \quad (V \text{ in ft./sec., } W \text{ in lbs., } S \text{ in sq. ft.})$$

$$\text{or} \quad V = 19.77 \sqrt{\frac{\left(\frac{W}{S}\right)}{C_L}} \cdot \sqrt{\frac{\rho_0}{\rho}} \quad (V \text{ in mi./hr.}) \quad (88)$$

The minimum speed, or "stalling speed" is obviously determined by using the maximum value of  $C_L$  in equation (88).

**Power required for horizontal flight.** The power required for horizontal flight is

$$T.Hp.r = \frac{DV}{550} \quad (V \text{ in ft./sec.})$$

$$\text{or,} \quad T.Hp.r = \frac{DV}{375} \quad (V \text{ in mi./hr.}) \quad (89)$$

$D$  being the total drag at the air speed  $V$ . The drag  $D$  can be divided into two parts, each with two subdivisions. The major components are *wing drag* and residual, or *parasite drag*. Wing drag is further divided into *induced* and *profile* (or section) drags. The coefficient of the former depends only on the lift coefficient and effective aspect ratio, while the coefficient of the latter varies with the wing section. Parasite drag must be divided into two parts, the first *variable with angle of attack* and designated here as  $P_1$ , the second *independent of angle of attack* and designated as  $P_2$ .  $P_1$  includes such items as nacelles, floats, hulls, tail surfaces, wing section drag, and fuselages having square or rectangular cross-sections.  $P_2$  includes struts, wires, fittings, wheels, tail skids, ordnance equipment, air-cooled engines, radiators, and fuselages having circular or elliptical cross-sections. If there is any doubt as to the classification of the fuselage drag, it should be put in  $P_1$ .

$P_1$  and  $P_2$  vary directly as  $V^2$ , while induced drag varies inversely as  $V^2$ .

**Wing drag, induced.** The *induced drag* is that part of the total wing drag which is due to a virtual inclination of the lift vector as a result of the downwash, the effect being the same as if the wing were operating along an upward slope. The induced drag is independent of wing section and varies only with lift coefficient  $C_L$  and effective aspect ratio  $n$ .

$$\text{Induced drag coefficient } C_{Di} = \frac{C_L^2 S}{\pi(kb)^2} = \frac{C_L^2}{\pi n} \quad (90)$$

where  $S$  is the total wing area,  $b$  the maximum wing span, and  $k$  Munk's factor for equivalent monoplane span. For a monoplane  $k = 1.0$ . For a biplane  $k$  varies with the ratio of gap/maximum-span, and with the ratio of spans.

Figures 26 and 27 (Chapter II) give the values of  $k$  for any conventional arrangement.

Equation (90) is most conveniently used in performance calculations if written in the form

$$D_i = \frac{2W^2}{\pi \rho V^2 (kb)^2} \quad (91)$$

$W$  being the gross weight.

For standard sea-level density this reduces to

$$D_i = \frac{267.7}{V^2} \left( \frac{W}{kb} \right)^2 \quad (V \text{ in ft./sec.})$$

$$\text{or} \quad D_i = \frac{124.5}{V^2} \left( \frac{W}{kb} \right)^2 \quad (V \text{ in mi./hr.}) \quad (92)$$

These equations reduce to the form  $D_i = \frac{K}{V^2}$  for any particular case.

It is often desirable to calculate directly the induced power required. The equations at sea-level density are:

$$T.H.p._i = \frac{D_i \cdot V}{550} = \frac{0.487}{V} \left( \frac{W}{kb} \right)^2 \quad (V \text{ in ft./sec.})$$

$$\text{or} \quad T.H.p._i = \frac{D_i \cdot V}{375} = \frac{0.332}{V} \left( \frac{W}{kb} \right)^2 \quad (V \text{ in mi./hr.}) \quad (93)$$

In any particular case, these reduce to the form  $T.H.p._i = \frac{K}{V}$ .

**Parasite drag  $P_i$  — variation with angle of attack.** A study of the drag of nacelles, floats, hulls, and fuselages, having square or rectangular cross-section, shows a very definite dependence on angle of attack. (See Figure 3, N. A. C. A. Technical Report No. 236.) A study of the profile drag of ten of the most widely used wing sections shows that at any given value of  $\frac{V}{V_s}$  the ratio  $\frac{(C_{D_o})}{(C_{D_o})_0}$  is practically independent of section. If the relation previously found for variation with angle is converted to average speed range, the agreement will be found very close, so that a

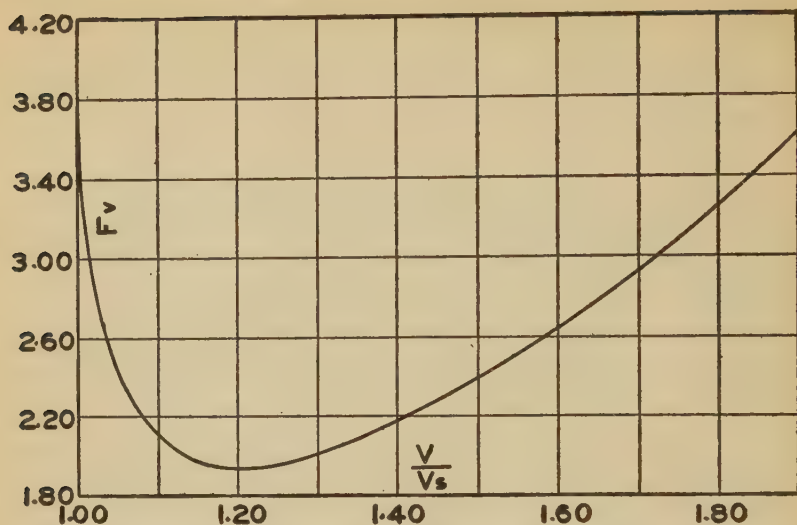


Figure 97. Factor  $F_v$  for Finding Value of Variable Parasite Drag  $P_1$  at Any Speed

single curve of  $\frac{D}{D_0}$  against  $\frac{V}{V_s}$  is sufficient, but in most cases it will be found more convenient to use  $F_v = \frac{D}{D_0} \cdot \left(\frac{V}{V_s}\right)^2$  which is plotted in Figure 97. The values most frequently used are as follows:

| $\left(\frac{V}{V_s}\right)$ | $\left(\frac{D}{D_0}\right)$ | $F_v = \left(\frac{D}{D_0}\right) \cdot \left(\frac{V}{V_s}\right)^2$ |
|------------------------------|------------------------------|-----------------------------------------------------------------------|
| 1.00                         | 4.100                        | 4.10                                                                  |
| 1.05                         | 2.210                        | 2.44                                                                  |
| 1.10                         | 1.750                        | 2.12                                                                  |
| 1.15                         | 1.500                        | 1.98                                                                  |
| 1.20                         | 1.350                        | 1.95                                                                  |
| 1.30                         | 1.190                        | 2.01                                                                  |
| 1.40                         | 1.110                        | 2.18                                                                  |
| 1.50                         | 1.060                        | 2.38                                                                  |
| 1.60                         | 1.025                        | 2.63                                                                  |
| 1.70                         | 1.007                        | 2.91                                                                  |
| 1.80                         | 1.000                        | 3.24                                                                  |
| 2.00                         | 1.000                        | 4.00                                                                  |
| 2.20                         | 1.000                        | 4.84                                                                  |
| 2.40                         | 1.000                        | 5.76                                                                  |

By the use of  $F_v$ , the accurate calculation of performance is considerably simplified since  $P_1 = (P_1)_0 \cdot F_v$ , where  $(P_1)_0$  is the value of  $P_1$  (without correction for angle) at stalling speed,  $V_S$ .  $P_1$  is usually calculated for the condition of thrust line horizontal.

An example of the calculation of power required for horizontal flight will follow.

**Example of calculation of power required.** Given an airplane of the following characteristics:

Gross weight:  $W = 2,100$  lbs.

Wing area:  $S = 290$  sq. ft.

Wing section: R. A. F.-15

Maximum span: Upper = 33.2 ft. Lower = 33.2 ft.

Span ratio = 1.00 Gap = 4.65 ft.

Ratio (gap/span) = 0.139 Chords equal.

Span factor  $k$  (from Figure 26) = 1.125

Effective aspect ratio =  $\frac{(kb)^2}{S} = 4.8$

Parasite drag coefficients:  $P_1 = .020$   $P_2 = .026$

The maximum lift coefficient of the R.A.F.-15 from wind tunnel tests is  $C_L = 1.02$ . The stalling speed is

$$V = 19.77 \sqrt{\frac{W}{S}} \sqrt{\frac{C_{L0}}{C_L}} = 52.6 \text{ mi./hr.}$$

The induced drag from equation (92) is

$$D_i = \frac{124.5}{V^2} \left( \frac{W}{kb} \right)^2 = \frac{397,000}{V^2} (\text{mi./hr.})$$

The basic value of  $P_1$  at stalling speed is

$$(P_1)_0 = .020 qS = .020 \times \frac{.002378}{2} \times (52.6 \times 1.467)^2 \\ \times 290 = 41.2 \text{ lbs.}$$

The value of  $P_1$  at any speed is

$$P_1 = (P_1)_0 \cdot F_v = 41.2 F_v$$

It is somewhat easier to obtain the basic value by the use of the dynamic pressure at 100 mi./hr., that is,  $q_{100} = 25.58$  lbs./ft.<sup>2</sup>, the formula being  $(P_1)_0 = C_{DP_1} \times q_{100} \times S \left( \frac{V}{100} \right)^2$ .

Using this method, the basic value of  $P_2$  is

$$(P_2)_0 = .026 \times 25.58 \times 290 \times (0.526)^2 = 53.4 \text{ lbs.}$$

The value of  $P_2$  at any speed is

$$P_2 = (P_2)_0 \left( \frac{V}{V_S} \right)^2 = 53.4 \left( \frac{V}{V_S} \right)^2$$

The calculations for thrust power required at sea level are given in Table II.

TABLE II. EXAMPLE OF CALCULATION FOR THRUST POWER REQUIRED AT SEA LEVEL

WEIGHT = 2,100 lbs.

| Speed<br>Ratio<br>$\frac{V}{V_S}$ | Air<br>Speed<br>$V$<br>mi./hr. | Factor<br>$F_\eta$ | DRAG          |               |               |             | Thrust<br>Power<br>Required<br>at<br>Sea<br>Level | $\frac{L}{D}$ |
|-----------------------------------|--------------------------------|--------------------|---------------|---------------|---------------|-------------|---------------------------------------------------|---------------|
|                                   |                                |                    | Parasite      |               | Induced       | Total       |                                                   |               |
|                                   |                                |                    | $P_1$<br>lbs. | $P_2$<br>lbs. | $D_i$<br>lbs. | $D$<br>lbs. |                                                   |               |
| 1.00                              | 52.6                           | 4.10               | 168           | 53            | 143           | 364         | 51.0                                              | 5.77          |
| 1.05                              | 55.2                           | 2.44               | 100           | 59            | 130           | 289         | 42.5                                              | 7.28          |
| 1.10                              | 57.8                           | 2.12               | 87            | 65            | 119           | 271         | 41.8                                              | 7.76          |
| 1.20                              | 63.1                           | 1.95               | 80            | 77            | 100           | 257         | 43.3                                              | 8.18          |
| 1.40                              | 73.7                           | 2.18               | 90            | 105           | 73            | 268         | 52.7                                              | 7.84          |
| 1.70                              | 89.4                           | 2.91               | 120           | 154           | 50            | 324         | 77.2                                              | 6.64          |
| 2.00                              | 105.2                          | 4.00               | 164           | 213           | 36            | 413         | 116.0                                             | 5.08          |
| 2.30                              | 121.0                          | 5.29               | 217           | 283           | 27            | 527         | 170.0                                             | 3.99          |

Note that  $\frac{L}{D}$  has been tabulated along with  $T.H.p._{r_0}$  in Table

II. This is not necessary, except as a general check on the calculations, unless the power required for two or more weights is to be calculated. If the change in weight does not affect the parasite drag coefficients, as for example with a varying fuel load,



the value of  $\frac{L}{D}$  at any given speed ratio  $\frac{V}{V_S}$  remains constant, and it is unnecessary to carry out the full calculation to find the new value of  $T.H.p._{ro}$ . At any given value of  $\frac{V}{V_S}$  and at constant density, the following relations hold

$$\frac{V_2}{V_1} = \sqrt[4]{\frac{W_2}{W_1}} \quad (94)$$

$$D_i = \frac{W_i}{\frac{L}{D}}, \quad \frac{D_2}{D_1} = \frac{W_2}{W_1} \quad (95)$$

$$\frac{(T.H.p._r)_2}{(T.H.p._r)_1} = \left(\frac{W_2}{W_1}\right)^{1.5} \quad (96)$$

Equations (94), (95), and (96) are often used in performance calculations.

**Maximum thrust power available at sea level.** The next step is to calculate the maximum thrust power available at various air speeds. To do this, the engine and propeller characteristics must be known. Let it be assumed that the engine is the Wright-Hispano, Type E, developing 195 B.Hp. at 1,800 r.p.m., and that the propeller is a two-bladed wooden type designed for general service which is a compromise between best speed and best climb. The constant  $K$  in the diameter formula of equation (75) will be 300. The maximum speed will be somewhat greater than 115 mi./hr. as shown by Figure 138. The diameter will therefore be

$$D = 300 \sqrt[4]{\frac{B.H.p.}{r.p.m.^2 V}} = 300 \sqrt[4]{\frac{195}{(1,800)^2 \times 115}} = 8.0 \text{ ft.}$$

For 10% slip the pitch must be

$$P = \frac{115 \times 1.467 \times 1.10}{30} = 6.2 \text{ ft.}$$

From Figure 82 the maximum efficiency of this propeller will occur at  $\frac{V}{nD} = 0.72$ , approximately, and this is the designed  $\frac{V}{nD}$ .

The corresponding maximum efficiency from Figure 83 is about 78%.

Using Figure 96, the thrust power is readily calculated for various speeds, by finding the values of  $\frac{T.Hp.}{T.Hp._o}$  at the corresponding values of  $\frac{V}{V_d}$  where  $V_d = 115$  mi./hr., the designed speed for maximum propeller efficiency. At this speed the value of  $T.Hp._o$  is  $T.Hp._o = B.Hp. \times \eta_m = 195 \times .78 = 152.0$ . Since 1,800 r.p.m. is well below the peak of the power curve for the Wright E engine, the 1,800 r.p.m. curve on Figure 96 should be used. The calculations for maximum  $T.Hp._o$  at sea level are given in Table 12.

TABLE 12. CALCULATION OF MAXIMUM THRUST HORSE-POWER AVAILABLE AT SEA LEVEL

| Air Speed<br>$V$<br>mi./hr. | $\frac{V}{V_d}$ | $\frac{T.Hp.}{T.Hp._o}$ | $T.Hp.$ |
|-----------------------------|-----------------|-------------------------|---------|
| 50                          | .435            | .633                    | 96.3    |
| 60                          | .522            | .713                    | 108.2   |
| 70                          | .608            | .770                    | 118.5   |
| 80                          | .696            | .846                    | 128.5   |
| 90                          | .782            | .897                    | 136.2   |
| 100                         | .870            | .945                    | 143.5   |
| 110                         | .957            | .984                    | 149.3   |
| 115                         | 1.000           | 1.000                   | 152.0   |
| 120                         | 1.043           | 1.013                   | 154.0   |

**Graphical calculation of performance.** If the values of  $T.Hp._r$  and  $T.Hp._o$  from Tables 11 and 12, are plotted against air speed, as on Figure 98, the sea-level performance is easily obtained by graphical solution. The intersection of the curves shows that the maximum speed is 116 mi./hr. The maximum difference between  $T.Hp._o$  and  $T.Hp._r$  occurs at 70 mi./hr. where  $(T.Hp._o - T.Hp._r) = 70 E.Hp.$  The rate of climb corresponding to any excess thrust power is

$$\frac{dh}{dt} = E.Hp. \times \frac{33,000}{W} \quad (97)$$

where  $W$  is the gross weight of the airplane. In this case  $E.H.p. = 70$  and  $W = 2,100$ , so that

$$\frac{dh}{dt} = 70 \times \frac{33,000}{2,100} = 1,100 \text{ ft./min.}$$

This is the rate of climb at sea level, usually designated as "initial rate of climb."

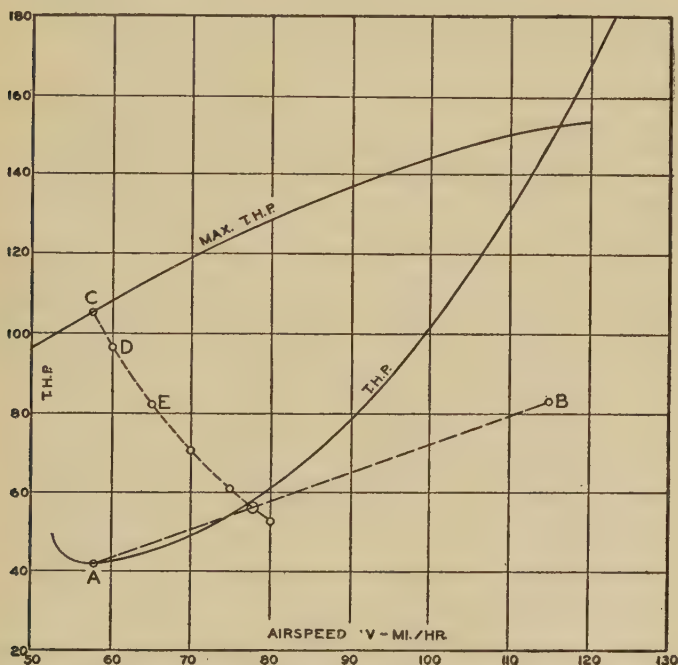


Figure 98. Curves of Power Available and Power Required, Illustrating Graphical Solution for Absolute Ceiling

*Absolute Ceiling* or the altitude at which the rate of climb is zero may now be determined accurately by a simple graphical construction. At the absolute ceiling, the airplane will be flying at an angle of attack near that giving minimum power. If flight were started at sea level at this angle of attack and continued all the way up to the absolute ceiling without change, the maximum rate of climb would not be obtained at all altitudes, but the

absolute ceiling would be the same as that obtained if the correct air speeds for best climb were used. In this climb at constant angle of attack, both the air speed and  $T.H\dot{p}$ . required will vary inversely as the square root of the density. That is

$$V = V_0 \sqrt{\frac{\rho_0}{\rho}}$$

and  $T.H\dot{p}_r = T.H\dot{p}_{r_0} \sqrt{\frac{\rho_0}{\rho}}$

or, when  $V = KV_0$ ,  $T.H\dot{p}_r = K T.H\dot{p}_{r_0}$ . Referring to Figure 98, it will be seen that the minimum value of  $T.H\dot{p}_{r_0}$  is 41.5 at  $V_0 = 57.5$  mi./hr. Hence at the altitude where  $\sqrt{\frac{\rho_0}{\rho}} = 2.0$ ,  $V = 115$  mi./hr. and  $T.H\dot{p}_r = 83$ . A straight line passing through this point  $B$  and the initial point ( $V = 57.5$  mi./hr.,  $T.H\dot{p}_r = 41.5$ , or the origin,  $V = 0$ ,  $T.H\dot{p}_r = 0$ ) is the locus of minimum power at all densities. Consider now the power available.

At  $V_0 = 57.5$  mi./hr.,  $T.H\dot{p}_a = 105.3$ . This is the initial value  $C$ . At the altitude where  $V = 60$  mi./hr.,  $\sqrt{\frac{\rho_0}{\rho}} = \frac{60}{57.5} = 1.043$ , and  $\frac{\rho}{\rho_0} = 0.918$ . The altitude and the corresponding ratio of  $\frac{T.H\dot{p}_r}{T.H\dot{p}_a}$  may thus be determined using the data in Table 10.

It is more convenient, however, to plot the ratio  $\frac{T.H\dot{p}_r}{T.H\dot{p}_a}$  against

$\sqrt{\frac{\rho_0}{\rho}} = \frac{V}{V_0}$  as in Figure 99, and eliminate the intermediate step. In either case the  $T.H\dot{p}_a$  at 60 mi./hr. and at the altitude where  $\sqrt{\frac{\rho_0}{\rho}} = \frac{60}{57.5}$  is determined. Figure 99 gives  $\frac{T.H\dot{p}_r}{T.H\dot{p}_a} = 0.895$  at  $\frac{V}{V_0} = 1.043$ . Figure 98 shows that  $T.H\dot{p}_{a_0} = 108$ . Hence,

$T.Hp._a = 108 \times 0.895 = 96.7$ , which is to be plotted at point  $D$  on Figure 98. Repeating this process for various speeds gives

| $V$  | $\frac{V}{V_o}$ | $\frac{T.Hp.}{T.Hp._o}$ | $T.Hp._o$ | $T.Hp.$ |
|------|-----------------|-------------------------|-----------|---------|
| 57.5 | 1.000           | 1.000                   | 105.3     | 105.3   |
| 60.0 | 1.043           | .895                    | 108.0     | 96.7    |
| 65.0 | 1.130           | .718                    | 113.6     | 81.6    |
| 70.0 | 1.217           | .588                    | 119.0     | 70.0    |
| 75.0 | 1.303           | .490                    | 123.6     | 60.6    |
| 80.0 | 1.391           | .409                    | 128.0     | 52.3    |

Plotting these values of  $T.Hp.$  against  $V$  on Figure 98 and passing the curve  $CDEO$  through the points, it is found to intersect the line  $AB$  at  $V = 78.0$  mi./hr. This signifies that at the density where the air speed for minimum power is 78 mi./hr., the curves of  $T.Hp._a$  and  $T.Hp._r$  are tangent, which is the condition for zero climb and absolute ceiling. This density ratio may be found from  $\sqrt{\frac{\rho_o}{\rho}} = \frac{V}{V_o} = \frac{78}{57.5} = 1.356$ , and the corresponding altitude obtained from standard atmosphere tables and charts, or the altitude may be read directly from Figure 99 where it has been included for convenience. This altitude is 19,500 ft.

The points  $C, D, E, O$ , just calculated are each a single point on a curve of  $T.Hp._a$  for the altitude represented by the relation

$\sqrt{\frac{\rho_o}{\rho}} = \frac{V}{V_o}$ . In the example given, 6 points were used to determine the curve  $CDEO$ , but 3 points usually are sufficient.

*Service Ceiling* is the altitude at which the rate of climb is 100 ft./min. If the rate of climb be assumed linear with altitude, that is

$$\frac{dh}{dt} = C_o - ah \quad (98)$$

then the service ceiling is given by

$$h_s = H \left[ \frac{C_o - 100}{C_o} \right] \quad (99)$$

where  $H$  is the absolute ceiling and  $C_0$  the initial rate of climb. For the case just calculated,  $H = 19,500$  ft. and  $C_0 = 1,100$  ft./min.

$$\therefore h_s = 19,500 \left[ \frac{1,100 - 100}{1,100} \right] = 17,730 \text{ ft.}$$

*Time of Climb to Any Altitude.* On the assumption that  $\frac{dh}{dt} = (C_0 - ah)$ , the time of climb to any altitude  $h$  is

$$T = \frac{H}{C_0} \log_e \left[ \frac{H}{H-h} \right] = 2.303 \frac{H}{C_0} \log_{10} \left[ \frac{H}{H-h} \right] \quad (100)$$

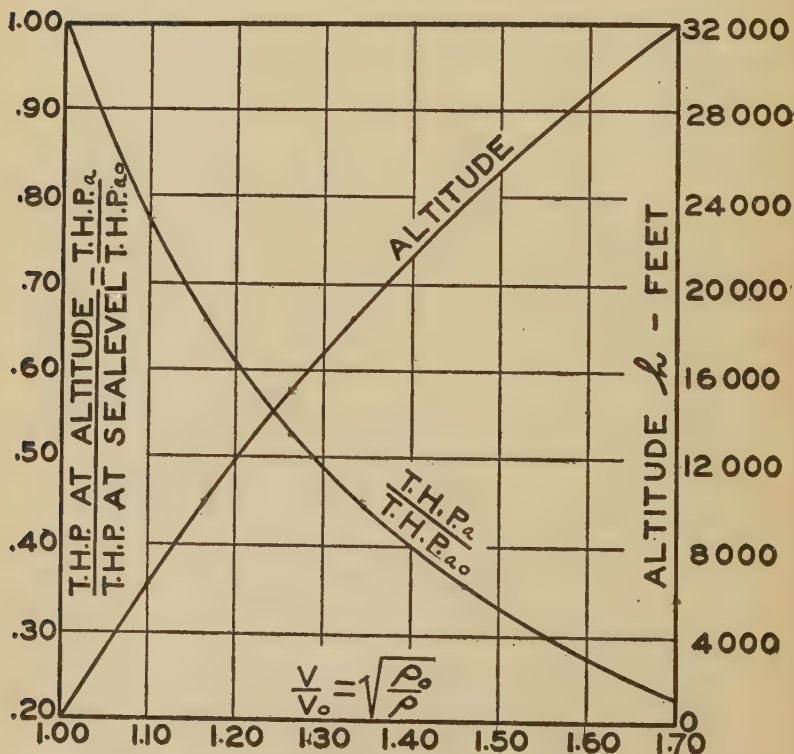


Figure 99. Relation Between T.H.p., Density and Standard Altitude



*Altitude Climbed in a Given Time, T.* The preceding equation may be converted to the form

$$h = H \left( 1 - e^{-\frac{C_0}{H}T} \right) \quad (101)$$

for calculating the altitude climbed in time  $T$ .

**Calculation of power required at altitude.** In steady horizontal flight  $C_L$  and  $C_D$  are constant at a given angle of attack, if the variation in slip stream effects on wing and tail is neglected, and this variation is normally negligible. Then at any given angle of attack,  $C_L$  and  $\frac{V}{V_S}$  being constant, the following ratios hold in passing from density  $\rho_1$  to  $\rho_2$ .

$$\frac{V_2}{V_1} = \sqrt{\frac{\rho_1}{\rho_2}} \quad (102)$$

$$D_2 = D_1 \quad (103)$$

$$\frac{(T.H.p.)_2}{(T.H.p.)_1} = \sqrt{\frac{\rho_1}{\rho_2}} \quad (104)$$

These equations supplement equations (94), (95), and (96); they are often used in calculating performance at altitude.

Table 13 contains the calculations for  $V$  and  $T.H.p.$ , at 5,000, 10,000, 15,000, and 20,000 ft., using the sea-level data from Table 11.

**Calculation of power available at altitude.** The decrease in power available with altitude is given by Figure 99. This decrease includes the effect of changes in B.H.p., r.p.m., and propeller efficiency at constant air speed. Hence, if  $T.H.p._a$  is known for a given speed at sea level, the corresponding value for the same speed at any other altitude is  $T.H.p. = T.H.p._{a_0} \times \frac{T.H.p.}{T.H.p._0}$ , where the value of  $\frac{T.H.p.}{T.H.p._0}$  for the altitude in question is read from Figure 99.

Table 14 contains the calculation for  $T.H.p._a$  at various altitudes, based on the data in Table 12.

TABLE 13. THRUST POWER REQUIRED AT ALTITUDE

| Sea Level |                      | 5,000 ft.<br>$\sqrt{\frac{\rho_0}{\rho}} = 1.077$ |                     | 10,000 ft.<br>$\sqrt{\frac{\rho_0}{\rho}} = 1.164$ |                     | 15,000 ft.<br>$\sqrt{\frac{\rho_0}{\rho}} = 1.260$ |                     | 20,000 ft.<br>$\sqrt{\frac{\rho_0}{\rho}} = 1.370$ |                     | $\frac{V}{V_S}$ |
|-----------|----------------------|---------------------------------------------------|---------------------|----------------------------------------------------|---------------------|----------------------------------------------------|---------------------|----------------------------------------------------|---------------------|-----------------|
| V         | T.H.p. <sub>ro</sub> | V                                                 | T.H.p. <sub>r</sub> | V                                                  | T.H.p. <sub>r</sub> | V                                                  | T.H.p. <sub>r</sub> | V                                                  | T.H.p. <sub>r</sub> |                 |
|           |                      |                                                   |                     |                                                    |                     |                                                    |                     |                                                    |                     |                 |
| 52.6      | 51.0                 | 56.7                                              | 54.9                | 61.2                                               | 59.4                | 66.3                                               | 64.3                | 72.1                                               | 69.9                | 1.00            |
| 55.2      | 42.5                 | 59.5                                              | 45.8                | 64.3                                               | 49.5                | 69.5                                               | 53.5                | 75.6                                               | 58.2                | 1.05            |
| 57.8      | 41.8                 | 62.3                                              | 45.0                | 67.3                                               | 48.7                | 72.8                                               | 52.7                | 79.2                                               | 57.3                | 1.10            |
| 63.1      | 43.3                 | 68.0                                              | 46.7                | 73.5                                               | 50.4                | 79.5                                               | 54.6                | 86.5                                               | 59.3                | 1.20            |
| 73.7      | 52.7                 | 79.4                                              | 56.8                | 85.8                                               | 61.4                | 92.9                                               | 66.4                | 101.0                                              | 72.2                | 1.40            |
| 89.4      | 77.2                 | 96.3                                              | 83.1                | 104.2                                              | 89.9                | 112.6                                              | 97.3                | 122.4                                              | 105.7               | 1.70            |
| 105.2     | 116.0                | 113.3                                             | 125.0               | 122.5                                              | 135.2               | 132.5                                              | 146.2               | .....                                              | .....               | 2.00            |
| 121.0     | 170.0                | 130.3                                             | 183.2               | .....                                              | .....               | .....                                              | .....               | .....                                              | .....               | 2.30            |

NOTE: Values of  $\sqrt{\frac{\rho_0}{\rho}}$  are taken from Figure 159.

TABLE 14. MAXIMUM THRUST HORSEPOWER AVAILABLE AT ALTITUDES

| Air Speed<br>$V$<br>mi./hr. | $T.Hp_{.ao}$<br>Sea Level | T.Hp. AT ALTITUDE        |                           |                           |                           |
|-----------------------------|---------------------------|--------------------------|---------------------------|---------------------------|---------------------------|
|                             |                           | 5,000 ft.<br>$F = 0.820$ | 10,000 ft.<br>$F = 0.663$ | 15,000 ft.<br>$F = 0.532$ | 20,000 ft.<br>$F = 0.425$ |
| 50                          | 96.3                      | 79.0                     | 63.9                      | 51.2                      | 40.9                      |
| 60                          | 108.2                     | 88.8                     | 71.8                      | 57.6                      | 46.0                      |
| 70                          | 118.5                     | 97.2                     | 78.6                      | 63.1                      | 50.3                      |
| 80                          | 129.5                     | 105.4                    | 85.2                      | 68.4                      | 54.6                      |
| 90                          | 136.2                     | 111.7                    | 90.3                      | 72.5                      | 57.9                      |
| 100                         | 143.5                     | 117.7                    | 95.2                      | 76.3                      | 61.0                      |
| 110                         | 149.3                     | 122.4                    | 99.0                      | 79.4                      | 63.5                      |
| 120                         | 154.0                     | 126.3                    | 102.2                     | ....                      | ....                      |

$$F = \frac{T.Hp_a}{T.Hp_{.ao}}$$

**Calculation of performance at altitude.** If the values of  $T.Hp_r$  and  $T.Hp_a$  for various altitudes now be plotted against speed as in Figure 100, speeds and climbs are readily obtained by the graphical solution. Following the method previously used, one obtains:

| Altitude | Maximum Speed | Maximum E.Hp. | Climbing Speed<br>$V_C$ | Rate of Climb |
|----------|---------------|---------------|-------------------------|---------------|
| 0        | 116.0         | 70.0          | 70                      | 1,100         |
| 5,000    | 112.8         | 49.0          | 72                      | 770           |
| 10,000   | 108.3         | 30.4          | 74                      | 478           |
| 15,000   | 100.6         | 14.0          | 78                      | 220           |
| 20,000   | (85)          | ....          | ..                      | ....          |

The rates of climb in the fifth column are plotted against altitude in Figure 101, and a smooth curve drawn through the points. This curve intersects the altitude axis (rate of climb = 0) at about 19,550 ft. The absolute ceiling obtained by plotting rates of climb, therefore, checks almost exactly with the 19,500 ft. obtained by the short method illustrated on Figure 98. Figure 101 shows, however, that in general it is not permissible

to assume a linear relation between rate of climb and altitude. This subject will be discussed at length in the following chapter.

Figure 102 is a plot of maximum air speed and air speed for best climb, against altitude. The variation of maximum speed

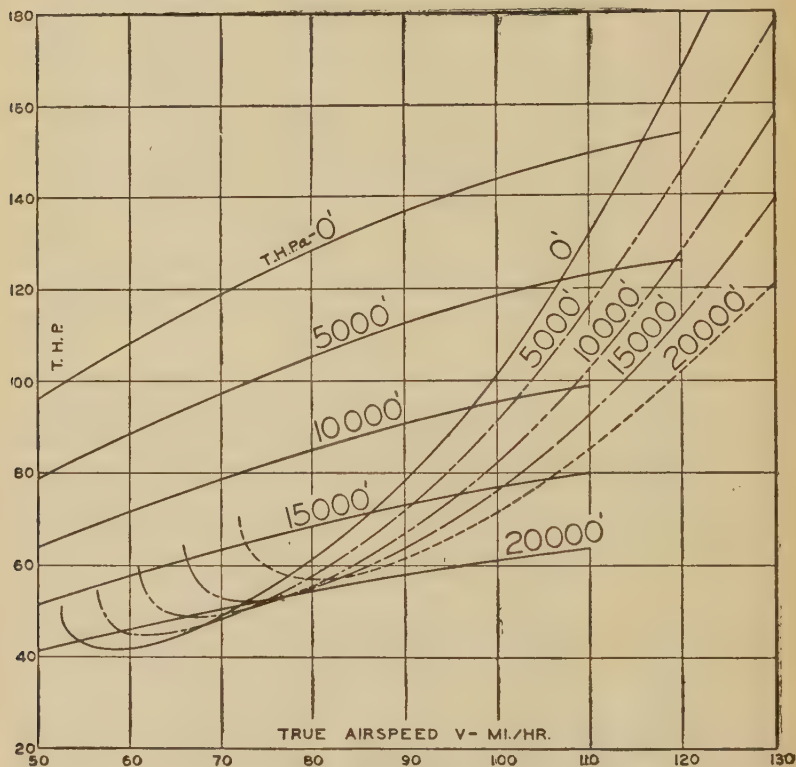


Figure 100. Curves of Power Available and Power Required at Various Altitudes

with altitude is much the same for all airplanes, but there may be considerable difference in the form of curve for best air speed in climb. Whether the best air speed in climb remains constant, or increases with altitude depends chiefly on the parasite resistance and only slightly on aspect ratio, as shown in Chapter IX.

**Effect of slip stream and miscellaneous corrections.** It is entirely unnecessary to apply any corrections other than those

given. In particular, the effects of slip stream on drag are either negligible or may better be handled by an appropriate reduction in propeller efficiency. It should be noted that the factor  $k$  (the ratio of the span of the equivalent monoplane to the span of a

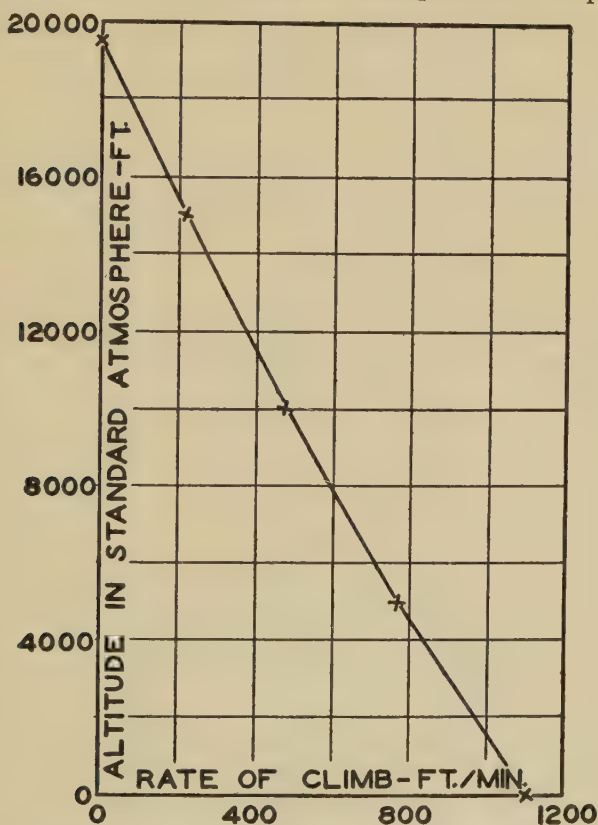


Figure 101. Variation of Rate of Climb with Altitude

biplane) includes the effect of the factors formerly listed as biplane interference, gap/chord ratio, and aspect ratio. The effect of stagger is considered negligible from a theoretical viewpoint.

The effectiveness of any method is in the consistency of the results obtained. The method outlined has been thoroughly tested and found far superior to any other so far developed.

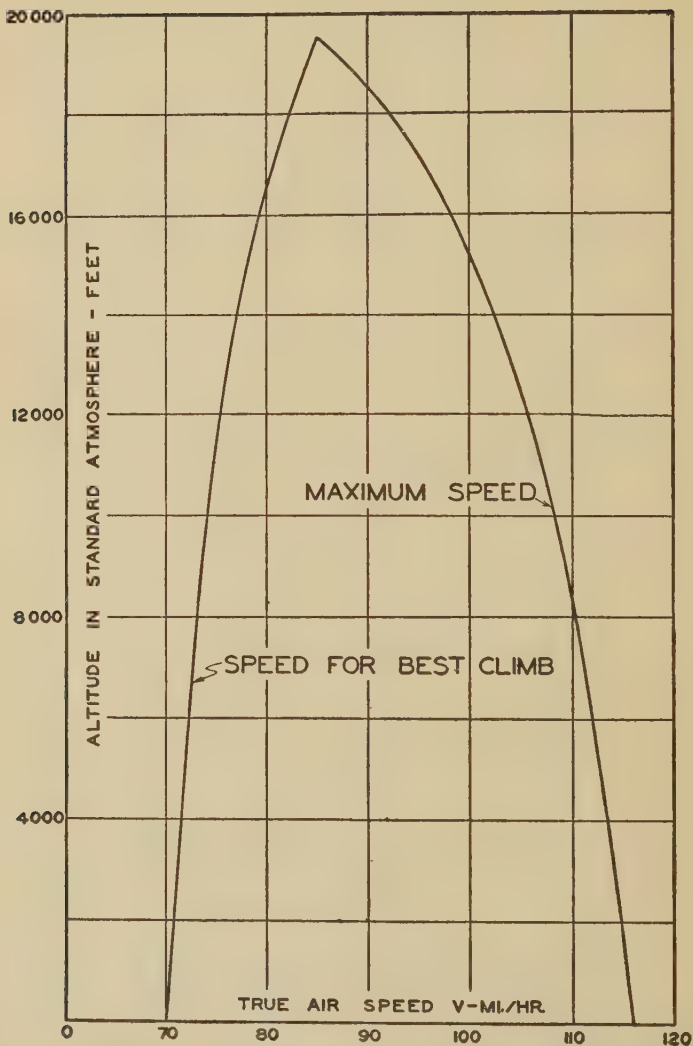


Figure 102. Variation of Maximum Speed and Best Climbing Speed with Altitude



## CHAPTER VIII

### VARIATION OF RATE OF CLIMB WITH ALTITUDE

In the past it has been customary to assume that the rate of climb decreased linearly with altitude, that is,

$$\frac{dh}{dt} = C_0 - ah \quad (98)$$

and this assumption was apparently justified by practically all of the early test data. In 1924 the author made a theoretical analysis which led to the conclusion that the relation should be of the form

$$\frac{d}{dh} \left( \frac{dh}{dt} \right) = -a + 2bh \quad (105)$$

which means physically that it is the *slope* of the rate of climb curve, instead of the curve itself, which is linear with altitude. Flight test data were analyzed and ample evidence of the existence of this relation found. Subsequently all of the accurately conducted Navy performance tests have been so analyzed and a large number of performances calculated for the same purpose. No exceptions being found, the conclusion is unavoidable that all future work must be on the basis of equation (105).

This does not lead to any great amount of extra work. Integrating equation (105) gives

$$\frac{dh}{dt} = C_0 - ah + bh^2 \quad (106)$$

A second integration gives

$$T = \frac{1}{\sqrt{a^2 - 4bC_0}} \log_e \left[ \frac{h(-a + \sqrt{a^2 - 4bC_0}) + 2C_0}{h(-a - \sqrt{a^2 - 4bC_0}) + 2C_0} \right] \quad (107)$$

$$\text{Letting } K = \frac{1}{\sqrt{a^2 - 4bC_0}}$$

$$C_1 = (-a + \sqrt{a^2 - 4bC_0})$$

and

$$C_2 = (-a - \sqrt{a^2 - 4bC_0})$$

the equation reduces to

$$T = K \log_e \left[ \frac{C_1 h + 2C_0}{C_2 h + 2C_0} \right] \quad (108)$$

At the absolute ceiling  $\frac{dh}{dt} = 0$ . Hence

$$bH^2 - aH + C_0 = 0$$

$$\text{and } H = \frac{+a - \sqrt{a^2 - 4bC_0}}{2b} \quad (109)$$

The time to climb to the absolute ceiling is infinite and this occurs when the denominator of the log term in equation (108) is zero. Hence

$$C_2 H + 2C_0 = 0$$

$$\text{or } H = \frac{-2C_0}{C_2} = \frac{2C_0}{+a + \sqrt{a^2 - 4bC_0}} \quad (110)$$

The *service ceiling* is given by  $bh_S^2 - ah_S + C_0 = \frac{dh}{dt} = 100$ , from which

$$h_S = \frac{+a - \sqrt{a^2 - 4b(C_0 - 100)}}{2b} \quad (111)$$

The time to climb to any altitude is readily obtained from equation (108) when written in the form

$$\frac{T}{e^K} = \frac{C_1 h + 2C_0}{C_2 h + 2C_0}$$

and solved for  $h$ , giving

$$h = \frac{2C_0 \left( \frac{T}{e^K} - 1 \right)}{C_1 - C_2 \frac{T}{e^K}} \quad (112)$$

**Values of the constants  $a$  and  $b$ .**  $a$  and  $b$  may be calculated from equation (105) if the slope to the rate of climb curve be measured at two or more altitudes, but this method is not practical because a slight error in reading the slope will be enormously magnified in the values of  $a$  and  $b$ . It is much better

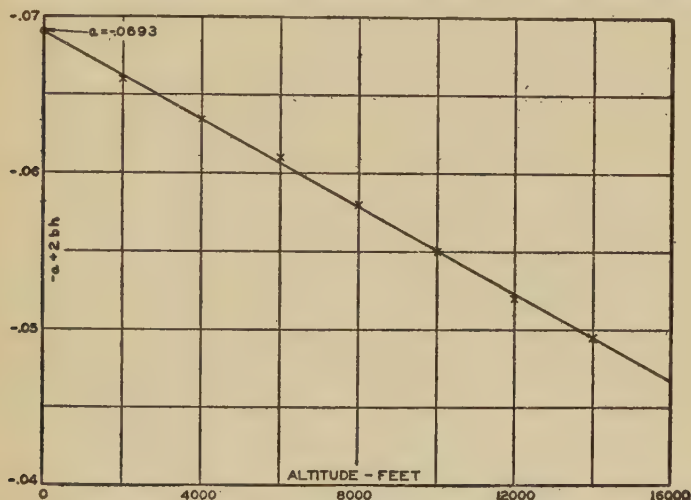


Figure 103. Graphical Solution for  $a$  and  $b$  in True Rate of Climb Equation

to plot the slopes against altitude and pass a straight line through the points. The intersection of this line with the slope axis ( $h = 0$ ) gives directly the value of  $a$ , while  $b$  is readily determined by the slope of the line.

To illustrate by an example, the values of  $\frac{d}{dh} \left( \frac{dh}{dt} \right)$  obtained from Figure 101 are as follows:

|              |        |        |        |        |        |        |        |
|--------------|--------|--------|--------|--------|--------|--------|--------|
| Altitude     | 2,000  | 4,000  | 6,000  | 8,000  | 10,000 | 12,000 | 14,000 |
| $(-a + 2bh)$ | -.0660 | -.0635 | -.0610 | -.0580 | -.0550 | -.0520 | -.0495 |

When  $(-a + 2bh)$  is plotted against altitude, Figure 103, a straight line passed through the points is found to intersect the axis at  $a = -.0693$ . To find  $b$ , take the reading at some altitude say 10,000 ft. where  $(-a + 2bh) = .0551$ . Hence

$$b = \frac{.0693 - .0551}{2 \times 10,000} = +.000000710. \quad \text{To obtain maximum ac-}$$

curacy this operation should be performed at the highest altitude on the plot; in this case 16,000 ft., where  $(-a + 2bh) = -.0466$ . Hence  $b = \frac{.0693 - .0466}{32,000} = +.000000710$ , checking exactly with the first value.

**Absolute ceiling.** Substituting these values,  $a = -.0693$  and  $b = .000000710$ , into equation (109) gives

$$H = \frac{+a + \sqrt{a^2 - 4bC_0}}{2b} = \frac{+.0693 - \sqrt{.00481 - .00312}}{2 \times .000000710} = 19,500 \text{ ft.}$$

which checks the value indicated on Figure 101.

**Service ceiling.** To illustrate the calculation for service ceiling, the same data will be used. The initial rate of climb was 1,100 ft./min. From equation (111) the service ceiling is

$$h_s = \frac{+a - \sqrt{a^2 - 4b(C_0 - 100)}}{2b} = \frac{.0693 - \sqrt{.00481 - .00284}}{.000000710 \times 2} = 17,500 \text{ ft.}$$

which also checks with the value indicated on Figure 101.

**Time of climb.** The values of the constants in equation (108) are:

$$K = \frac{1}{\sqrt{a^2 - 4bC_0}} = \frac{1}{\sqrt{.00481 - .00312}} = \frac{1}{.0411} = 24.3$$

$$C_1 = (-a + \sqrt{a^2 - 4bC_0}) = -.0693 + .0411 = -.0282$$

$$C_2 = (-a - \sqrt{a^2 - 4bC_0}) = -.0693 - .0411 = -.1104$$

The time of climb to any altitude  $h$  is, therefore, given by

$$T = 24.3 \times \log_e \left[ \frac{2,200 - .0282 h}{2,200 - .1104 h} \right]$$

Figure 105 contains curves for the true time of climb curve as calculated by this equation. On the same figure are given the climb curves for the two straight-line assumptions shown on Figure 104. Assumption No. 1 is that the rate of climb is linear

with altitude from the initial climb to the absolute ceiling. This assumption results in an entirely too optimistic time of climb curve and must be rejected. Assumption No. 2 is that the rate of climb is linear with altitude and that the slope is determined

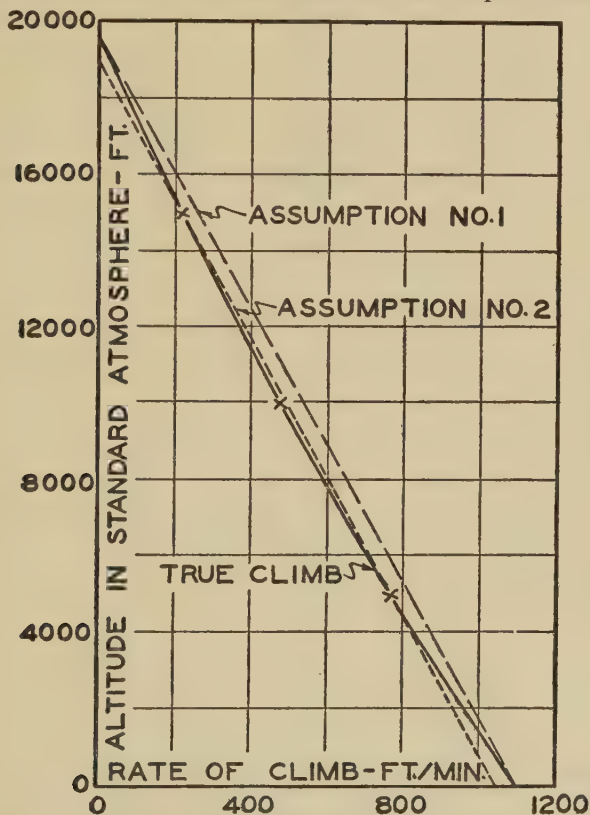


Figure 104. Comparison of True Rate of Climb with Assumed Rates

by the rates of climb at moderate altitudes. This assumption is very common in reducing performance to standard conditions, for obvious reasons. It leads to a reasonable approximation to the true times of climb, but the initial climb, absolute ceiling, and service ceiling are reduced 5% or more. If accuracy is desired, the true rate of climb curve must be drawn and the values of  $a$  and  $b$  determined for each case.

At this time it is desired to point out that the curvature in the rate of climb curve which has so far been considered, is due entirely to the relations between power available and power

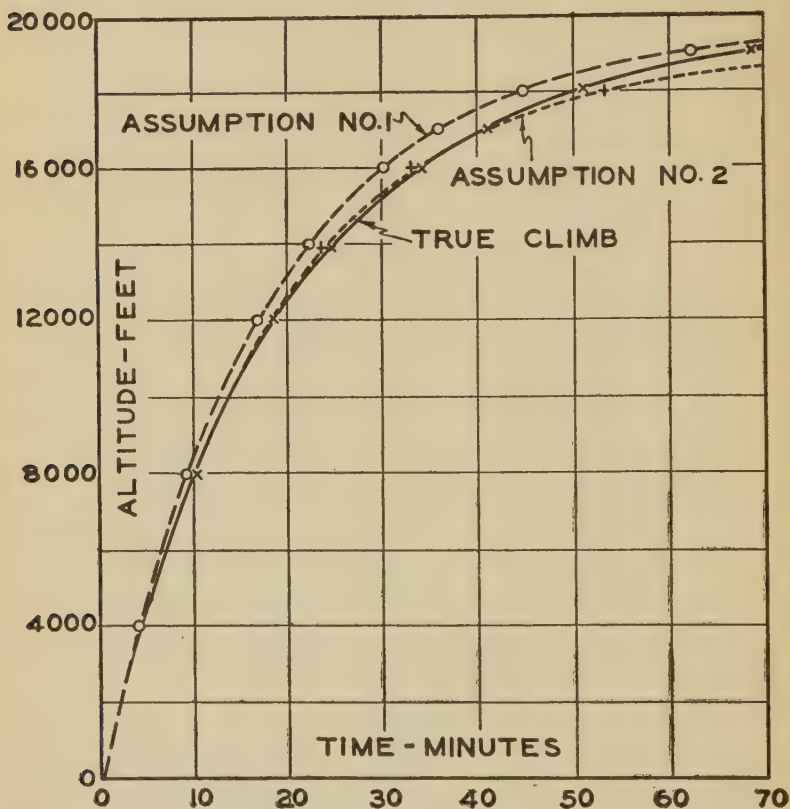


Figure 105. Comparison of True Time of Climb with Times Obtained from Assumptions Illustrated in Figure 104

required, and not to decrease in weight from use of fuel in the climb. The decrease in fuel load is normally sufficient to increase the service ceiling but slightly. The two effects are combined in the flight test data.

**Average values of  $a$  and  $b$ .** The value of  $a$  depends largely on the relation between absolute ceiling  $H$  and initial rate of



climb  $C_0$ , and may vary from .040 to .011, according to data now available. The value of  $b$  varies from about  $4.0 \times 10^{-7}$  to  $9.0 \times 10^{-7}$  with an average value around  $7.0 \times 10^{-7}$ .

When  $H$ , and  $C_0$  are given,  $a$  is readily calculated from the equation

$$a = \frac{C_0}{H} + b H$$

In performance estimates the value

$$b = 7.0 \times 10^{-7} \text{ may be used.}$$

## CHAPTER IX

### ASPECT RATIO AND PARASITE DRAG

The performance of an airplane is determined almost entirely by aspect ratio and parasite drag, yet few designers have more than a very general understanding of the relative importance of

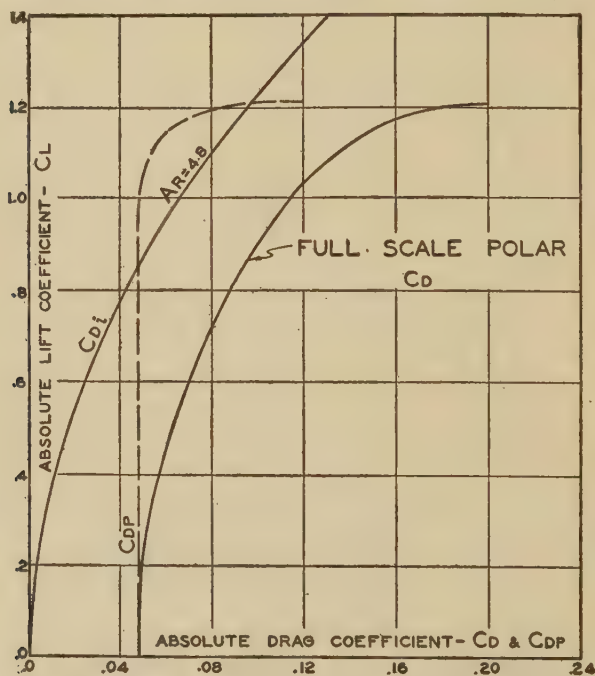


Figure 106. Full-Scale Polar for VE-7 Airplane

these two variables. Since much could be written on the subject, this short chapter supplies only an outline. In order to obtain the most striking results, the procedure adopted is as follows: An airplane for which the full-scale polar is available, is selected,

and the performance calculated for a series of aspect ratios with the original parasite coefficient  $C_{DP} = .046$ . This process is repeated with the total parasite coefficient reduced to  $C_{DP} = .031$ , corresponding to a reduction of about one-half in the original

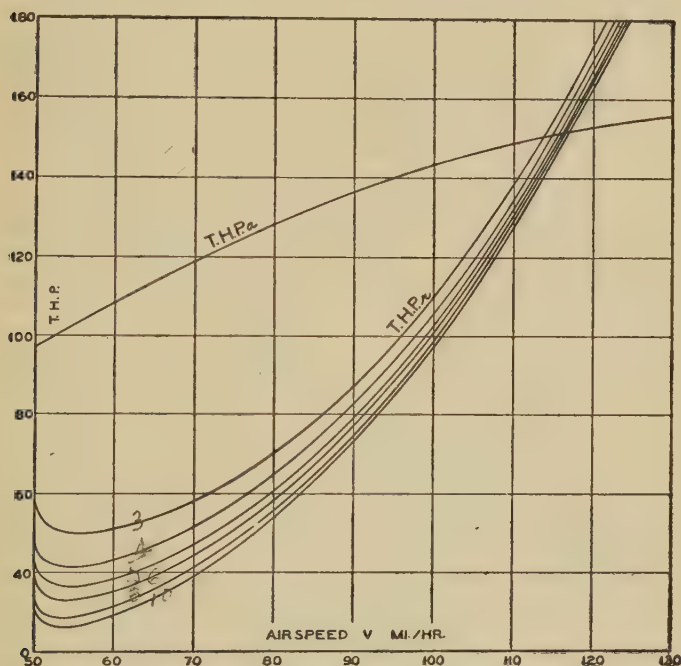


Figure 107. Curves of Power Required at Sea Level for Various Aspect Ratios with Normal Parasite

structural parasite, and also with the total parasite increased to  $C_{DP} = .076$ , corresponding to twice the original structural parasite. "Aspect ratio," here as elsewhere in this volume, is to be the "effective aspect ratio,"  $n = \frac{(kb)^2}{S}$ , as explained in Chapters

II and VII. The parasite coefficients vary with  $C_L$  and speed range but in referring to any condition, the constant coefficient at low lifts will be used. It includes the wing section drag.

The VE-7 airplane has been selected as the basis for the

calculations because the full-scale polar is available,<sup>†</sup> and also because it lies near an average value for both aspect ratio and parasite. Figure 106 is the full-scale polar as given on Figure 11 of N. A. C. A. Technical Report No. 220. On this figure are

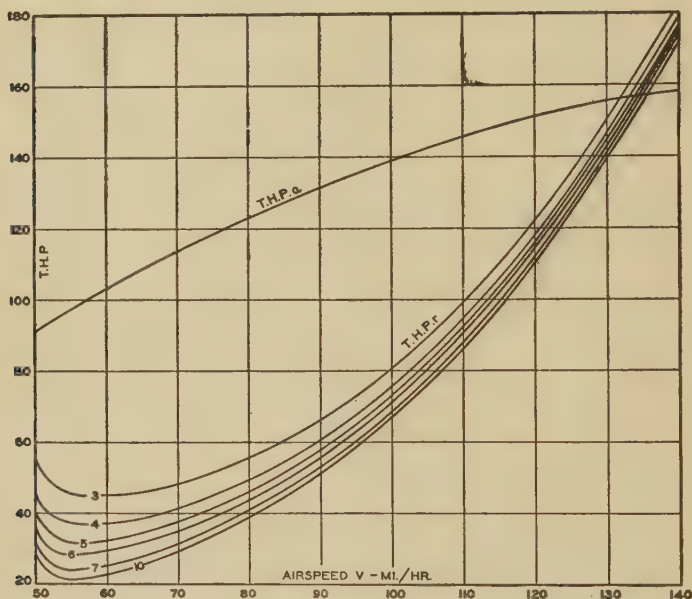


Figure 108. Curves of Power Required at Sea Level for Various Aspect Ratios with Low Parasite

also drawn the curve of induced drag coefficient for the VE-7 effective aspect ratio,  $n = 4.8$ , and the curve of parasite drag coefficient  $C_{DP} = C_D - C_{Di}$ . The full-scale maximum lift coefficient is approximately  $C_L = 1.20$ . The wing area is 290 sq. ft. With a gross load of 2,230 lbs., the stalling speed is 50 mi./hr.

The method of calculation was as follows: At a number of selected air speeds the corresponding values of  $C_L$  were calculated and  $C_{DP}$  read from Figure 106. The parasite drag and parasite  $T.H.p.$ , were then calculated from

<sup>†</sup> W. F. Durand and E. P. Lesley, "Comparison of Tests on Air Propellers in Flight with Wind Tunnel Model Tests on Similar Forms," N. A. C. A. Technical Report No. 220 (1925).

$$D_P = C_{DP} \frac{\rho}{2} S V^2$$

$$\text{and } T.H\dot{p}_{.rP} = \frac{D_P \times V}{375}$$

The same values of  $T.H\dot{p}_{.rP}$  were used with all aspect ratios. Since induced drag was not required, the induced power required, was calculated directly from the equation

$$T.H\dot{p}_{.ri} = \frac{.332}{V} \left( \frac{W}{kb} \right)^2 = \frac{.332 W^2}{n S V} = \frac{5,690}{n V}$$

$n = \frac{(kb)^2}{S}$  being the effective aspect ratio, and  $V$  the air speed in mi./hr. At any air speed, total  $T.H\dot{p}_{.r} = T.H\dot{p}_{.rP} + T.H\dot{p}_{.ri}$ .

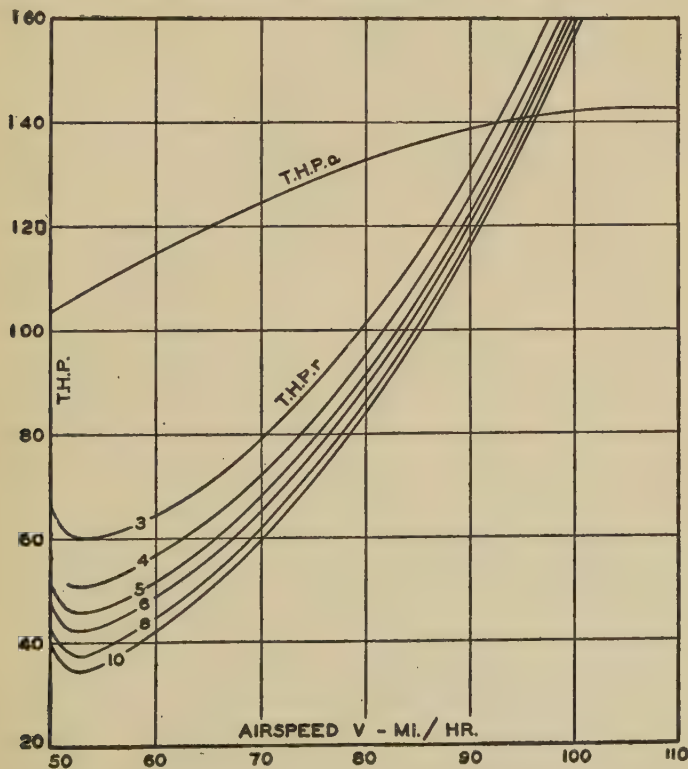


Figure 109. Curves of Power Required at Sea Level for Various Aspect Ratios with High Parasite

Values of  $T.H.p.$ , were calculated at various speeds for aspect ratios 3, 4, 5, 6, 8, and 10, with each of the parasite coefficients. Power curves were drawn for a series of altitudes so that speeds, rates of climb, and ceilings could be accurately determined.

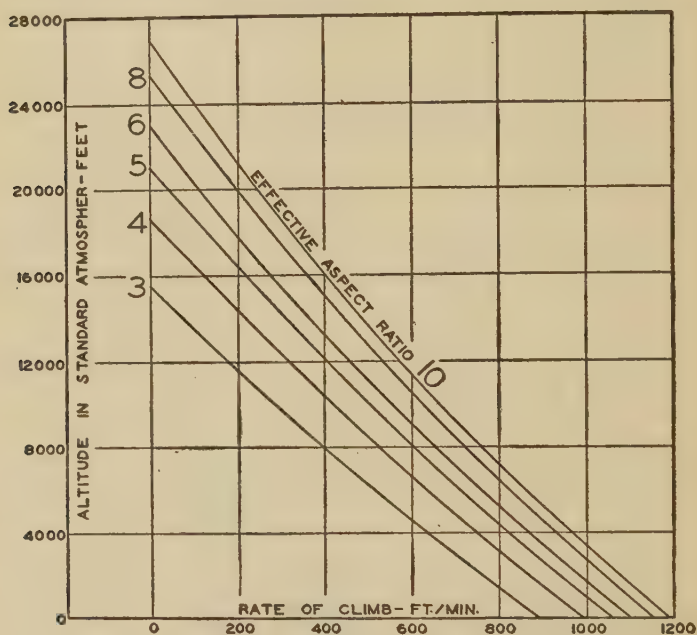


Figure 110. Rates of Climb for Various Aspect Ratios with Normal Parasite

In these calculations the Wright E engine developing 195 B.Hp. at 1,800 r.p.m. was used. (See Figure 10 of N. A. C. A. Technical Report No. 220.) Figures 107, 108, and 109 contain the sea-level power curves while Figures 110, 111, and 112 contain the rates of climb against altitude. Let us now consider each item of performance.

**High speed at sea level.** It is immediately apparent from an inspection of Figures 107, 108, and 109, that the effect of parasite is tremendously greater than that of aspect ratio. When  $C_{DP} = .046$ , the high speed increases from 114.0 to 117.3 mi./hr.



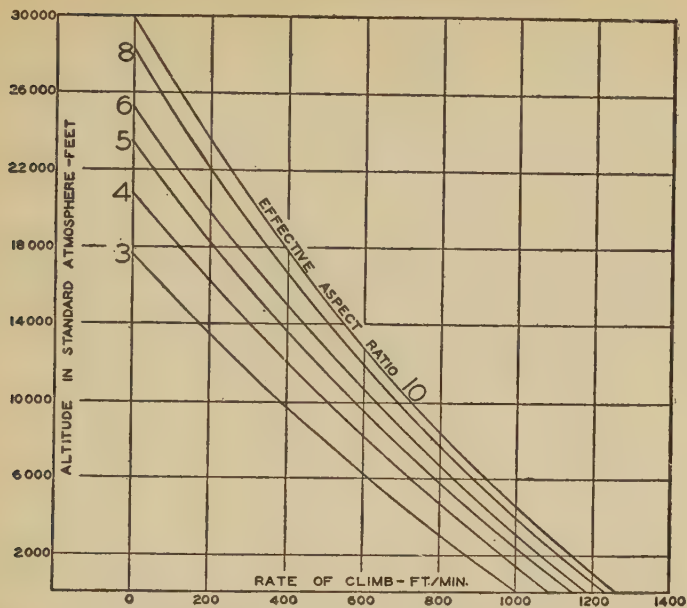


Figure III. Rates of Climb for Various Aspect Ratios with Low Parasite

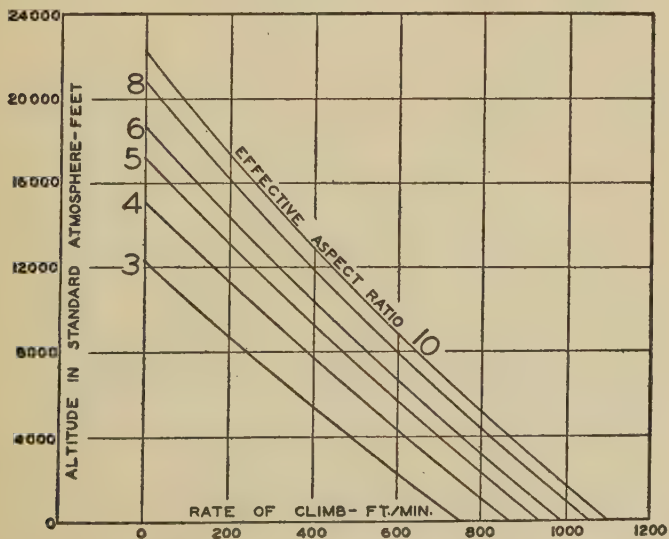


Figure II2. Rates of Climb for Various Aspect Ratios with High Parasite

as the effective aspect ratio is increased from 3 to 10 and a similar increase is obtained at the other parasite values. In contrast to the small effect of aspect ratio, the effect of changing  $C_{DP}$  from .046 to .031 is to increase the high speed (at aspect ratio 5) from 115.7 to 134 mi./hr. The relative effects are more clearly brought out by Figure 113.

It must be understood that these curves merely show the effect of aspect ratio and parasite on the high speed of an airplane having 290 sq. ft. of wing area, weighing 2,230 lbs. and fitted with an engine developing 195 B.Hp. at 1,800 r.p.m. They do not apply to any other wing area, weight or power.

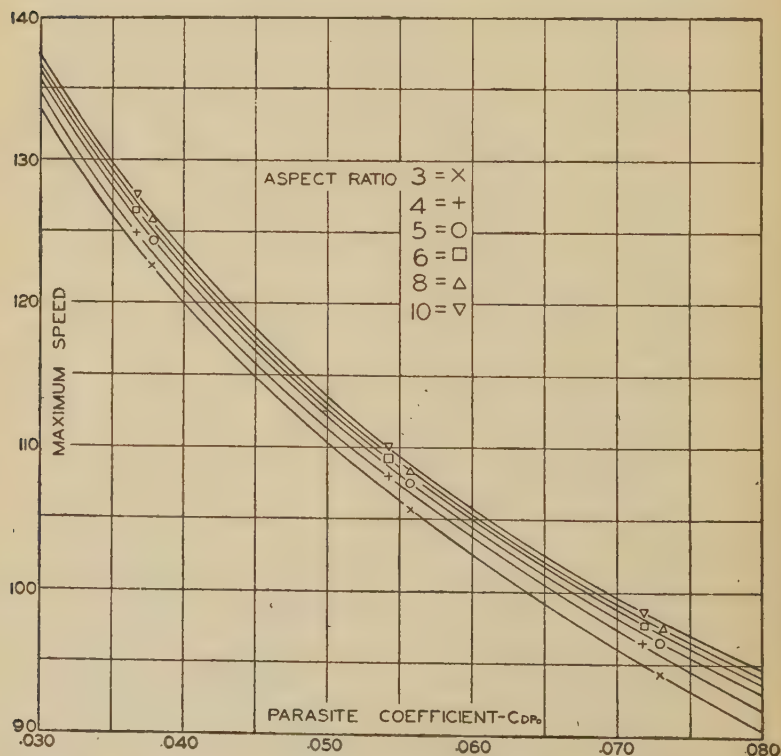


Figure 113. Comparative Effect of Aspect Ratio and Parasite on High Speed at Sea Level

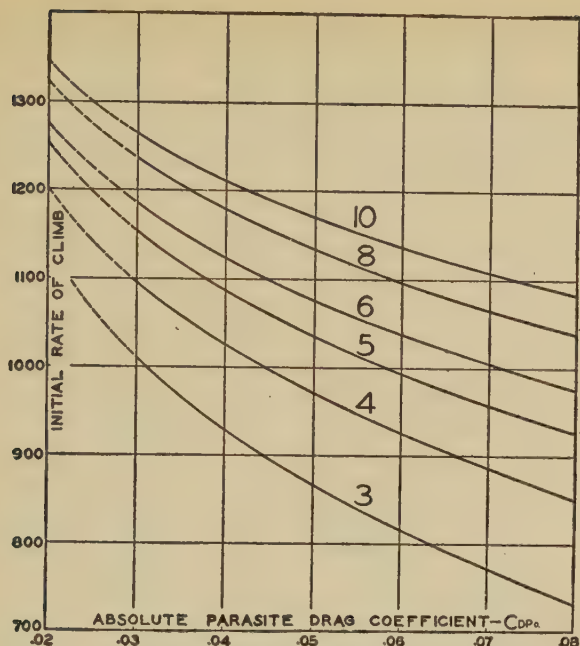


Figure 114. Comparative Effect of Aspect Ratio and Parasite on Initial Rate of Climb

**Initial rate of climb.** Figure 114 is a plot of initial rate of climb against aspect ratio and parasite coefficient. This figure shows that the higher the parasite, the greater the effect of aspect ratio. In general, aspect ratio has somewhat more effect than parasite on the initial rate of climb.

The percentage changes shown on this figure may be deceptive. The actual *increments* shown will hold approximately for any basic rate of climb. For example, if the power be increased so that the initial climb is 2,000 ft./min. instead of 1,000 ft./min., then all of the other climbs will be increased by approximately 1,000 ft./min. Consequently aspect ratio becomes increasingly important as the initial rate of climb is decreased.

**Absolute ceiling.** Figure 115 is a plot of absolute ceiling against aspect ratio and parasite coefficient. The effect of a

change in aspect ratio is to add or subtract a constant amount from the ceiling regardless of the parasite coefficient. Aspect ratio is of more importance than parasite coefficient, and particularly so when the normal ceiling is low.

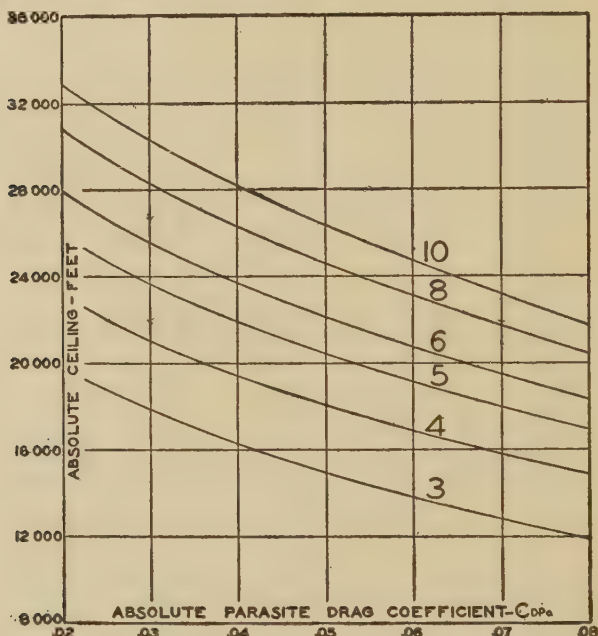


Figure 115. Comparative Effect of Aspect Ratio and Parasite on Absolute Ceiling

These curves, like the high-speed curves, apply only to the one particular wing area, weight, and power, used in the calculations. They cannot be used directly to estimate the absolute ceiling for any other weight, wing area, and power.

Maximum  $\frac{L}{D}$ . Values of  $\frac{L}{D}$  may be calculated from the relation

$$\frac{L}{D} = \frac{WV}{375 \cdot T.H\dot{p}_r} \quad (113)$$

where  $T.H\dot{p}_r$  is the total thrust power required for horizontal

flight at the air speed  $V$  mi./hr. Values of  $\frac{L}{D}$  were obtained in this manner and plotted against  $V$  in order to determine the maximum and the corresponding air speed. These maxima are plotted against aspect ratio and parasite coefficient in Figure 116, which

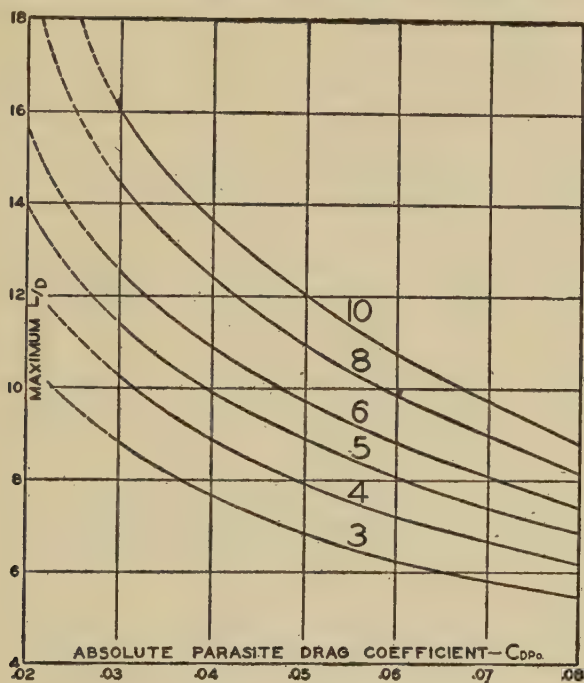


Figure 116. Variation of Maximum  $\frac{L}{D}$  with Aspect Ratio and Parasite

is of a general nature and holds true within narrow limits for all airplanes having a stalling speed of approximately 50 mi./hr.

Figure 117 is a plot of the ratio (air speed at maximum  $\frac{L}{D}$ )/(stalling speed), against aspect ratio and parasite coefficient.

**Speed for minimum power.** Figure 118 is a plot of the ratio (speed for minimum power)/(stalling speed), against aspect ratio and parasite coefficient. This information is of considerable value in estimating maximum endurance and absolute ceilings.

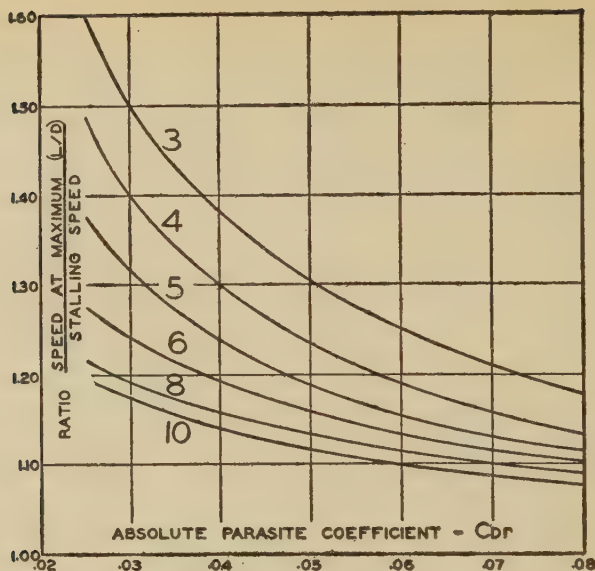


Figure 117. Effect of Aspect Ratio and Parasite on Air Speed for Maximum  $\frac{L}{D}$

**Speed range constant  $K$ .** The writer showed in N. A. C. A. Technical Report No. 173,<sup>2</sup> that the speed range of the average airplane was given with considerable accuracy by

$$\frac{V_m}{V_s} = \frac{K \left( \frac{L}{D} \cdot \eta_m \right)^{\frac{1}{2}}}{\left( V_s \cdot \frac{W}{H\rho} \right)^{\frac{1}{2}}} \quad (114)$$

where  $K$  had an average value of 10.2 with negligible variation due to changes in wing loading and power loading, at constant aspect ratio, and parasite coefficient. As a by-product of the present investigation, the variation of  $K$  with aspect ratio and parasite coefficient has been determined and is plotted on Figure 119. The original value ( $K = 10.2$ ) need not be changed for the average airplane, but the extreme cases likely to be met

<sup>2</sup> W. S. Diehl, "Reliable Formula for Estimating Airplane Performance and the Effects of Changes in Weight, Wing Area or Power," (1923).



range from about 9.5 to 10.8, with a normal variation from about 10.0 to 10.4.  $\frac{L}{D}$  may be obtained from Figure 116.

**Initial climbing speed factor.** It was shown in N. A. C. A. Technical Report No. 173, that the initial rate of climb was given by

$$C_0 = 33,000 \left[ \frac{K_2 \eta_m}{\frac{W}{H p.}} - \frac{V_S + K_3 (V_m - V_S)}{375 \frac{L}{D}} \right] \quad (115)$$

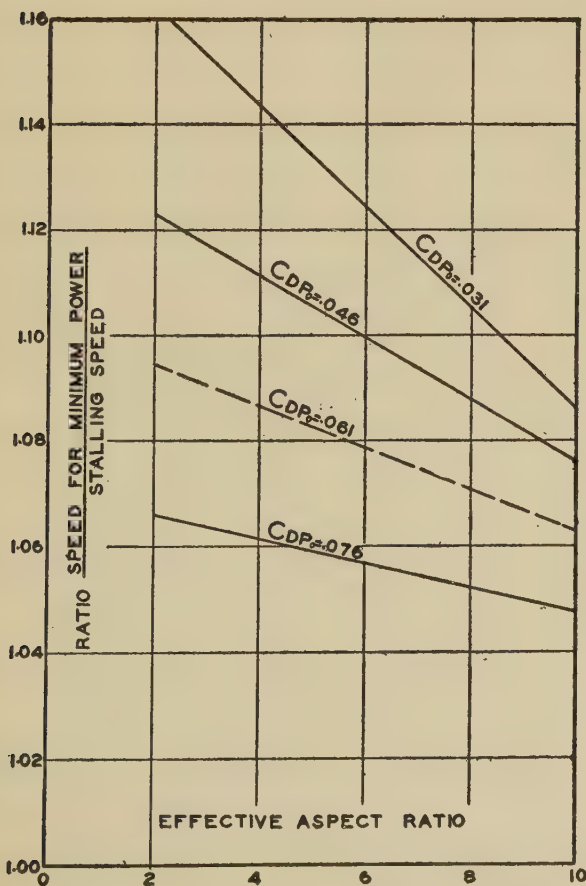


Figure 118. Effect of Aspect Ratio and Parasite on Speed for Minimum Power

where  $K_3$  had an average value of about 0.30 for the cases then considered. As a matter of convenience it was assumed that  $K_3 = \frac{1}{3}$  so that the climbing speed  $V_C = V_S + \frac{1}{3} (V_m - V_S)$ .

Values of  $K_3$  have been determined as a part of this study and are plotted against aspect ratio and parasite in Figure 120. It appears that  $K_3$  may be expected to vary from 0.25 to about 0.33 under normal conditions and between 0.20 and 0.35 under extreme conditions. 0.28 is a better average value than 0.33.

Figure 120 may be used to estimate the sea-level air speed for best climb.

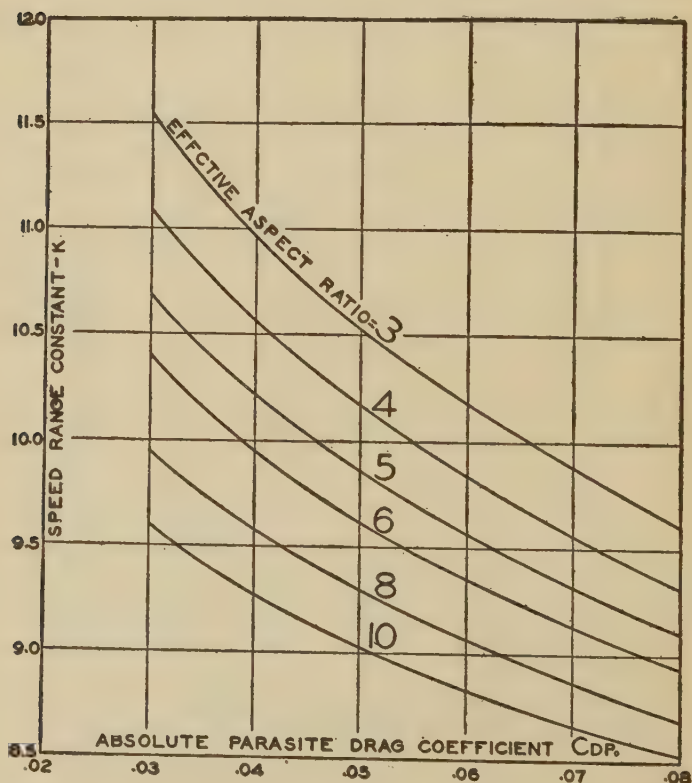


Figure 119. Effect of Aspect Ratio and Parasite on Speed Range Constant  $K$

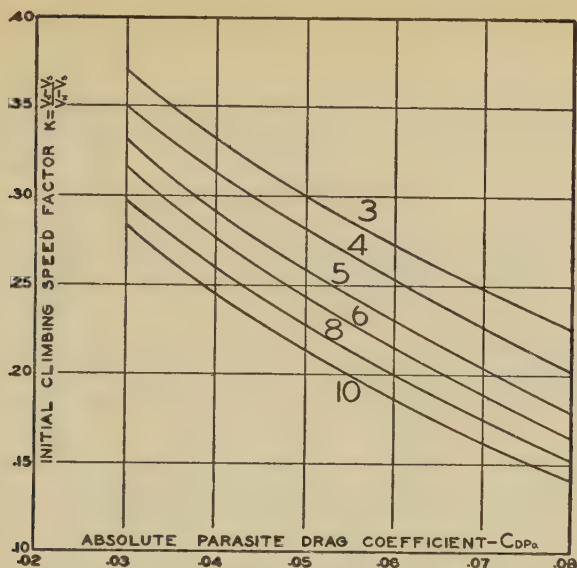


Figure 120. Effect of Aspect Ratio and Parasite on Air Speed for Best Climb at Sea Level

**Climbing speeds at altitude.** This study will be completed by Figure 121 showing the effect of aspect ratio and parasite on the air speed for best climb at various altitudes. This and the preceding Figure should be of some value to test pilots in laying out air speeds for best climb.

The maximum speeds at altitude will be analyzed in Chapter X.

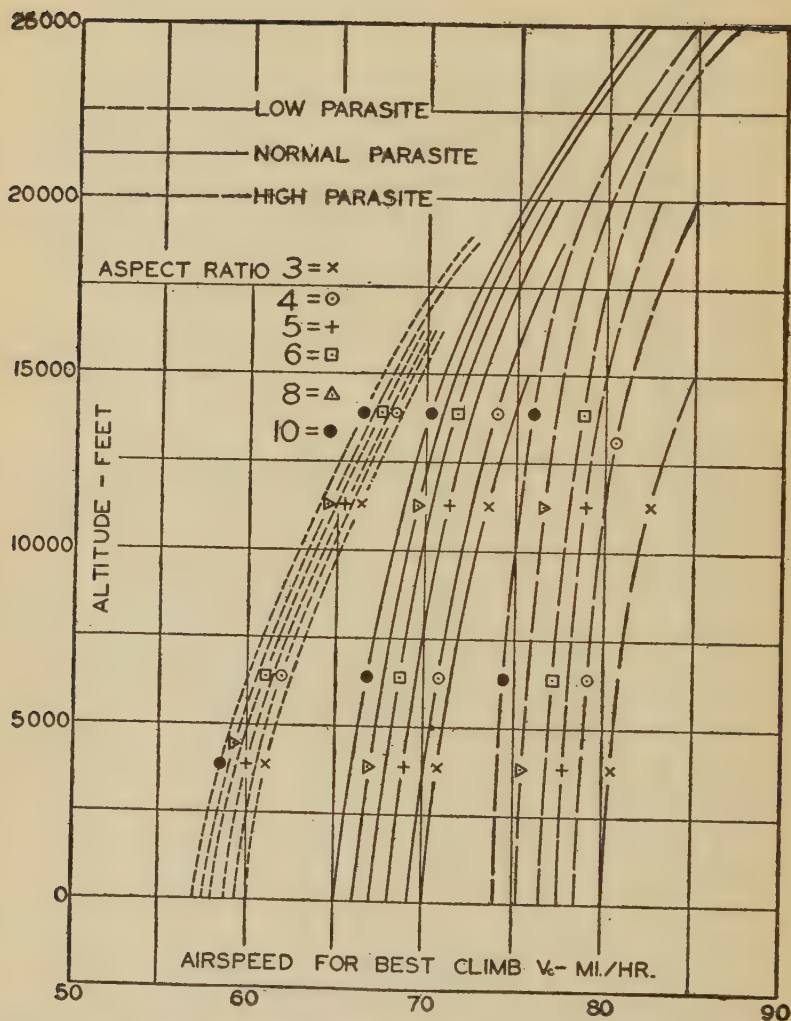


Figure 121. Effect of Aspect Ratio and Parasite on Air Speed for Best Climb at Altitudes

## CHAPTER X

### REDUCTION OF OBSERVED PERFORMANCE TO STANDARD CONDITIONS

Since the power required for horizontal flight at a constant angle of attack varies as  $\sqrt{\frac{\rho_0}{\rho}}$  regardless of the actual pressure and temperature while the thrust power available varies approximately as  $P^{1.15}$  and  $T^{-0.5}$ , it follows that the performance obtained at any given pressure varies with the temperature at that pressure. "Reduction to standard conditions" is descriptive of the corrections that must be applied to observed performance in order to bring into agreement test data obtained under different temperature-pressure relations. The standard condition generally agreed on is an arbitrary specified variation of pressure and free-air temperature with altitude, designated as the "standard atmosphere." If the method of reduction is correct, the reduced performance in standard atmosphere will be identical, regardless of the temperature-pressure conditions during the flights, providing necessarily that the temperatures are within reasonable limits imposed by the engine-cooling system.

Two general methods have been widely used in performance reduction. In the older of the two, the "density" method, an observed rate of climb or an observed air speed is plotted at the altitude in standard atmosphere at which the density is that determined by the observed pressure and temperature. In the other, the "pressure" method, the altitude is determined by the pressure only. Neither method is satisfactory, but in general, the pressure method is the better of the two. If the

temperatures do not depart more than a few degrees from standard, both methods give very nearly identical results, but as the temperatures diverge from standard, the reduced data become unreliable. Consider two climbs, one made with temperature  $10^{\circ}\text{C.}$  above normal, the other with temperature  $10^{\circ}\text{C.}$  below normal at all altitudes. On the density basis, the reduction gives two approximately parallel climb curves, one starting at  $+1,170$  ft. and the other at  $-1,230$  ft. On the pressure basis, the reduction gives two diverging curves, both starting at the same point. It can, therefore, be definitely stated that the density method will not give consistent results, and that the pressure method is the better, but not entirely satisfactory.

Several modifications to each method have been used at one time or another. A method recently proposed and investigated in England is<sup>1</sup> based on the use of an engine power-factor varying as  $P^{\frac{1}{3}}\rho^{\frac{1}{3}}$  and according to the British reports this method is superior to the pressure method. It is of interest to note that the relation  $P^{\frac{1}{3}}\rho^{\frac{1}{3}}$  is equivalent to  $PT^{-\frac{1}{3}}$  which may be compared with the relation  $P^{1.15}T^{-0.5}$  used by the author.

Instead of trying to fit various theoretical or empirical methods to observed performance, it appears more logical to determine the variation of performance with temperature and pressure and thus determine an accurate method of performance reduction. An extensive series of systematic performance calculations have been made for this purpose and very definite results obtained. Owing to the great number of calculations and curves involved, it will be possible to give only a brief outline of the methods followed and the final results.

In the first series of calculations, use was made of the performance data for aspect ratio 4.8 and normal parasite, as calculated for standard conditions in the study of aspect ratio and parasite in Chapter IX. Powers required and available

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<sup>1</sup> R. & M. No. 1080, "The Reduction of Performance Tests on the Standard Atmosphere," by R. S. Capon (1927); and R. & M. No. 1099, "A Discussion of the Law of Variation of Engine Power with Height," by H. Glauert (1927).



were calculated for absolute temperatures 83.3%, 90.9%, 110%, and 120% normal at constant pressures corresponding to altitudes of 4,000, 8,000, 10,000, 12,000, and 16,000 ft. The particular low temperatures used were selected on account of the simplification in slide-rule calculations. Powers available were assumed to vary as  $P^{1.15}T^{-0.5}$ . The data were plotted on a fairly large scale so that maximum speeds, climbing speeds, and actual rates of climb could be determined accurately.

In the second series of calculations the data on the six aspect ratios and three parasite coefficients considered in Chapter IX were used. This investigation was limited to temperatures 83.3% and 120% normal at 10,000 ft. only, the first series having shown the effects to be linear with temperature and independent of altitude. The results of this investigation are outlined in the following paragraphs.

**The reduction of climb to standard.** Table 15 contains a summary of the results of the first series of calculations at 12,000 ft. pressure altitude. It was immediately apparent from an inspection of the preliminary data that a definite relation existed between the pressure altitude  $h_p$ , the density altitude  $h_d$ , and what may be called the "equivalent altitude" in standard atmosphere  $h$ . (At an "equivalent altitude," under standard conditions, the rate of climb is the same as that obtained under a given non-standard condition.) As shown by Table 15 the final relation was found to be

$$h = h_p - K (h_p - h_d) \quad (116)$$

The values of  $K$  found at altitudes other than 12,000 ft. are as follows:

|                  | $\frac{T}{T_s}$ | .833 | .909 | 1.10 | 1.20 |
|------------------|-----------------|------|------|------|------|
| Values of $K$ at |                 |      |      |      |      |
| 4,000'           | .....           | .354 | .353 | .350 | .362 |
| 8,000'           | .....           | .337 | .328 | .331 | .382 |
| 10,000'          | .....           | .336 | .331 | .399 | .377 |
| 16,000'          | .....           | .393 | .371 | .347 | .367 |

TABLE 15. EFFECT OF TEMPERATURE ON ACTUAL RATE OF CLIMB AT 12,000 FT.

| Temperature/Standard Temp.....                                   | 0.833  | 0.909  | 1.000  | 1.100  | 1.200  |
|------------------------------------------------------------------|--------|--------|--------|--------|--------|
| Pressure Altitude, $h_p$ ft. ....                                | 12,000 | 12,000 | 12,000 | 12,000 | 12,000 |
| Density Ratio, $\rho/\rho_0$ .....                               | 0.832  | 0.762  | 0.6931 | 0.630  | 0.577  |
| Density Altitude, $h_d$ .....                                    | 6,150  | 8,990  | 12,000 | 14,950 | 17,620 |
| Max. Speed, $V_m$ mi./hr. ....                                   | 103.3  | 104.7  | 104.7  | 105.6  | 105.7  |
| Climbing Speed, $V_C$ mi./hr. ....                               | 66.5   | 68.0   | 70.0   | 73.0   | 76.0   |
| Excess $THP$ (max).....                                          | 33.4   | 30.2   | 26.4   | 22.7   | 19.3   |
| Actual Rate of Climb, ft./min.....                               | 500    | 447    | 390    | 336    | 285    |
| Altitude in Stand. Atm. for Actual Rate of Climb, $h-h$ ft. .... | 9,850  | 10,850 | 12,000 | 13,050 | 14,100 |
| $(h_p-h)$ .....                                                  | 2,150  | 1,150  | 0      | -1,050 | -2,100 |
| $(h_p-h_d)$ .....                                                | 5,850  | 3,010  | 0      | -2,950 | -5,620 |
| $K = \frac{h_p-h}{h_p-h_d}$ .....                                | 0.368  | 0.382  |        | 0.356  | 0.373  |
| $\Delta V_m$ .....                                               | -1.4   | -0.5   | 0      | +0.9   | +1.0   |

TABLE 16. VALUES OF  $K = \frac{h_p - h}{h_p - h_d}$  FOR VARIOUS ASPECT RATIOS AND PARASITE COEFFICIENTS

| Effective<br>Aspect<br>Ratio | Normal Parasite          |                         |         | Low Parasite             |                         |         | High Parasite            |                         |         | General<br>Average |
|------------------------------|--------------------------|-------------------------|---------|--------------------------|-------------------------|---------|--------------------------|-------------------------|---------|--------------------|
|                              | $\frac{T}{T_S}$<br>0.833 | $\frac{T}{T_S}$<br>1.20 | Average | $\frac{T}{T_S}$<br>0.833 | $\frac{T}{T_S}$<br>1.20 | Average | $\frac{T}{T_S}$<br>0.833 | $\frac{T}{T_S}$<br>1.20 | Average |                    |
|                              |                          |                         |         |                          |                         |         |                          |                         |         |                    |
|                              |                          |                         |         |                          |                         |         |                          |                         |         |                    |
| 3                            | .369                     | .450                    | .407    | .361                     | .406                    | .382    | .391                     | .450                    | .418    | .401               |
| 4                            | .344                     | .423                    | .379    | .327                     | .388                    | .355    | .374                     | .432                    | .400    | .378               |
| 5                            | .327                     | .406                    | .363    | .336                     | .362                    | .348    | .357                     | .389                    | .371    | .360               |
| 6                            | .319                     | .388                    | .350    | .304                     | .362                    | .331    | .341                     | .389                    | .363    | .348               |
| 8                            | .302                     | .371                    | .333    | .294                     | .344                    | .317    | .332                     | .381                    | .354    | .335               |
| 10                           | .304                     | .335                    | .319    | .285                     | .318                    | .300    | .307                     | .355                    | .329    | .316               |

Values of  $K$  found in the second series are given in Table 16. The variation in  $K$  is so slight that the value 0.36 may be used without appreciable error for any normal airplane.

Two methods of plotting are available. A curve of equivalent altitude  $h = h_p - 0.36 (h_p - h_d)$  against time may be drawn and the rates of climb at various altitudes determined from the slope of the curve, or the geometrical rate of climb may be calculated and plotted against  $h$ .

**The reduction of maximum speed to standard conditions.** Referring to Table 15, it is seen that the maximum speed at 12,000 ft. pressure altitude is decreased 1.4 mi./hr. by temperatures 83.3% normal and increased 1.0 mi./hr. by temperatures 120% normal. These two conditions correspond to density altitudes of 6,150 ft. and 17,620 ft., respectfully. It is therefore obvious that speeds should not be plotted on the density basis.

Table 17 contains the maximum speed data for the six aspect ratios and three parasite coefficients at temperatures 83.3% and 120% normal at 12,000 ft. pressure altitude. These two temperatures correspond to density altitudes of 4,040 ft. and 15,670 ft. respectively. A study of the speed data shows that the effect of a temperature change at a given pressure increases very slightly with increase in aspect ratio and with decrease in parasite coefficient. Low temperatures decrease the maximum speed, high temperatures increase it, at a given pressure altitude. Up to 70% of the absolute ceiling, the change in high speed for normal temperatures is less than the experimental error, so that actual measured true air speed may be plotted, without correction, against the pressure altitude. A considerable error might easily be possible at altitudes above 70% of the absolute ceiling, if the measured speeds are followed too closely. The relations are such in this region that in all probability no satisfactory method of exact reduction can be found. It is recommended that this part of the data be obtained by extrapolation, to the absolute ceiling, of the curves of maximum speed and climbing speed plotted against altitude.

TABLE 17. EFFECT OF TEMPERATURE ON MAXIMUM SPEED AT 10,000 FT. PRESSURE ALTITUDE

| Effective Aspect Ratio | $\frac{T}{T_S}$          | Normal Parasite |            |               | Low Parasite  |            |               | High Parasite |           |              |
|------------------------|--------------------------|-----------------|------------|---------------|---------------|------------|---------------|---------------|-----------|--------------|
|                        |                          | = 1.00          |            | = 1.20        | = 1.00        |            | = 1.20        | = 1.00        |           | = 1.20       |
|                        |                          | = 0.833         |            |               | = 0.833       |            |               | = 0.833       |           |              |
| 3                      | $\frac{V_M}{\Delta V_M}$ | 101.2<br>-1.0   | 102.2<br>0 | 100.0<br>-2.2 | 118.7<br>-1.5 | 120.2<br>0 | 120.6<br>+0.4 | 83.6<br>+0.5  | 83.1<br>0 | 81.1<br>-2.0 |
| 4                      | $\frac{V_M}{\Delta V_M}$ | 103.8<br>-1.9   | 105.7<br>0 | 106.2<br>+0.5 | 121.0<br>-2.8 | 123.8<br>0 | 126.4<br>+2.6 | 85.9<br>-1.5  | 87.4<br>0 | 87.5<br>+0.1 |
| 5                      | $\frac{V_M}{\Delta V_M}$ | 105.2<br>-2.4   | 107.6<br>0 | 109.1<br>+1.5 | 122.5<br>-2.9 | 125.4<br>0 | 128.1<br>+2.7 | 87.3<br>-1.5  | 88.8<br>0 | 90.2<br>+1.4 |
| 6                      | $\frac{V_M}{\Delta V_M}$ | 106.2<br>-2.4   | 108.6<br>0 | 110.8<br>+2.2 | 123.1<br>-2.5 | 126.6<br>0 | 129.8<br>+3.2 | 88.4<br>-1.9  | 90.3<br>0 | 91.9<br>+1.6 |
| 8                      | $\frac{V_M}{\Delta V_M}$ | 107.6<br>-2.8   | 110.4<br>0 | 112.8<br>+2.4 | 124.4<br>-3.6 | 128.0<br>0 | 131.3<br>+3.3 | 89.4<br>-2.4  | 91.8<br>0 | 93.7<br>+1.9 |
| 10                     | $\frac{V_M}{\Delta V_M}$ | 108.2<br>-3.0   | 111.2<br>0 | 114.2<br>+3.0 | 125.0<br>-3.6 | 128.6<br>6 | 132.5<br>+3.9 | 90.0<br>-2.6  | 92.6<br>0 | 94.8<br>+2.2 |

**Example of climb reduction to standard.** Tables 18 and 19 contain the observed data and calculated altitudes for two climbs on a typical airplane. In climb No. 1 the ground temperature was  $26.5^{\circ}\text{C}$ . and at all altitudes the temperature was higher than normal. In climb No. 2 the ground temperature was  $12^{\circ}\text{C}$ . and the temperature at altitudes was sometimes below, sometimes above normal. The altitudes  $h$ , calculated from equation (116), are plotted on Figure 122 and the rates of climb determined by the slopes of the curves. In spite of the considerable difference in initial conditions, the rates of climb are in excellent agreement when reduced by the new method.

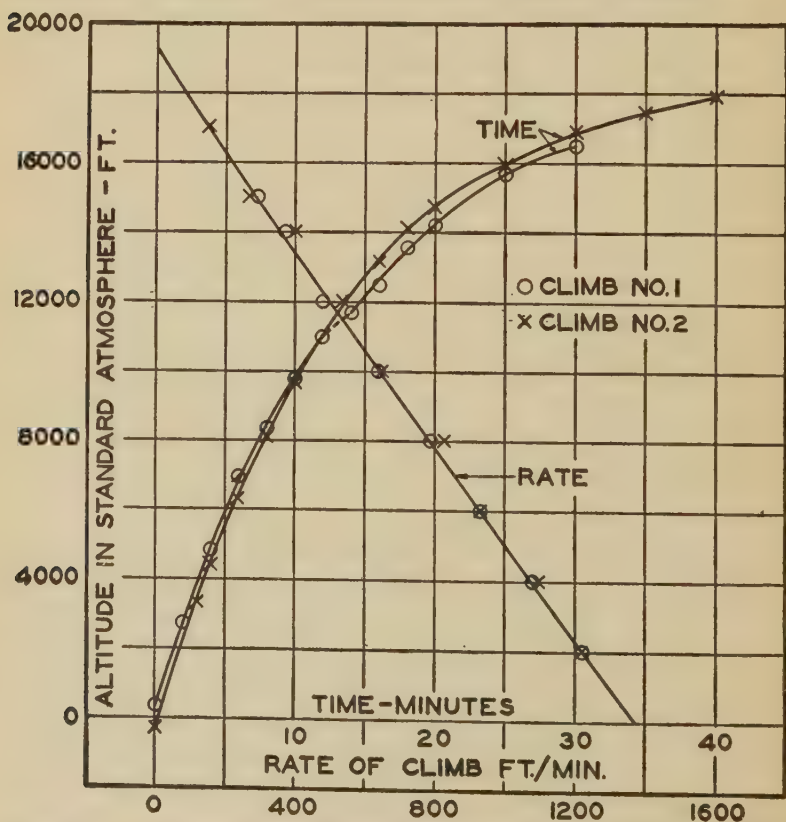


Figure 122. Comparison of Reduced Performance from Two Climbs by Different Pilots



TABLE 18. REDUCTION OF CLIMB TO STANDARD, CLIMB NO. 1

| OBSERVED DATA |                        |                     | ALTITUDES (ft.)   |                  |             |            |        |
|---------------|------------------------|---------------------|-------------------|------------------|-------------|------------|--------|
| Time<br>Min.  | Pressure<br>Mm.<br>Hg. | Air<br>Temp.<br>°C. | Pressure<br>$h_p$ | Density<br>$h_d$ | $h_p - h_d$ | $\Delta h$ | $h$    |
| 0             | 762                    | 26.5                | -70               | 1,170            | -1,240      | -446       | +376   |
| 2             | 697                    | 18.5                | 2,370             | 3,330            | -960        | -345       | 2,715  |
| 4             | 642                    | 12.0                | 4,590             | 5,290            | -700        | -252       | 4,842  |
| 6             | 594                    | 8.0                 | 6,660             | 7,430            | -770        | -277       | 6,937  |
| 8             | 563                    | 5.0                 | 8,070             | 8,780            | -710        | -255       | 8,325  |
| 10            | 533                    | 3.0                 | 9,490             | 10,270           | -780        | -280       | 9,770  |
| 12            | 509                    | 0.5                 | 10,680            | 11,460           | -780        | -280       | 10,960 |
| 14            | 493                    | -3.0                | 11,490            | 12,050           | -560        | -202       | 11,692 |
| 16            | 477                    | -6.0                | 12,330            | 12,730           | -400        | -144       | 12,474 |
| 18            | 458                    | -7.0                | 13,350            | 13,890           | -540        | -194       | 13,544 |
| 20            | 446                    | -8.5                | 14,020            | 14,470           | -450        | -162       | 14,182 |
| 25            | 421                    | -10.0               | 15,450            | 16,140           | -690        | -248       | 15,698 |
| 30            | 408                    | -11.0               | 16,230            | 16,950           | -720        | -259       | 16,489 |

$$\Delta h = 0.36 (h_p - h_d) \quad h = h_p - \Delta h$$

TABLE 19. REDUCTION OF CLIMB TO STANDARD CLIMB NO. 2

| OBSERVED DATA |                        |                     | ALTITUDES (ft.)   |                  |             |            |        |
|---------------|------------------------|---------------------|-------------------|------------------|-------------|------------|--------|
| Time<br>Min.  | Pressure<br>Mm.<br>Hg. | Air<br>Temp.<br>°C. | Pressure<br>$h_p$ | Density<br>$h_d$ | $h_p - h_d$ | $\Delta h$ | $h$    |
| 0             | 764                    | 12.0                | -140              | -520             | +380        | +137       | -277   |
| 3             | 670                    | 6.0                 | 3,440             | 3,180            | +260        | +93        | 3,347  |
| 4             | 645                    | 5.0                 | 4,470             | 4,340            | +130        | +47        | 4,423  |
| 6             | 605                    | 5.0                 | 6,170             | 6,470            | -300        | -108       | 6,278  |
| 8             | 565                    | 1.0                 | 7,970             | 8,160            | -190        | -68        | 8,038  |
| 10            | 531                    | -3.0                | 9,580             | 9,700            | -120        | -43        | 9,623  |
| 12            | 507                    | -3.0                | 10,780            | 11,150           | -370        | -133       | 10,913 |
| 16            | 464                    | -6.5                | 13,030            | 13,330           | -300        | -108       | 13,138 |
| 18            | 447                    | -9.5                | 13,960            | 14,330           | -370        | -133       | 14,093 |
| 20            | 435                    | -10.5               | 14,640            | 14,950           | -310        | -111       | 14,751 |
| 25            | 414                    | -15.5               | 15,870            | 15,980           | -110        | -40        | 15,910 |
| 30            | 398                    | -17.5               | 16,830            | 16,950           | -120        | -43        | 16,873 |
| 35            | 388                    | -19.5               | 17,460            | 17,460           | 0           | 0          | 17,460 |
| 40            | 381                    | -20.5               | 17,900            | 17,880           | 20          | +7         | 17,907 |

$$\Delta h = 0.36 (h_p - h_d) \quad h = h_p - \Delta h$$

## CHAPTER XI

### NOTES ON FLIGHT TESTING

The following notes on flight testing have been prepared for the information of test pilots. Some of the material is not new but appears desirable in explanation of the principles involved. Other parts, such as the conditions for best climb and methods of specifying these conditions, are not available elsewhere.

**Calibration of air-speed indicators.** In general, air-speed indicators must be calibrated by runs up and down wind over a measured course. Special methods, such as calibration against the reading of an instrument suspended well below the airplane<sup>1</sup> or against previously calibrated readings in a second airplane, are not available except at well-equipped flight test organizations. Calibration runs should not be attempted when the cross course component of the wind exceeds 14% ( $\frac{1}{4}th$ ) of the airplane speed, since this value reduces the measured speed 1%.

Assume that a series of runs in pairs has been made over a measured course, each pair consisting of a run up and down wind, during which the following data are recorded: time over course, air-speed indicator reading, r.p.m., air temperature, and pressure. The time over the course gives the ground speed. Averaging ground speeds (not the times over course) in each pair of runs gives the true air-speed for the given r.p.m. and indicator reading. The indicator reading is a function of  $\frac{1}{2}\rho V^2$  and  $V$  is the true air speed. The curve of readings plotted against  $\frac{1}{2}\rho V^2$  is the calibration curve. True speed may be obtained from the instrument reading and this curve, when  $\rho$  is known. While the true speed is determined by the density, the speed itself must be plotted against pressure altitude.

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<sup>1</sup> See W. G. Brown, "Measuring an Airplane's True Speed in Flight Testing," N. A.-C. A. Technical Note No. 135 (1923).

**Maximum speed.** In order to determine the actual maximum speed in horizontal flight, care must be taken to avoid errors due to following sources: (1) failure to maintain horizontal flight, (2) starting over a speed course before the airplane has settled down to steady flight, (3) variable winds, or cross wind component too large. There are many other precautions to be observed, but those enumerated constitute the chief sources of error.

A change in altitude of 40 ft./mi. will give 1% change in the measured high speed of an average airplane. For this reason high-speed runs are usually made at a very low altitude and, if available, a statoscope is carried, in order that the flight path may be maintained horizontal. If the ratio of maximum to stalling speed is greater than 1.5, the error due to ground effect is negligible.

All high-speed runs should be started at a distance from the first marker on the course which will allow from 30 seconds to 60 seconds for settling down to steady conditions before crossing the line. This precaution is very important. The last two or three miles of high speed are attained by a comparatively slow acceleration, and any control movements or changes of altitude and direction will affect the high speed. In the first stages of the approach to the starting line of the course, the pilot should fix his altitude and direction. Under no conditions would a diving start be permissible.

High-speed runs should be made in a calm if practicable, although good results can be obtained in any steady wind which does not have a cross course component greater than  $\frac{1}{4}$  of the speed to be measured. Tests should never be made with gusty, variable, or unsteady winds, no matter how light.

**Importance of correct air speed in climb.** Figure 123 gives the calculated variation of rate of climb with true air speed and altitude for a typical airplane. Each heavy curve gives the variation of rate of climb with air speed at constant standard altitude. The central cross curve (long dashes) passes through

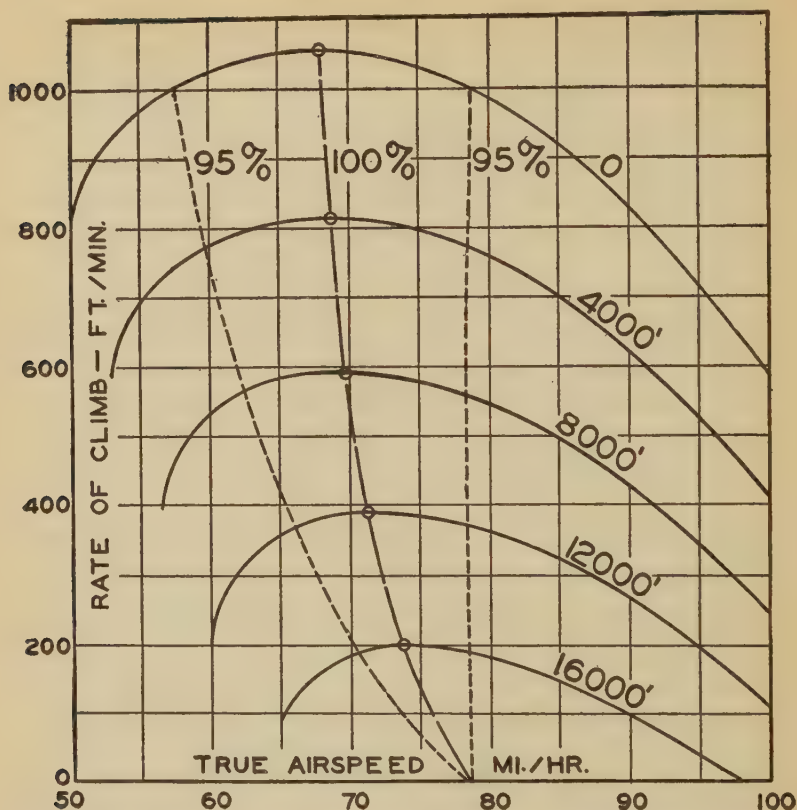


Figure 123. Variation of Climb with Air Speed and Altitude, Showing Necessity for Making Climb at Correct Air Speeds

the maximum rate of climb at each altitude and therefore gives the variation of best climbing speed with altitude. The two outside cross curves (short dashes) are marked 95%, and pass through rates of climb 95% of maximum at all altitudes. These curves show very strikingly the importance of correct air speed in climb.

**Variation of best climbing air speed with pressure and temperature.** It is very important that the test pilot know the proper variation of climbing air speed with altitude. How to express the variation of climbing air speed with altitude so as to

allow for variation from standard atmosphere conditions, has long been an insistent question. The answer to this question now appears simple.

In the study of performance reduction, the best climbing speeds for a typical airplane were determined at various pressure altitudes for temperatures 83.3%, 90.9%, 110%, and 120% normal. A large effect was found as follows:

| Temperature, % Normal.....                         |            | 83.3 | 90.9 | 100  | 110  | 120  |
|----------------------------------------------------|------------|------|------|------|------|------|
| Best climbing air speed at given pressure altitude | 4,000'...  | 66.0 | 67.5 | 70.0 | 70.5 | 71.0 |
|                                                    | 8,000'...  | 66.0 | 68.0 | 70.0 | 73.0 | 74.0 |
|                                                    | 10,000'... | 65.0 | 68.0 | 70.0 | 72.0 | 74.0 |
|                                                    | 12,000'... | 66.5 | 68.0 | 70.0 | 73.0 | 76.0 |
|                                                    | 16,000'... | 67.0 | 70.0 | 72.0 | 76.0 | 79.0 |

Taking the ratio of these climbing speeds to the corresponding stalling speeds gives:

| Temperature, % Normal.....                                                                  |            | 83.3 | 90.9 | 100  | 110  | 120  |
|---------------------------------------------------------------------------------------------|------------|------|------|------|------|------|
| Ratio $\frac{\text{Best climbing speed}}{\text{Stalling speed}}$ at given pressure altitude | 4,000'...  | 1.36 | 1.33 | 1.32 | 1.27 | 1.22 |
|                                                                                             | 8,000'...  | 1.28 | 1.26 | 1.24 | 1.23 | 1.20 |
|                                                                                             | 10,000'... | 1.22 | 1.22 | 1.20 | 1.18 | 1.16 |
|                                                                                             | 12,000'... | 1.19 | 1.19 | 1.17 | 1.16 | 1.16 |
|                                                                                             | 16,000'... | 1.14 | 1.15 | 1.14 | 1.13 | 1.13 |

It is obvious that for all practical purposes the ratio of best climbing speed to stalling speed may be considered constant at any given pressure altitude. This means that the relation between angle of attack and pressure altitude is not affected by temperature variations. Consequently, the conditions for best climb at altitude must be specified as the relation between the readings of a sensitive angle of attack indicator and an aneroid. The common method of specifying the indicated air speeds at various indicated altitudes may give erroneous results. It is satisfactory, however, to use indicated air speeds and aneroid pressures.

**Determination of best climbing air speeds.** The procedure formerly followed was to make a series of "saw-tooth" climbs at various air speeds at a given altitude and thus determine the air speed giving the greatest rate of climb. This was repeated at various altitudes and a curve of climbing air speed plotted



against altitude. Such a curve holds true only for the particular pressure and temperature conditions existing during the saw-tooth climbs. In order to obtain a general relation, it is necessary to specify either angles of attack or indicated air speeds in terms of aneroid pressure.

The recommended procedure is to determine the angle of attack, or indicated air speed, which gives the greatest rate of climb at each aneroid reading, and plot this against aneroid reading. For example, the angle of attack that gives the least time required to climb between aneroid readings of  $p = 510$  mm. and  $P = 490$ , would be taken as the best angle of attack for  $P = 500$ . The actual increment in pressure used to determine the best reading should be so varied according to the rate of climb as to give a minimum time interval of at least 30 seconds, and preferably more. Otherwise experimental errors may affect the results. The desirable increment in pressure will vary with altitude (or pressure) but it is constant in all of the runs at a given pressure.

**Climbing tests.** After the relation between angle of attack and pressure altitude has been determined, the actual climbs may be made. Under average conditions, the best method of starting a climb is to fly horizontally in a convenient direction and at a low altitude, long enough for the airplane to settle down to steady conditions at the proper angle of attack for the existing ground pressure. The time required is determined by the lag in the aneroid and air thermometer, and the climb should not be started until these instruments record ground conditions. At the proper time, the climb is started by opening the throttle while holding the angle of attack constant.

The climb should be made with a minimum number of turns, which should be rather wide for best results. It is advisable that a climb be repeated by one or more different pilots, if possible. The composite results of three climbs should be quite definite.



## CHAPTER XII

### RANGE AND ENDURANCE

**Maximum range.** The maximum range for a given fuel load is obtained by continuously flying, until the fuel is exhausted, at the air speed giving the most miles per pound (or the minimum pounds per mile). For a given airplane this air speed, usually designated as the "most economical air speed," or "economical speed," varies almost linearly with gross weight. The fuel consumption in lbs./mile also varies almost linearly with gross weight. It therefore follows that the most economical speed and corresponding fuel consumption need be calculated for only three gross weights in order to determine the maximum range. These weights should obviously be at, or near, full load, half-fuel load, and no-fuel load.

Fuel consumption is obtained at a given speed and weight by calculating the *r.p.m.* and *B.Hp.*, using the data given in Chapter VI. The various steps in the calculation may be tabulated as follows:

1. Air speed  $V$ .
2. Ratio  $\frac{V}{V_d}$ .  $V_d$  = propeller design speed. In absence of specific data, assume  $V_d = V_M$ .
3.  $T.Hp.$  required at speed  $V$ . Read from curve of  $T.Hp.r.$
4. Ratio  $\frac{T.Hp.}{T.Hp.o.}$ .  $T.Hp.o.$  =  $T.Hp.r.$  at speed  $V_d$ .
5.  $\frac{N}{N_o} = \frac{r.p.m.}{r.p.m.o.}$ . Read from Figure 94 or 95.
6.  $r.p.m. = \frac{N}{N_o} \times r.p.m.o.$   $r.p.m.o.$  designed, or rated  $r.p.m.o.$  at speed  $V_d$ .

7.  $\frac{V}{ND} / \left( \frac{V}{ND} \right)_0 = \frac{V}{V_d} \times \frac{N_0}{N}$ .
8.  $\frac{\eta}{\eta_m}$  from Figure 83.
9.  $\eta = \eta_m \times \frac{\eta}{\eta_m}$ .  $\eta_m$  is the maximum propeller efficiency.
10.  $B.Hp. = \frac{T.Hp.}{\eta}$ .
11. Specific fuel consumption  $C$ . Obtain from engine data or from Figure 79.
12. Fuel consumption, lbs./hr. =  $B.Hp. \times C$ .
13. Fuel consumption, lbs./mile =  $\frac{B.Hp. \times C}{V}$ .

The fuel consumption in lbs./mile is plotted against speed in order to obtain the minimum consumption and the corresponding speed (most economical speed). This operation is repeated for the other gross loads. The minimum fuel consumptions in lbs./mile and the corresponding economical speeds are then plotted against gross weight. If there is a pronounced curvature in the line passed through the minimum fuel consumption points on this plot, the range must be determined either by a step-by-step integration, or by a planimeter. In the step-by-step method the average fuel consumptions are determined for convenient increments in weight, say 500 lbs. or 1,000 lbs. The distance flown for the  $\Delta W$  under consideration is obviously  $\Delta R = \frac{\Delta W \text{ lbs.}}{C_a}$  where  $C_a$  is the average lbs./mile for the increment  $\Delta W$ . The endurance may be calculated along with the range from  $\Delta E \text{ hrs.} = \frac{\Delta R}{V_a}$ , where  $V_a$  is the average speed for the increment  $\Delta R$ .

If it is desired to use a planimeter, the reciprocal of the minimum fuel consumption, that is, the maximum miles per pound of fuel must be plotted against gross weight. The area under this curve has the dimensions of  $\frac{\text{mi.}}{\text{lb.}} \times \text{lbs.} = \text{miles}$ , and the

distance flown while using a given weight of fuel is proportional to the area under the curve and between the two corresponding gross weights. If the curve is plotted to the scales of  $1'' = 1,000$  lbs. weight, and  $1'' = 0.10$  (mi./lb.), each square inch under the curve will represent 100 miles. The endurance may be found in the same manner by plotting (hrs. endurance / lb. fuel) against gross weight and measuring the area under the curve.

The range may be calculated very quickly by formula when the fuel consumption in lbs./mile plots as a straight line against gross weight. If the slope of the fuel consumption curve, lbs./mile, against weight is  $b$ , and the equation of fuel consumption  $C = a + bW$ , then

$$\begin{aligned} R &= \int_{W_0}^{W_1} \frac{dW}{(a + bW)} \\ &= \frac{2.303}{b} \log_{10} \left[ \frac{a + bW_0}{a + bW_1} \right] \text{ miles} \end{aligned} \quad (117)$$

Since  $(a + bW_0)$  = initial fuel consumption,  $C_0$  (lbs./mile), and  $(a + bW_1)$  = final fuel consumption,  $C_1$  this becomes

$$R = \frac{2.303}{b} \log_{10} \left[ \frac{C_0 \text{ lbs./mile}}{C_1 \text{ lbs./mile}} \right] \quad (118)$$

The endurance is found directly from the fuel consumption in lbs./hr., if  $C$  is of the form  $(m + nW)$ :

$$\begin{aligned} T &= \int_{W_0}^{W_1} \frac{dW}{(m + nW)} \\ &= \frac{2.303}{n} \log_{10} \left[ \frac{m + nW_0}{m + nW_1} \right] \\ &= \frac{2.303}{n} \log_{10} \left[ \frac{C_0 \text{ lbs./hr.}}{C_1 \text{ lbs./hr.}} \right] \end{aligned} \quad (119)$$

Theoretically, the specific fuel consumption should include both gas and oil consumption, and the final weight should be based on the amount of oil remaining when all of the gasoline is

gone. Practically, the oil consumption may be neglected when modern engines are used and the final weight  $W_1$  taken as  $W_0$  less the weight of gasoline  $W_g$ . That is,  $W_1 = W_0 - W_g$ .

**Calculation of maximum endurance.** Maximum range is obtained by flying at a speed about 40% greater than the stalling speed, but maximum endurance is obtained at a considerably lower speed, 15% to 20% greater than the stalling speed. The calculation for maximum endurance is exactly like that for maximum range except that the factor now desired is the maximum (hrs./lb. fuel), at a given gross weight. Plotting (max. hrs./lb. fuel) against gross weight, the area under the curve will be proportional to the maximum endurance in hours.

The maximum (hrs./lb. fuel) should be obtained by calculating (hrs./lb.) at a number of speeds from 1.05  $V_S$  to 1.50  $V_S$  and plotting against speed.

**Range and endurance formulas.** Breguet's formulas for range and endurance are:

$$\text{Range} = 863.5 \frac{L}{D} \cdot \frac{\eta}{C} \cdot \log_{10} \left( \frac{W_0}{W_1} \right) \text{ miles} \quad (120)$$

$$\text{Endurance} = 750 \cdot \frac{V_C}{\sqrt{W}} \cdot \frac{L}{D} \cdot \frac{\eta}{C} \left[ \frac{1}{\sqrt{W_1}} - \frac{1}{\sqrt{W_0}} \right] \text{ hrs.} \quad (121)$$

where  $\frac{L}{D}$  is the ratio of lift to drag for the entire airplane,  $\eta$  the average propeller efficiency,  $C$  the *average* specific fuel consumption,  $W_0$  the initial gross weight,  $W_1$  the final gross weight, and  $V_C$  the cruising speed at the gross weight  $W_0$ .

The author has shown<sup>1</sup> that the range is given quite accurately by

$$R = 833.3 \frac{\eta}{C_1} \cdot \frac{L}{D} \cdot \left[ 1 - \left( \frac{W_1}{W_0} \right)^{0.45} \right] \quad (122)$$

when the mixture control is not used, and by

$$R = 892.9 \frac{\eta}{C_1} \cdot \frac{L}{D} \cdot \left[ 1 - \left( \frac{W_1}{W_0} \right)^{0.42} \right] \quad (123)$$

<sup>1</sup> W. S. Diehl, "Three Methods of Calculating Range and Endurance of Airplanes," N. A. C. A. Technical Report No. 234 (1925).

when mixture control is used. The corresponding equations for endurance in hours, are

$$T = 7,500 \frac{\eta}{C_i} \cdot \frac{L}{D} \cdot \frac{1}{V_{C_0}} \left[ \left( \frac{W_0}{W_i} \right)^{.05} - 1 \right] \quad (124)$$

when mixture control is not used, and

$$T = 4,687.5 \frac{\eta}{C_i} \cdot \frac{L}{D} \cdot \frac{1}{V_{C_0}} \left[ \left( \frac{W_0}{W_i} \right)^{.08} - 1 \right] \quad (125)$$

when mixture control is used. In equations (122) to (125),  $C_i$  is the *initial* instead of the average, specific fuel consumption, and  $V_{C_0}$  is the initial cruising speed in mi./hr.

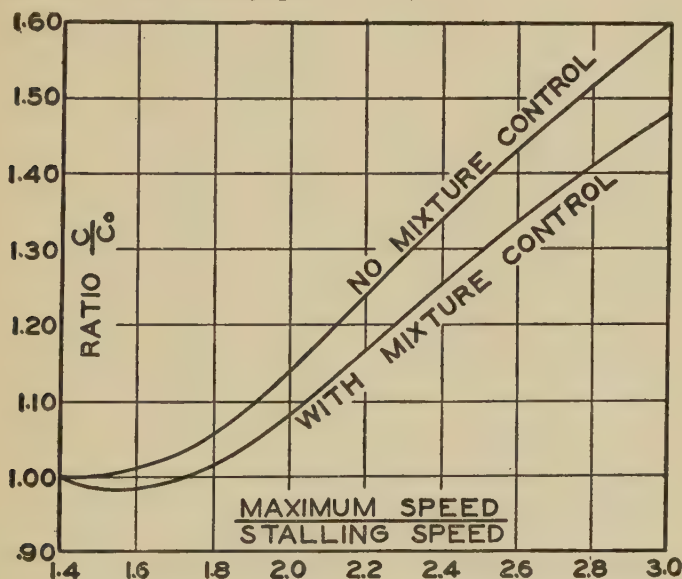


Figure 124. Variation of Specific Fuel Consumption with Speed Ratio  $\frac{V_M}{V_S}$

Ranges and endurance calculated by these formulas are in excellent agreement with step-by-step integration, provided that the proper values of  $C$  and  $C_i$  are selected. Figure 124 enables the values of either  $C$  or  $C_i$  to be calculated accurately. To calculate the average specific fuel consumption for a given fuel load, find the average ratio of maximum to minimum speed

$\frac{V_M}{V_S}$  and read the value of  $\frac{C}{C_0}$  from Figure 124.  $C_1$  is obtained from the ratio  $\frac{C}{C_0}$  corresponding to the initial value of  $\frac{V_M}{V_S}$ .

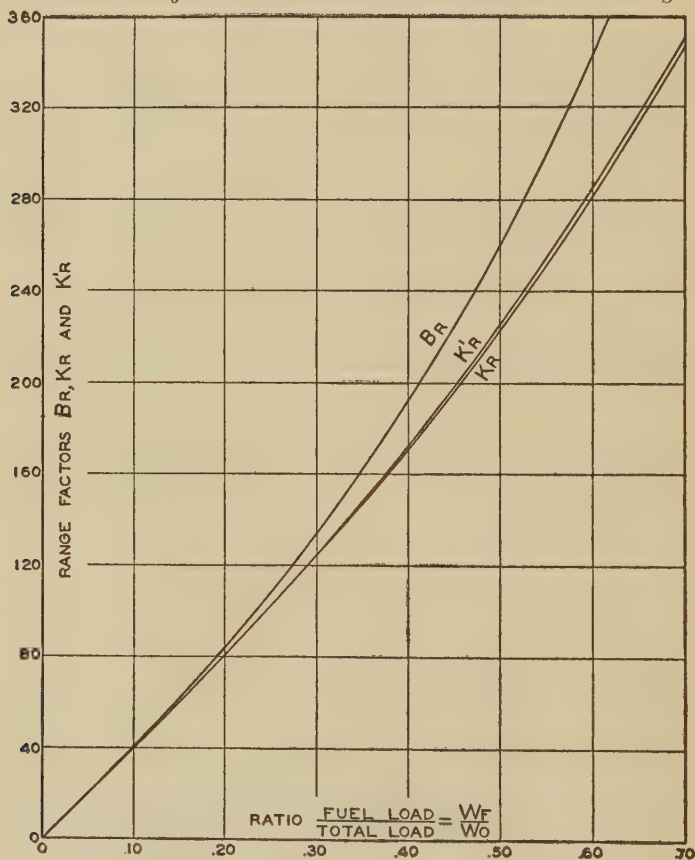


Figure 125. Factors for Simplified Range Formulas

Equations (120) to (125) may be written

$$R = B_R \cdot \frac{L}{D} \cdot \frac{\eta}{C} \quad (126)$$

$$T = B_E \cdot \frac{L}{D} \cdot \frac{\eta}{C} \cdot \frac{1}{V_{C_0}} \quad (127)$$

$$R = K_R \cdot \frac{L}{D} \cdot \frac{\eta}{C_1} \quad (\text{No mixture control}) \quad (128)$$



$$T = K_E \cdot \frac{L}{D} \cdot \frac{\eta}{C_I} \cdot \frac{1}{V_{Co}} \quad (\text{No mixture control}) \quad (129)$$

$$R = K'_R \cdot \frac{L}{D} \cdot \frac{\eta}{C_I} \quad (\text{With mixture control}) \quad (130)$$

$$T = K'_E \cdot \frac{L}{D} \cdot \frac{\eta}{C_I} \cdot \frac{1}{V_{Co}} \quad (\text{With mixture control}) \quad (131)$$

The factors  $B_R$ ,  $B_E$ ,  $K_R$ ,  $K_E$ ,  $K'_R$ , and  $K'_E$  will be found on Figures 125 and 126.

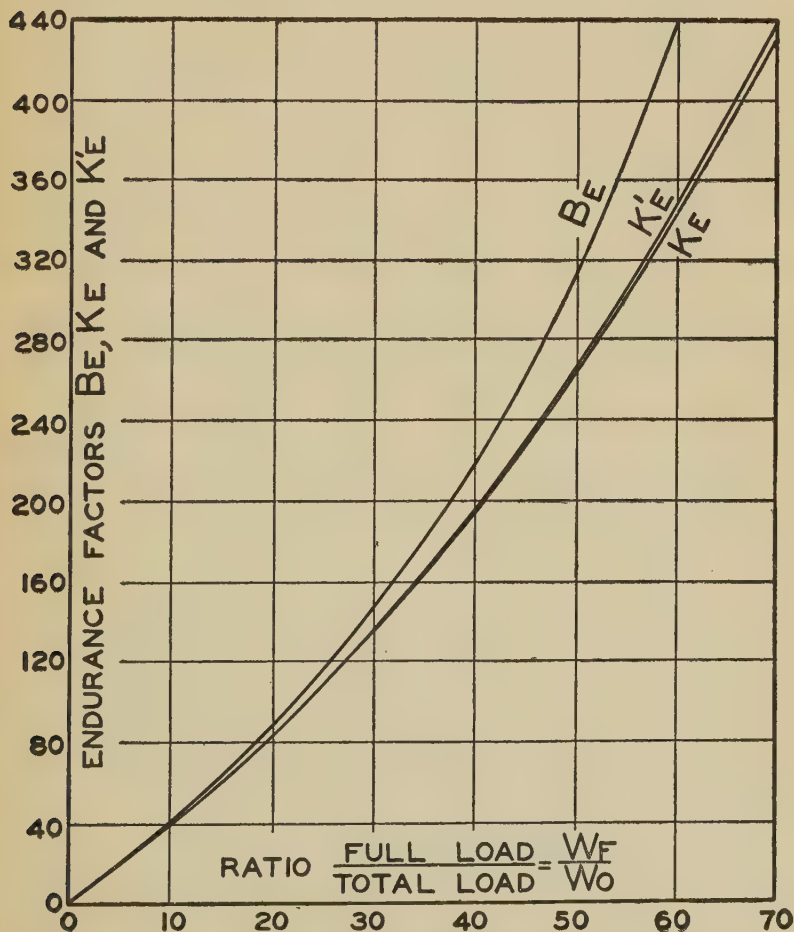


Figure 126. Factors for Simplified Endurance Formulas

In using these equations, the maximum  $\frac{L}{D}$  and maximum propeller efficiency may be used if maximum range is desired. The specific fuel consumption at full throttle  $C_o$  is obtained either from engine test data or estimated from general engine data. The oil consumption may be included in  $C_o$  if it is so desired,

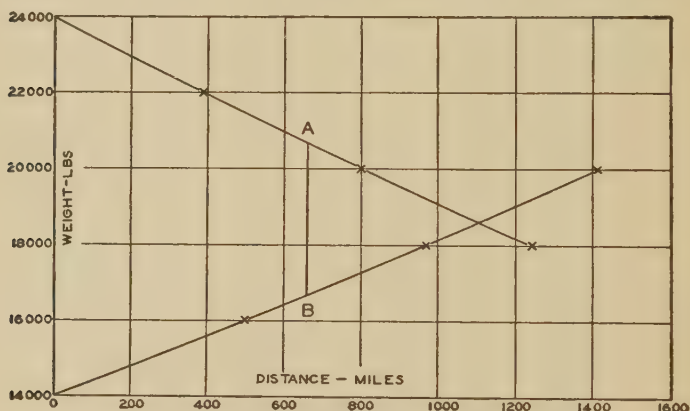


Figure 127. Graphical Solution for Bombing Range

but the final weight  $W_1$  must then allow for the oil remaining when all of the gasoline is exhausted. The difference in range obtained by allowing for oil consumption is hardly worth the trouble if the engine is average and modern. Some of the early engines used enough oil to make an appreciable difference.

**Bombing range.** The distance that an airplane can fly with a given fuel load, drop a bomb, and return to the starting point, is easily obtained by a simple graphical solution. Let  $W_o$  be the initial gross load including full bomb and fuel loads, and  $W_1$  the final load without fuel or bombs. Calculate the range for two or three fuel loads assuming constant initial load  $= W_o$ . The largest fuel load should be somewhat more than  $\frac{1}{2} (W_o - W_1)$ . Repeat the process assuming constant final load  $= W_1$ . Plot as two curves of range against weight, one starting from  $W_o$  and showing increasing range with decreasing weight, the other

starting from  $W_1$  and showing increasing range with increasing weight. The intersection of these curves gives the maximum radius with all fuel and no bombs. Figure 127 illustrates how the range is found for any desired bomb load for an airplane weighing 24,000 lbs. and carrying 10,000 lbs. of fuel and bombs.

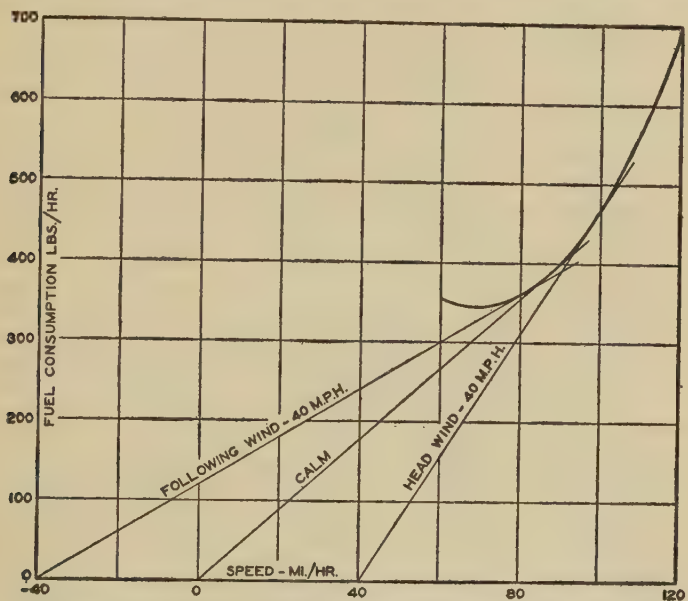


Figure 128. Effect of Wind on Most Economical Air Speed

For 10,000 lbs. of fuel and no bombs the curves intersect at 1,110 miles; hence the range is  $2 \times 1,110 = 2,220$  miles. For a 4,000-lb. bomb load and 6,000 lbs. of fuel, the airplane can fly out to the point  $A$  at 660 miles, drop the bomb load  $AB = 4,000$  lbs. and return to the point of departure.

**Effect of wind on range.** The most economical speed is affected considerably by wind direction and force, being increased by head winds and decreased by following winds. The most economical speed is readily found for any wind velocity, by a simple graphical solution. If the fuel consumption in lbs./hr. is plotted against air speed in mi./hr., with both scales starting

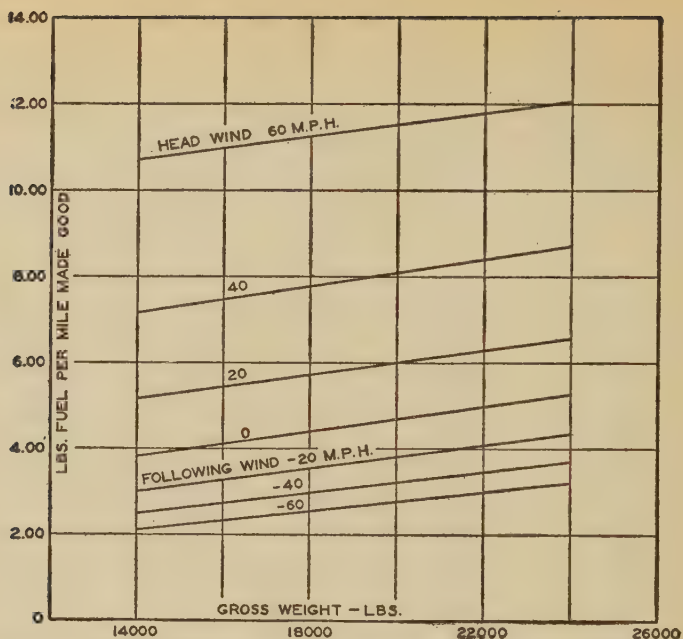


Figure 129. Effect of Wind on Fuel Consumption for a Typical Flying Boat

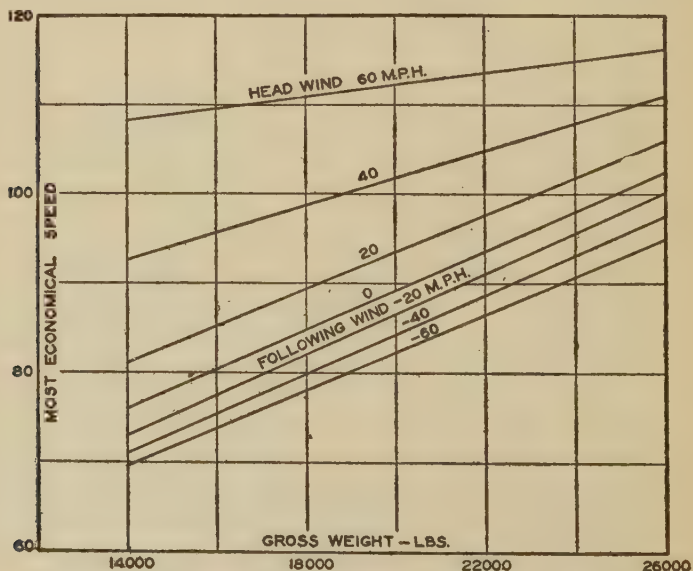


Figure 130. Effect of Wind on Most Economical Speed for a Typical Flying Boat

from zero, as in Figure 128, then a line drawn from any point on the speed axis is tangent to the curve at the most economical speed for the wind condition determined by the starting point. Since the ground speed is the difference between the air speed and the wind speed, the starting point or origin for ground speed is

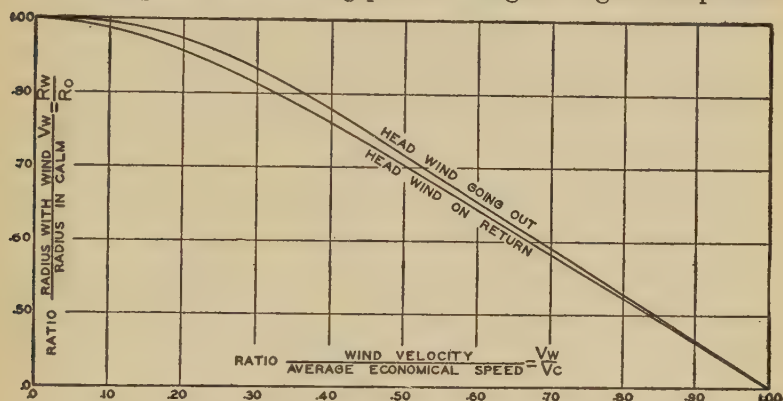


Figure 131. Reduction of Range in a Steady Wind

moved to the right or to the left for head winds and following winds respectively. That is, with a 40 mi./hr. head wind, the ground speeds will be 40 mi./hr. less than the air speeds, and the origin will be at 40 mi./hr. A line drawn from this point in Figure 128 is tangent to the curve at 99 mi./hr., which is the economical speed for a 40 mi./hr. head wind, at the gross weight represented by the fuel consumption curve. Similar tangents drawn for zero wind and 40 mi./hr. following wind indicate economical speeds of 86 mi./hr. and 81 mi./hr. respectively. It is obvious that this operation determines the minimum pounds of fuel per ground mile.

It is somewhat longer, but perhaps more accurate, to calculate the lbs. fuel per ground-mile for various air speeds and wind speeds. If the lbs. fuel per ground-mile at a given wind speed be plotted against air speed, the most economical air speed is accurately determined. Figure 129 shows the variation in fuel consumption and Figure 130 shows the variation in the most economical speed for a typical large flying boat.

A head wind of a given velocity reduces range more than a following wind of the same velocity increases it. The maximum distance that can be flown out and return to starting point is less in a *steady* wind than in a calm, and decreases rapidly as the steady wind velocity increases. The radius is greater if the head wind is encountered going out with full load than if the head wind is met on the return flight. Figure 131 gives the average effect of a steady wind on radius of action.



## CHAPTER XIII

### SPECIAL FLIGHT PROBLEMS

There are a number of problems of considerable interest and some practical value connected with special flight conditions. For most solutions extreme accuracy is not required, and reasonable simplifying assumptions are allowable. A few of the more important problems in this class will be outlined briefly.

**Gliding flight without power.** For equilibrium in a glide without power, at an angle  $\theta$  to the horizontal

$$W \sin \theta = D \quad (132)$$

$$L \cos \theta + D \sin \theta = W \quad (133)$$

and  $L \sin \theta = D \cos \theta \quad (134)$

From equation (134)

$$\frac{L}{D} = \cot \theta \quad (135)$$

or the angle of glide is equal to the angle whose cotangent is  $\frac{L}{D}$ .

Since  $L = K_L \frac{\rho}{\rho_0} V^2$  and  $D = K_D \frac{\rho}{\rho_0} V^2$ , it follows from equation (133) that the speed along the flight path is

$$V = \sqrt{\frac{\rho_0}{\rho}} \sqrt{\frac{W}{K_D \left( \frac{L}{D} \cos \theta + \sin \theta \right)}} \quad (136)$$

For the limiting speed in a vertical dive  $K_L \cos \theta$  is negligible and  $\sin \theta$  is substantially unity. Hence

$$D = K_D \frac{\rho}{\rho_0} V_L^2 = W$$

$$V_L = \sqrt{\frac{\rho_0}{\rho}} \sqrt{\frac{W}{K_D}} \quad (137)$$

The altitude lost per minute in a glide without power at a fixed angle of attack must be such that the rate of change in potential energy is equal to the power required for horizontal flight at the same angle of attack. That is

$$V_v = \frac{dh}{dt} = \frac{33,000 T.Hp.r}{W} \text{ ft./min.} \quad (138)$$

From these equations it is possible to solve practically all of the problems of gliding flight.

**Gliding flight with power.** Assuming that the thrust  $T$  acts along the flight path and letting the angle  $\theta$  between the flight path and the horizontal be positive upward, the equations of equilibrium are

$$T - W \sin \theta = D \quad (139)$$

$$L \cos \theta - (D - T) \sin \theta = W \quad (140)$$

$$\text{and} \quad L \sin \theta = (D - T) \cos \theta \quad (141)$$

The inclination of the thrust to the flight path may be allowed for, but little is to be gained in accuracy.

Assuming that the thrust varies with velocity and that the drag is  $D = KV^2$ , it can be shown <sup>1</sup> that the velocity along the flight path is

$$V = V_M \left[ -\frac{1}{2} + \sqrt{2.25 - \left(\frac{L}{D}\right)_0 \sin \theta} \right] \quad (142)$$

if the thrust at  $V = 2V_M$  is zero. This condition applies to propellers of low pitch/diameter ratio. For propellers of high pitch/diameter ratio the thrust holds up better. Assuming that  $T = T_0/2$  when  $V = 2V_M$ , ( $T$  = thrust at  $V = V_M$ ), the velocity along the flight path is

$$V = V_M \left[ -\frac{1}{4} + \sqrt{1.5625 - \left(\frac{L}{D}\right)_0 \sin \theta} \right] \quad (143)$$

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<sup>1</sup> W. S. Diehl, "The Effect of Flight Path Inclination on Airplane Velocity," N. A. C. A. Technical Report No. 238.

In these equations  $\left(\frac{L}{D}\right)_0$  is the value of  $\frac{L}{D}$  for the airplane at the maximum horizontal speed  $V_M$  and has the value

$$\left(\frac{L}{D}\right)_0 = \frac{V_M}{375 \eta} \cdot \frac{W}{B.Hp.} \quad (144)$$

$\left(\frac{L}{D}\right)_0$  has no direct relation to the maximum value of  $\frac{L}{D}$ . Figure 132 is a plot of  $\frac{V}{V_M}$  against  $\theta$  for various values of  $\left(\frac{L}{D}\right)_0$  according

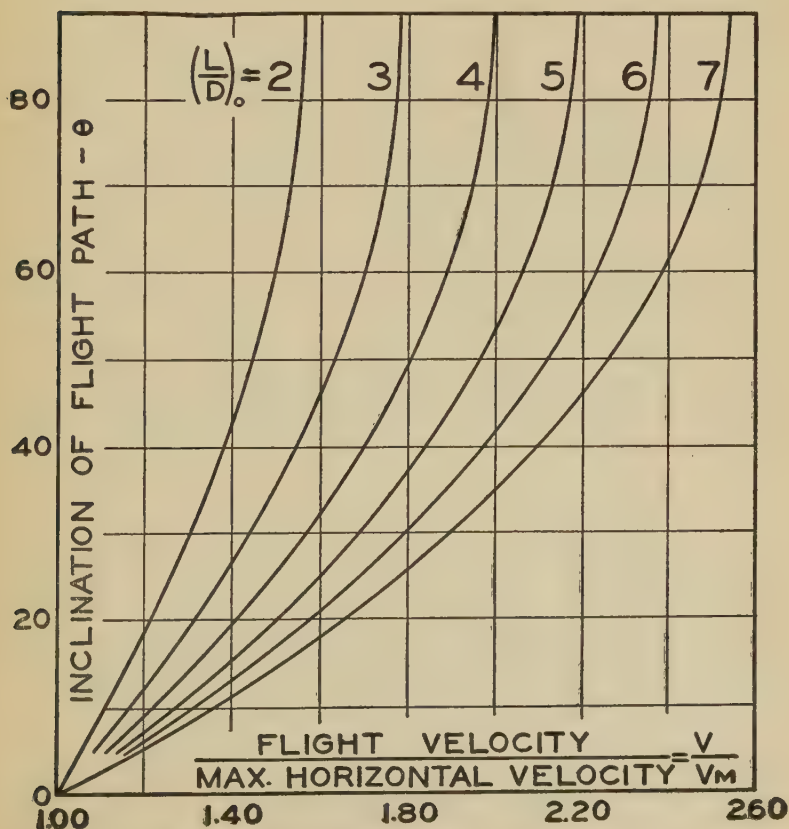


Figure 132. Air Speed in a Dive with Power On. Thrust Zero at  $V = 2 V_M$

to equation (142), and Figure 133 is a similar plot according to equation (143). The effect of thrust on diving speed is comparatively small.

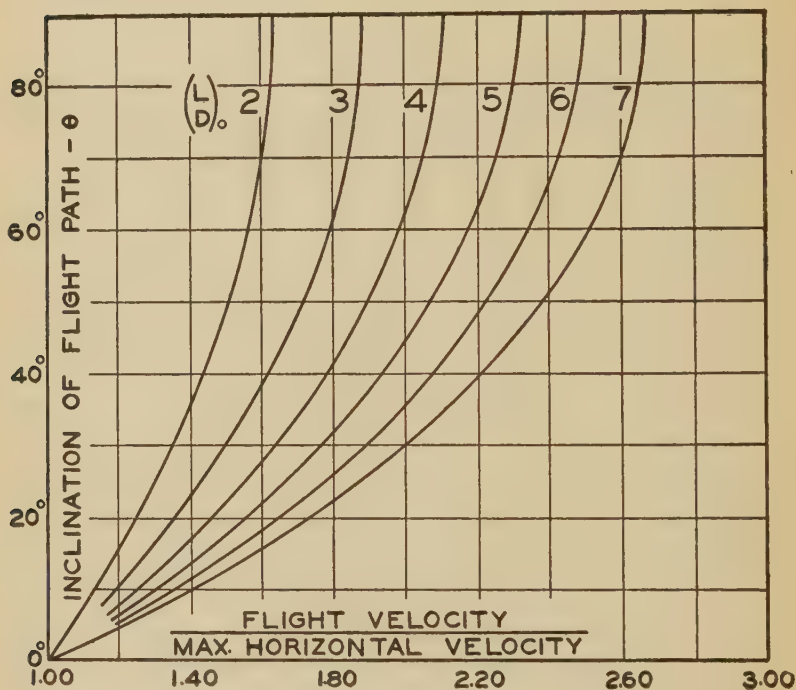


Figure 133. Air Speed in a Dive with Power On,  $T = \frac{T_0}{2}$  at  $V = 2 V_M$

**Circling flight.** For equilibrium in a horizontal turn, the weight must be balanced by the vertical component of the lift and the centrifugal force must be balanced by the horizontal component of the lift. If  $\varphi$  is the angle of bank, the equations for equilibrium are

$$W = L \cos \varphi \quad (145)$$

$$\text{and centrifugal force} = \frac{WV^2}{gr} = L \sin \varphi \quad (146)$$

$$\text{from which} \quad \tan \varphi = \frac{V^2}{gr} \quad (147)$$

$V$  and  $r$  must be consistent units, e.g., ft./sec. and ft. Since  $W$  is constant,  $L$  must vary as  $\sec \varphi$  and the acceleration in a turn is

$$\frac{L}{W} = \sec \varphi \quad (148)$$

Since  $L$  varies as  $V^2$ , the relation between the speed in straight horizontal flight and the speed in a turn at constant lift coefficient or angle of attack is

$$\frac{V_t}{V} = \sec \varphi \quad (149)$$

At constant lift coefficient, or angle of attack, the drag will vary as  $V^2$  and the thrust horsepower required as  $V^3$ . If curves of T.Hp. available and T.Hp. required are plotted against velocity on logarithmic scales as in Figure 134, the solution for minimum radius of turn at any given lift coefficient is made very simple. The power required varies as  $V^3$ , along straight lines  $AA'$  and  $BB'$ . The intersection of such a line with the curve  $T.Hp._a$  gives the maximum air speed at which turning flight can be made with the initial lift coefficient. Consider the line  $BB'$ .  $B$  is at 100 mi./hr. and  $B'$  at 192 mi./hr. Hence the angle of bank is

$$\sec \varphi = \left( \frac{V_t}{V} \right)^2 = \left( \frac{192}{100} \right)^2 = 3.68$$

$$\text{or} \quad \varphi = 74^\circ - 13'$$

From equation (147) the radius of the turn is

$$r = \frac{V^2}{g} \cot \varphi = \frac{(192 \times 1.467)^2}{32.2} \times 0.282 = 695 \text{ ft.}$$

This process may be repeated for as many points, lift coefficients, speeds, or angles of attack, as desired. The results would be as tabulated on page 219.

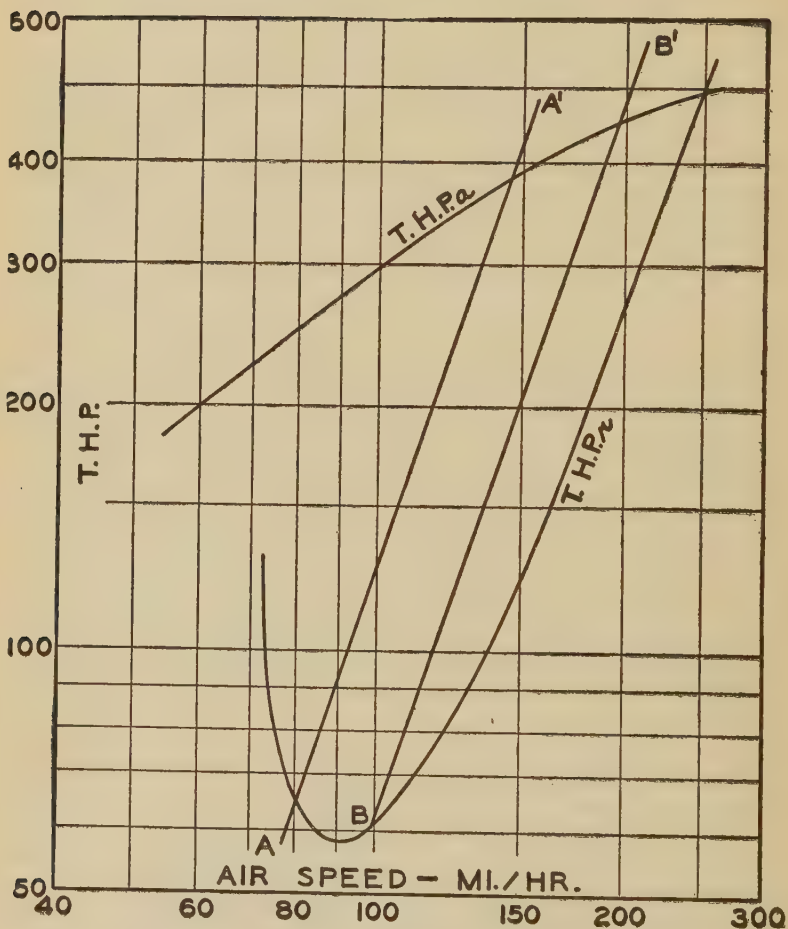


Figure 134. Graphical Solution for Speed in a Turn



| Angle of<br>Attack<br>Degrees | Air Speed in mi./hr. |              | Angle of<br>Bank<br>$\varphi$<br>deg.-min. | Radius of<br>Turn<br>ft. |
|-------------------------------|----------------------|--------------|--------------------------------------------|--------------------------|
|                               | Horizontal<br>Flight | In a<br>Turn |                                            |                          |
| 1                             | 198.2                | 250.0        | 51 - 06                                    | 3,370                    |
| 2                             | 166.8                | 244.7        | 62 - 22                                    | 2,100                    |
| 4                             | 130.3                | 227.5        | 70 - 52                                    | 1,200                    |
| 6                             | 111.0                | 207.5        | 73 - 21                                    | 861                      |
| 8                             | 98.5                 | 190.0        | 74 - 25                                    | 673                      |
| 10                            | 89.2                 | 171.5        | 74 - 17                                    | 553                      |
| 12                            | 82.7                 | 152.0        | 72 - 48                                    | 478                      |
| 14                            | 77.4                 | 132.3        | 69 - 58                                    | 427                      |
| 18                            | 73.8                 | 103.8        | 59 - 42                                    | 422                      |
| 20                            | 73.5                 | 93.6         | 51 - 59                                    | 457                      |

If the time required to turn through a given angle is calculated, it is found that the minimum time corresponds with the maximum bank. The foregoing data are based on the wind tunnel tests of a 1924 racing airplane.

"Squashing" in a turn is due to an attempt to fly under a condition requiring more power than is available. For example, any shorter radius of turn than 422 ft. at  $\alpha = 18^\circ$ , in the case calculated above, will require more power than is available and the airplane will lose flying speed and "squash" from centrifugal force since the centripetal force cannot be obtained from the reduced lift. The same condition exists in a pull-up from a dive. The lighter the load and wing loading and the greater the power, the more difficult to "squash" on a turn. High aspect ratio and low parasite resistance give low minimum power required and therefore reduce the tendency to "squash."

**Spiral gliding flight.** The spiral glide is a combination of gliding and circling, the path of the center of gravity being a true helix. If the angle of bank is  $\varphi$  and the helix angle is  $\theta$ , then assuming that the radius  $r$  is large in comparison with the span, so that velocity over wings is substantially uniform, it follows that

$$L \cos \varphi \cos \theta + D \sin \theta = W \quad (150)$$

$$W \sin \theta = D \quad (151)$$

$$\text{and} \quad W \cos \theta = L \cos \varphi \quad (152)$$

$$\text{from which} \quad \frac{L}{D} = \frac{\cot \theta}{\cos \varphi} \quad (153)$$

which reduces to  $\frac{L}{D} = \cot \theta$ , equation (135), when the angle of bank  $\varphi = 0$ , that is, in a straight glide.

The proper angle of bank is determined as in simple circling flight by the balancing of the centrifugal force with the radial component of lift. That is,

$$\text{centrifugal force} = \frac{WV^2 \cos^2 \theta}{gr} = L \cos \theta \sin \varphi \quad (154)$$

which reduces to equation (148) when  $\theta = 0^\circ$ .

**Effect of a diving start on speed over a measured course.** If an airplane is dived at high speed and pulled out into level flight, it will have a speed in excess of that which can be maintained in horizontal flight. If a high-speed flight is made over a measured course after a diving start, the speed will asymptotically approach the normal maximum, and the average over the course will be greater than the normal maximum. This method was formerly used extensively in races, and sometimes by test pilots in order to get the maximum speed possible.

Assuming that the propeller thrust is constant and the flight over the course is at constant altitude, the effect of a diving start on the average speed may be calculated.<sup>2</sup> The speed at any time  $t$  is

$$V = \frac{C_1 e^{at} + C_2}{C_3 e^{at} + C_4} \quad (155)$$

and the distance flown is

$$S = \frac{W}{gK} \log_e (C_3 e^{at} + C_4) - V_0 t - \frac{W}{gK} \log_e (2\sqrt{TK}) \quad (156)$$

<sup>2</sup> W. S. Diehl, "The Effect of a Diving Start on Airplane Speed," N. A. C. A. Technical Report No. 228 (1925).

where  $V_0$  is the speed (ft./sec.) at the time of crossing the starting line ( $t = t_0$ ),  $V$  the speed at time  $t$ ,  $S$  the distance flown in  $t$  seconds,  $W$  the gross weight,  $T$  the thrust at normal maximum speed  $V_M$ , and  $K$  the drag coefficient ( $K = \frac{T}{V_M^2}$ ). The other constants have the values

$$C_1 = T + V_0 \sqrt{TK}$$

$$C_2 = V_0 \sqrt{TK} - T$$

$$C_3 = \sqrt{TK} + KV_0$$

$$C_4 = \sqrt{TK} - KV_0$$

$$\text{and } a = \frac{2g \sqrt{TK}}{W}$$

The persistence of excess velocity depends largely on the value of  $a$  which does not vary over as wide a range as might be expected. Representative values for various types of airplanes are:

| Type             | $W$ lbs. | $V_M$ mi./hr. | $a$  |
|------------------|----------|---------------|------|
| Racing.....      | 2,100    | 250           | .050 |
| Pursuit.....     | 2,800    | 160           | .046 |
| Observation..... | 2,300    | 125           | .076 |
| Bomber.....      | 7,000    | 120           | .101 |

A racing or pursuit airplane requires about two minutes to settle down to normal speed after a diving start and a 10% increase in velocity at the start of a one-mile course will give about 6% increase over the normal maximum speed.

**Limiting speed in a dive.** The maximum speed in a dive with full throttle can be obtained from Figure 132 or Figure 133. If the propeller is idling, the thrust will be negative, and the value will depend on the pitch/diameter ratio. Data on propeller drag are given in Chapter VI. When propeller thrust is neglected, the solution gives the limiting speed at sea level

$$V_L = \sqrt{\frac{W}{K}} \quad (157)$$

where  $K$  is the drag coefficient defined by  $D = KV^2$ , and  $W$  is the gross weight.

While the time required to attain the theoretical limiting velocity is infinite, the initial approach is very rapid. If the fall be from rest, as from a stall or a loop, the relation between velocity and time is

$$V = V_L \left[ \frac{e^{at} - 1}{e^{at} + 1} \right] \quad (158)$$

where  $a = \frac{2g}{V_L}$ .

The time required to attain any speed  $V$  is obtained by solving equation (158) for  $t$ , and is

$$t = \frac{V_L}{2g} \log_e \left[ \frac{V_L + V}{V_L - V} \right] \quad (159)$$

The relation between velocity and altitude is

$$V = V_L \sqrt{(1 - e^{-bh})} \quad (160)$$

where  $b = \frac{2g}{V_L^2}$  and  $h$  is the total altitude lost. If  $h$  is in feet and  $t$  is in seconds,  $V_L$  must be in ft./sec. to determine  $a$  and  $b$ .

It is of interest to find how quickly an airplane picks up speed in a dive. Figure 135 gives the speed against altitude lost for limiting speeds of 200, 300, 400, and 500 ft./sec. The following table may be used in plotting similar curves for particular airplanes:

|                     |        |       |        |        |       |       |       |
|---------------------|--------|-------|--------|--------|-------|-------|-------|
| $\frac{V}{V_L} =$   | 0.2    | 0.4   | 0.5    | 0.6    | 0.7   | 0.8   | 0.9   |
| $\frac{h}{V_L^2} =$ | .00064 | .0027 | .00448 | .00693 | .0105 | .0159 | .0258 |

It is possible to calculate the effect of density on diving speeds but the result is of little practical value.

At some moderate altitude, an airplane can attain a speed in excess of the limiting velocity at that altitude if the dive is started at a very high altitude. If the dive is continued to lower altitudes, the speed will decrease slightly owing to the increasing air density.

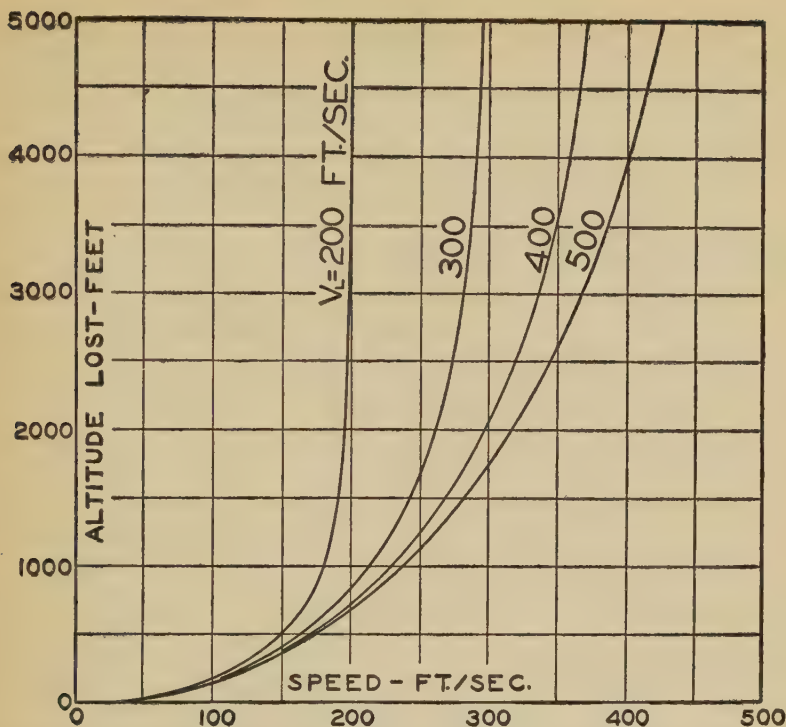


Figure 135. Variation of Air Speed with Limiting Velocity and Altitude Lost in a Vertical Dive

**Take-off run.** The distance run by an airplane in taking off is readily calculated if the following assumptions are made:

Air forces vary as  $V^2$ . That is,  $L = K_L V^2$  and  $D = K_D V^2$ . Average coefficients may be used when the angle of attack is variable.

Thrust increases uniformly with velocity.  $T = T_o' - aV$  ( $a$  = thrust gradient,  $T_o'$  = static thrust).

Friction drag varies as load on wheels and skid with constant coefficient of traction  $\mu$ .

The derivation of the equations from these assumptions is straightforward but too lengthy to be given here. The velocity  $V$  (ft./sec.) at the time  $t$  (sec.) from the start is

$$V = \frac{2T_0 (e^{bt} - 1)}{(C_1 e^{bt} - C_2)} \quad (161)$$

and the total distance run is

$$S = \frac{2T_0}{C_2} t + \frac{W}{gK} \log_e \left[ \frac{C_1 e^{bt} - C_2}{2 \sqrt{a^2 + 4KT_0}} \right] \quad (162)$$

The values of the constants are

$$T_0 = T_0' - \mu W. \quad T_0' = \text{static thrust, lbs.}$$

$$a = \text{thrust gradient} = \frac{T_0' - T}{V}$$

$$C_1 = a + \sqrt{a^2 + 4KT_0}$$

$$C_2 = a - \sqrt{a^2 + 4KT_0}$$

$$W = \text{gross weight, lbs.}$$

$$K = K_D - \mu K_L$$

$$b = \frac{g}{W} \sqrt{a^2 + 4KT_0}$$

The time to reach any velocity  $V$  may be obtained by solving equation (161) for  $t$ .

$$t = \frac{1}{b} \cdot \log_e \left[ \frac{2 T_0 - C_2 V}{2 T_0 - C_1 V} \right] \quad (163)$$

The foregoing equations are for calm. When the take-off is into a wind velocity  $V_W$ , the time required is  $t = t_2 - t_1$ , where  $t_1$  is the time required to attain the speed  $V = V_W$ , and  $t_2$  is the time of the getaway speed, both in a calm. The air distance traveled is  $S = S_2 - S_1$ , where  $S_1$  and  $S_2$  are the distances covered in times  $t_1$  and  $t_2$  respectively. The ground distance, or ground-run is  $S_g = S - V_W \cdot t$ .

If  $V$  and  $S$  are calculated at various intervals and plotted against  $t$ , the times and distances may be read directly for any wind speed and getaway speed. A convenient method of presenting these data for any given airplane is in the form of a curve of ground-run against wind speed. The calculations for this curve may be tabulated as follows:



1. Wind velocity,  $V_W$  ft./sec.
2. Time saved by wind,  $\Delta t$ . Read from curve of  $V$  against  $t$ .
3. Time to take off,  $t = t_0 - \Delta t$ .  $t_0$  is time to take off in calm.
4. Air distance at start of take-off,  $\Delta S$ . Read from curve of  $S$  against  $t$ .
5. Air distance during take-off,  $S' = S_0 - \Delta S$ .  $S_0$  is total distance in a calm.
6. Wind effect,  $S_W = V_W \cdot t$ .
7. Ground-run,  $S = S' - S_W$ .

The static thrust is calculated by the empirical equation

$$T_0 = 6,000 \left[ 18.7 - 9.5 \frac{p}{D} \right] \frac{B.Hp_0}{(r.p.m.) \cdot D} \quad (79)$$

where  $p$  is the pitch and  $D$  is the diameter. The thrust at any speed  $V$  may be found from

$$T = \frac{375 T.Hp.}{V \text{ mi./hr.}}$$

in which T.Hp. is the thrust power at the speed  $V$ . This can be obtained from Figure 96.

**Approximate take-off run.** A very close approximation to the take-off run may be obtained from a simple formula. The distance  $S$  required to attain a speed  $V_S$  with an average acceleration  $a$  is

$$S = \frac{V_S^2}{2a} \quad (164)$$

The average acceleration force on an airplane in take-off is proportional to the thrust power, or  $F = K\eta \cdot B.Hp$ . Hence

$$a = \frac{g}{W} K\eta \cdot B.Hp. = \frac{gK\eta}{\left( \frac{W}{Hp.} \right)} \quad (165)$$

The average efficiency in take-off is not greatly affected by propeller characteristics so that

$$S = KV_S^2 \left( \frac{W}{Hp.} \right) \quad (166)$$

Values of  $K$  have been determined from a number of observed and calculated take-off runs. If allowance be made for the known B.Hp. instead of the nominal B.Hp., and if values of  $S$  are taken from Figure 10 instead of Table I in N. A. C. A. Technical Report No. 249,<sup>3</sup> the observed value of  $K$  varies from .0119 to .0132 with an average value of about .0125. Values of  $K$  from eight calculated take-off runs range between .0110 to .0126 with an average value of .0118. The following values may be used:

|                    |             |
|--------------------|-------------|
| Pursuit.....       | $K = .0125$ |
| Observation.....   | $K = .0122$ |
| Light Bombers..... | $K = .0120$ |
| Heavy Bombers..... | $K = .0118$ |

If the engine is geared or if the propeller does not have the normal pitch and diameter for maximum efficiency at or just below high speed, it is better to write equation (166) in the form

$$S = .0215 V_S^{2.25} \frac{W}{T_0} \quad (166a)$$

Where  $V_S$  is the take-off speed in mi./hr.,  $W$  the gross weight and  $T_0$  the static thrust (See equation (79)). Equation (166a) is more accurate than equation (166).

**General curve for take-off run.** If the ground-run required to attain the getaway speed  $V_S$  is  $S_0$  in a calm, and  $S_W$  in a wind velocity  $V_W$ , then the ratio  $\frac{S_W}{S_0}$  is substantially the same at any given value of  $\frac{V_W}{V_S}$  for all airplanes. Figure 136 is a plot of  $\frac{S_W}{S_0}$  against  $\frac{V_W}{V_S}$  as determined from six calculated take-off runs for widely varying types of airplanes. This curve may be used to construct a curve of ground-run against wind velocity for any airplane for which the ground-run in a calm,  $S_0$ , and the stalling speed,  $V_S$ , are known. If great accuracy is not required,  $S_0$  may be obtained from equation (166).

<sup>3</sup> Thomas Carroll, "A Comparison of the Take-off and Landing Characteristics of a Number of Service Airplanes" (1927).

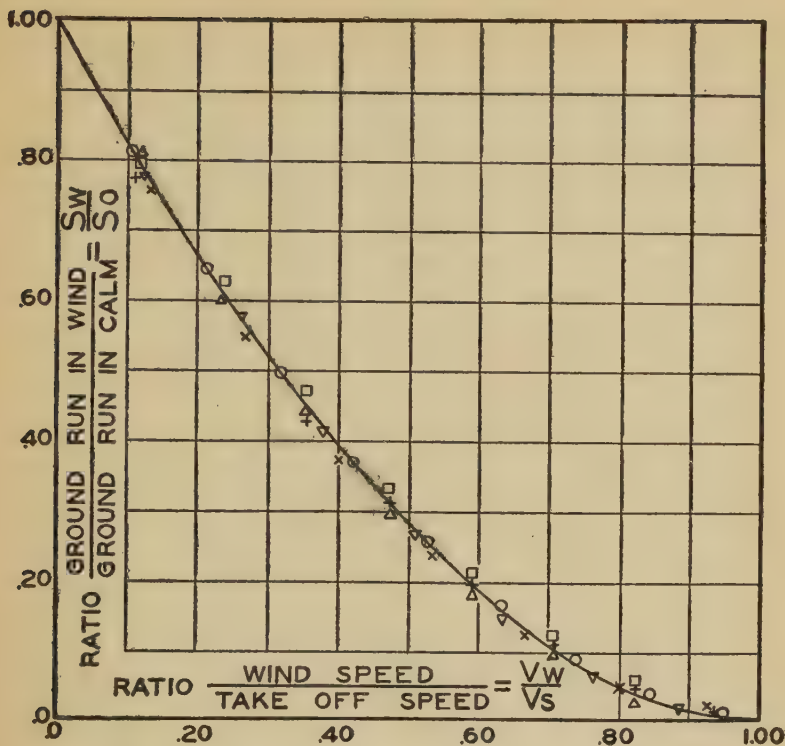


Figure 136. General Curve of Effect of Wind on Take-off Run

**Landing run.** It may be shown<sup>4</sup> that the landing run is given by

$$S = \frac{V_L^2}{2g \left( \frac{D}{L} - \mu \right)} \log_e \left( \frac{\left( \frac{D}{L} \right)}{\mu} \right) \quad (167)$$

where  $V_L$  is the landing speed in feet per second,  $\frac{D}{L}$  the reciprocal of the  $\frac{L}{D}$  at the landing angle, and  $\mu$  the coefficient of friction.

<sup>4</sup> H. Glauert, "The Landing Speed of Airplanes," Br. A. C. A., R. & M. No. 666.

The value of  $\mu$  is higher than that in the take-off owing to the high value of the tail skid drag. According to Glauert, the value of  $\mu$  for the skid alone is 0.50, and the average effective value of  $\mu$  for both skid and wheels is about 0.12. Obviously, this value is much affected by the shape of the skid and the load

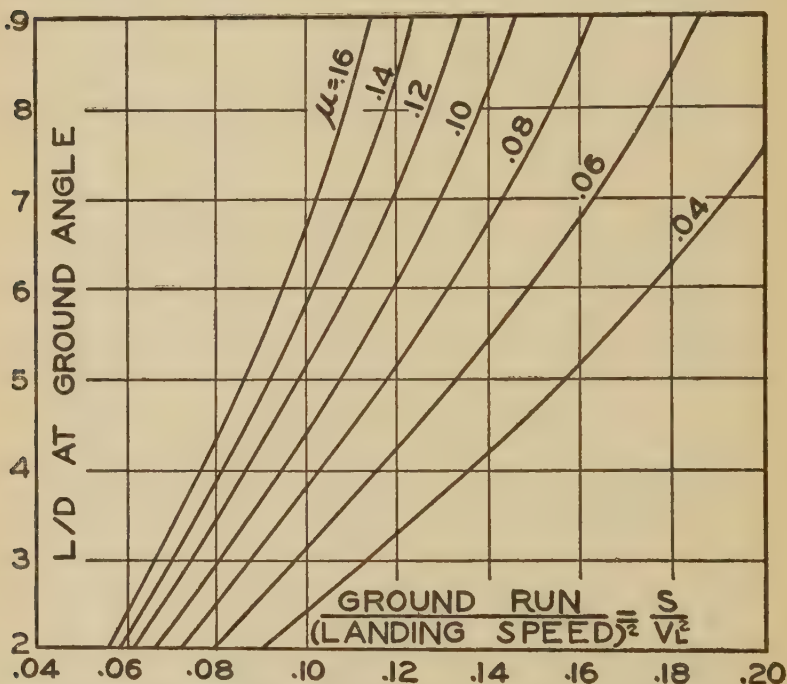


Figure 137. Relation Between Landing Run, Landing Speed, Coefficient of Friction and  $\frac{L}{D}$

that it carries. Since there is considerable variation in landing speed according to the method of landing, a similar variation in run is to be expected.

Figure 137 is a plot of  $\frac{S}{V_L^2}$  against  $\frac{L}{D}$  and  $\mu$  as calculated from equation (167). A study of this figure in connection with known landing runs leads to the conclusion that the average value of  $\mu$  is probably between .06 and .10, and that the value

of  $\frac{S}{V_L^2}$  is normally between .008 and .0140. The value of  $\frac{S}{V_L^2}$  is considerably reduced by increasing the angle of attack of the wings when the skid is on the ground.

**Descent of a parachute.** The terminal velocity of a parachute, in ft./sec., is

$$V = \sqrt{\frac{W}{C_D \rho S}} = \sqrt{\frac{W}{K \frac{\rho}{\rho_0} S}} \quad (168)$$

where  $W$  is the gross weight in lbs.,  $\rho$  the air density (lbs.-ft.-sec. units), and  $S$  the total area in sq. ft.  $C_D$  varies according to the shape of the parachute, between 0.70 and 0.80. The corresponding values of  $K$  are about .0017 and .0019. (See Chapter IV.)

The usual method of determining  $K$  by timing the drop from a known altitude may lead to values as much as 10% too high unless correction is made for air density and its variation with altitude. A simple method of making the correction is available. For any increment less than 6,000 ft. in altitude, the density ratio  $\frac{\rho}{\rho_0}$  is linear with altitude

$$\frac{\rho}{\rho_0} = (1 - ah) \quad (169)$$

The parachute may be assumed to descend with its instantaneous limiting velocity at all altitudes. The equation of motion is

$$\frac{dV}{dt} = g - \frac{KS}{W} \frac{\rho}{\rho_0} V^2 = g - \frac{KS}{W} (1 - ah) V^2 \quad (170)$$

$\frac{dV}{dt}$  is very small in comparison with other terms since it has a value of the order of .01. Substituting  $\frac{dh}{dt}$  for  $V$ , neglecting  $\frac{dV}{dt}$  and integrating, gives

$$t = \sqrt{\frac{KS}{W}} \cdot \frac{2}{3a} \left[ (1 - ah_2)^{1.5} - (1 - ah_1)^{1.5} \right] \quad (171)$$

$h_1$  being the higher altitude.

The value of  $a$  must be determined by the densities at  $h_1$  and  $h_2$ , that is, from pressure and temperature readings. Since  $\frac{\rho}{\rho_0} = 0.3789 \frac{P}{T}$ , where  $P$  is the pressure in mm. of Hg and  $T$  is the absolute temperature in °C., the value of  $a$  may be obtained from

$$a = \frac{0.3789}{h_1 - h_2} \left[ \frac{P_2}{T_2} - \frac{P_1}{T_1} \right] \quad (172)$$

In the standard atmosphere below 6,000 ft.,  $a = .000027$ . For a parachute having a limiting velocity  $\sqrt{\frac{W}{KS}} = 20$  ft./sec. in air of standard density, the time of fall from  $h_1 = 2,000$  ft., to  $h_2 = 0$  ft., (sea level), is 98.6 sec. giving an average velocity of  $V = 20.28$  ft./sec., which reduces  $K$  about 3%. The error is approximately proportional to the altitude.

It is interesting to note that limiting velocity of a parachute may vary more than 5% with the normal sea-level temperature changes.

**The "flat spin."** To the casual observer, the only difference between a conventional spin and the so-called "flat spin" is in the attitude of the airplane. In a normal spin, the longitudinal axis is inclined well downward and is nearer vertical than horizontal. In the flat spin the longitudinal axis may be almost horizontal and the motion almost pure yawing. Very few airplanes can be kept in a flat spin and once in a flat spin recovery is always difficult. The reasons for this are now well known as a result of independent and correlated investigations carried out by the Army, Navy, and National Advisory Committee for Aeronautics.<sup>5</sup> These investigations included a large number of actual spins carried out by the Army and Navy.

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<sup>5</sup> Montgomery Knight, "Wind Tunnel Tests on Autorotation and the Flat Spin," N. A. C. A. Technical Report No. 273 (1927).



The flat spin is due chiefly to three major factors and perhaps to two or more minor factors. The major factors are wing arrangement, mass distribution, and center of gravity location. The minor factors are control size, control efficiency, etc.

In order that a flat spin may be maintained, it is necessary that rotation at high angles of attack be a stable condition. This condition apparently exists in all biplanes having wings of equal chord without stagger. On the other hand, it is probable that no monoplane will autorotate at very large angles of attack.

The second major condition is mass distribution. A solid body always tends to rotate about its principal axis having the greatest moment of inertia. An airplane which has a much larger moment of inertia about its normal axis than about its longitudinal and lateral axes is subject to large precessional moments in the combined roll and yaw of a spin. The major precessional moment is a stalling moment which holds up the nose of the airplane. If the c.g. is well aft, say beyond 35% of the mean chord, the precessional moment may be greater than the elevator moment, and recovery from a spin is very difficult.

The obvious remedies in biplane design are to use stagger and overhang if possible, preferably with some taper in the wings, and to keep the wing weights low in order to reduce the moment of inertia in yaw. Large tail surfaces are of considerable assistance in preventing a dangerous spin or in assuring quick recovery from any spin. A majority of the airplanes which have been reported to recover slowly from spins are deficient in horizontal tail surface area.

The results of the British investigation of spinning are given in R. & M. No. 1001, "The Spinning of Aeroplanes," by W. B. Gates and L. W. Bryant (1927). This is a very extensive and valuable report, covering all phases of the spin.

## CHAPTER XIV

### PERFORMANCE ESTIMATION

The aeronautical engineer frequently finds it desirable to estimate the change in performance due to slight changes in design characteristics, not great enough to justify a complete performance calculation. Methods for the rapid estimating of performance are also of value for other purposes where an approximate solution will serve. An experienced estimator can usually obtain results which agree very well with accurate calculations or flight test data. Numerous methods for estimating performance have been published from time to time, but none can be considered as a substitute for the complete calculation, if accuracy is essential. The methods described in this chapter have been thoroughly tried by the author and are believed to be sufficiently reliable for most purposes.

**Stalling speed.** The stalling speed, in miles per hour at sea level is calculated from the equation

$$V_S = 19.77 \sqrt{\frac{\left(\frac{W}{S}\right)}{C_{L \max}}} = \sqrt{\frac{\left(\frac{W}{S}\right)}{\bar{K}_y \max}} \quad (88)$$

where  $\frac{W}{S}$  is the wing loading in lbs./sq. ft. If the wing section is normal in thickness and camber, the maximum value of  $C_L$  or  $\bar{K}_y$  may be read directly from wind tunnel test data, but the model chord should be greater than 4 in. and the wind speed greater than 50 ft./sec. in the test. If the section is thin, or "double cambered," the lift coefficient may increase considerably with Reynolds Number. If the section is thick or deeply cambered, the lift may decrease with Reynolds Number. (See Chapter I for additional information.)

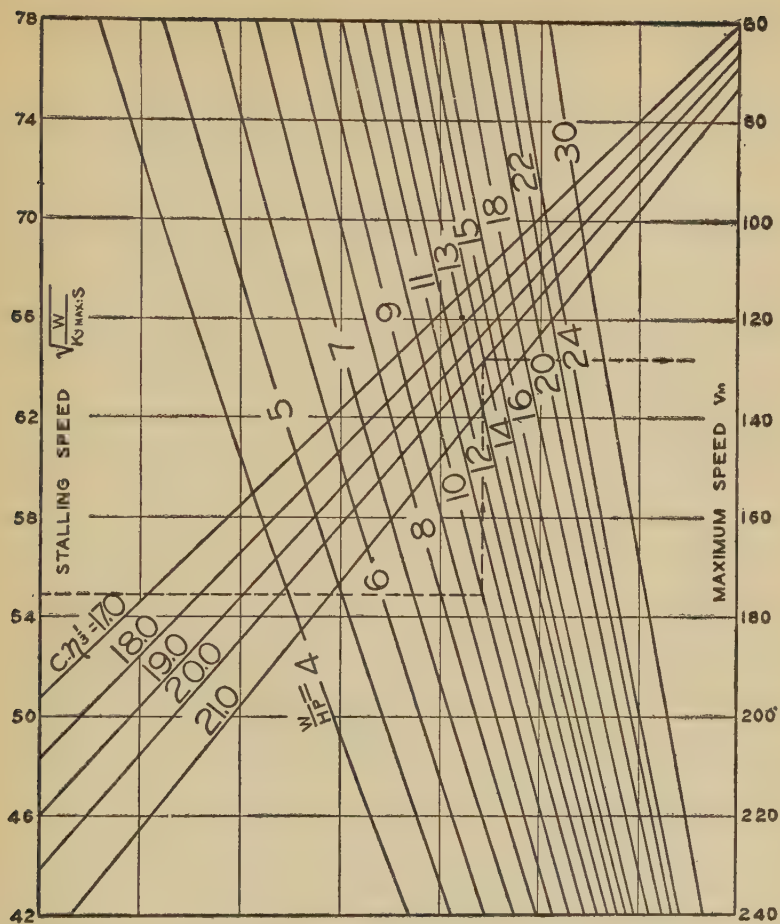


Figure 138. Graphical Solution of Equation (114) for Maximum Speed in mi./hr.

**Maximum speed.** Maximum speed may be calculated from the formula:<sup>1</sup>

$$\frac{V_M}{V_S} = \frac{K \left( \frac{L}{D} \cdot \eta_m \right)^{\frac{1}{3}}}{\left( V_S \cdot \frac{W}{H \cdot p} \right)^{\frac{1}{3}}} \quad (114)$$

<sup>1</sup> For derivation of this and other performance formulas see W. S. Diehl, "Reliable Formula for Estimating Airplane Performance, etc.," N. A. C. A. Technical Report No. 173 (1923).

in which  $V_M$  is the maximum speed,  $V_S$  the stalling speed,  $\frac{L}{D}$  the maximum overall lift/drag ratio,  $\eta_m$  the maximum propeller efficiency,  $\frac{W}{Hp}$  the power loading in lbs. per B.Hp., and  $K$  a constant which depends on the effective aspect ratio and parasite resistance as shown on Figure 119, Chapter IX. The average value of  $K$  is about 10.2, and the average value of  $K\left(\frac{L}{D} \cdot \eta_m\right)^{\frac{1}{3}}$  is between 19.0 and 20.0. Figure 138 is a plot of equation (114) enabling maximum speeds to be read directly.

**Initial rate of climb.** The initial rate of climb in ft./min. may be calculated from the formula

$$\left(\frac{dh}{dt}\right)_0 = C_0 = 33,000 \left[ \frac{K_2 \eta_m}{\left(\frac{W}{Hp}\right)} - \frac{V_C}{375 \frac{L}{D}} \right] \quad (173)$$

in which  $\frac{W}{Hp}$  is the power loading in lbs./B.Hp.,  $\eta_m$  the maximum propeller efficiency,  $V_C$  the climbing speed in mi./hr.,  $\frac{L}{D}$  the maximum overall ratio of lift to drag, and  $K_2$  the ratio of power available at climbing speed to power available at maximum speed. Analysis of power curves for a number of airplanes shows that an average value of  $K_2$  is

$$K_2 = \left(\frac{V_M}{V_S}\right)^{-0.27} \quad (174)$$

$K_2$  is plotted against  $\frac{V_M}{V_S}$  in Figure 139. This average value of  $K_2$  is based on a climbing speed  $V_C = V_S + \frac{1}{3}(V_M - V_S) = \frac{2V_S + V_M}{3}$ .

Actually, the climbing speed varies considerably with aspect ratio and parasite as shown by Figure 120, but the effect on the rate of climb is not great. However, if desired, a more exact value of  $V_C$  may be used and the value of  $K_2$  determined from

Figure 96, using the ratio  $\frac{V_C}{V_M}$ .

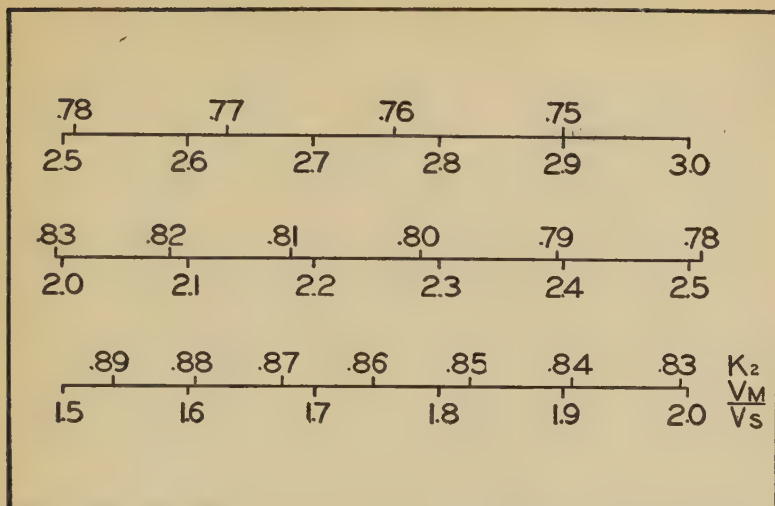


Figure 139. Thrust Power Coefficient  $K_2$ , in Equation (173). Values of  $K_2$  are Above and Values of  $\frac{V_M}{V_S}$  are Below the Base Lines

**Absolute ceiling.** When plotted against speed, the curves of  $T.Hp_a$  and  $T.Hp_r$  are tangent at the absolute ceiling. The point of tangency is very near the minimum power required, and normally at a speed approximately  $\sqrt{\frac{\rho_0}{\rho}} \times V_{MP}$ , where  $V_{MP}$  is the speed for minimum power at sea level. These relations enable the absolute ceiling to be given in terms of the ratio  $\frac{Hp_{ao}}{Hp_{ro}}$  where  $Hp_{ro}$  is the minimum power required at sea level, and  $Hp_{ao}$  is the power available at sea level and at the air speed of the tangency. It may be shown that

$$\frac{Hp_{ao}}{Hp_{ro}} = \frac{K_3' \eta_m \cdot Hp}{\left[ \frac{W \cdot V_{MP}}{375 \left( \frac{L}{D} \right)} \right]} = \frac{K_3 \eta_m \left( \frac{L}{D} \right)}{K_5 V_S \left( \frac{W}{Hp} \right)} \quad (175)$$

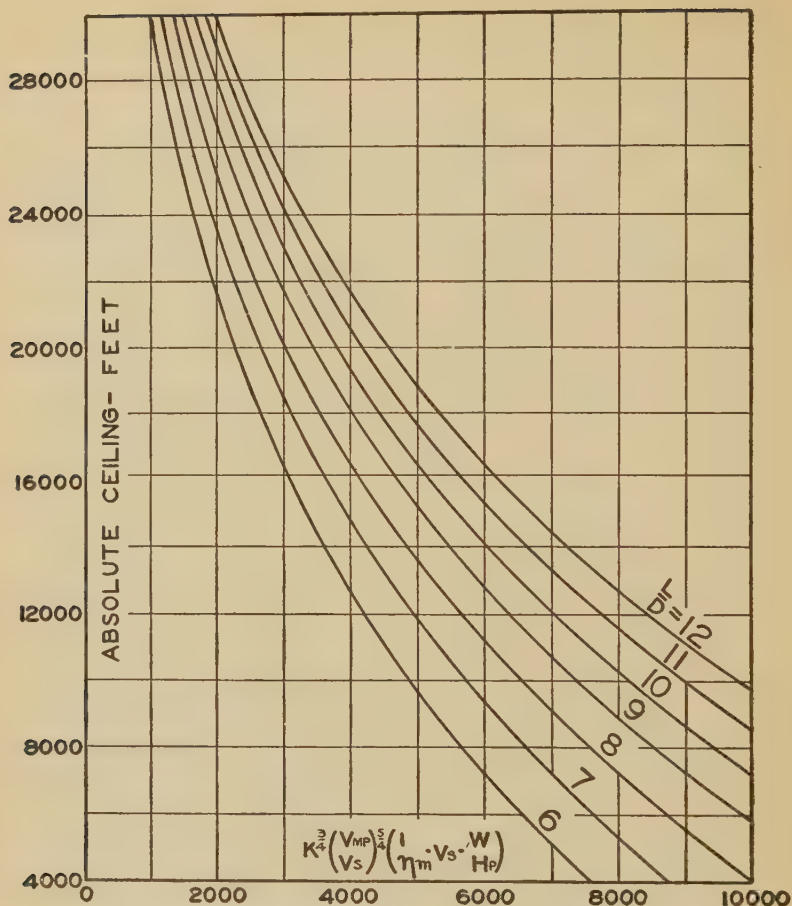


Figure 140. Graphical Solution of Equation (176) for Absolute Ceiling

Analysis of power curves shows that  $K_3 = 375 \left( \frac{V_M}{V_S} \right)^{-0.60}$ . Substituting the value of  $\frac{V_M}{V_S}$  from equation (114) gives:

$$K_3 = \frac{375}{K^{0.60}} \times \left[ \frac{\frac{1}{\eta_m} \cdot V_S \cdot \frac{W}{H_p}}{\frac{L}{D}} \right]^{0.20}$$



substituting this into the equation (175) gives

$$\frac{H\dot{p}_{.a_0}}{H\dot{p}_{.r_0}} = 375 \left[ \frac{\frac{L}{D}}{K^{\frac{1}{2}} (K_5)^{\frac{1}{2}} \cdot \frac{1}{\eta_m} \cdot V_S \cdot \frac{W}{H\dot{p}_{.}}} \right]^{0.80} \quad (176)$$

Neither  $K$  nor  $K_5$  vary enough to affect the value of the expression in the brackets, so that the average values of  $K^{\frac{1}{2}} = (10.2)^{\frac{1}{2}} = 5.7$  and  $(K_5)^{\frac{1}{2}} = (1.10)^{\frac{1}{2}} = 1.13$  may be used. The product  $K^{\frac{1}{2}} (K_5)^{\frac{1}{2}}$  then has the average value 6.40.

Equation (176) is most conveniently used when plotted as in Figure 140. To use this plot, first calculate the value of the

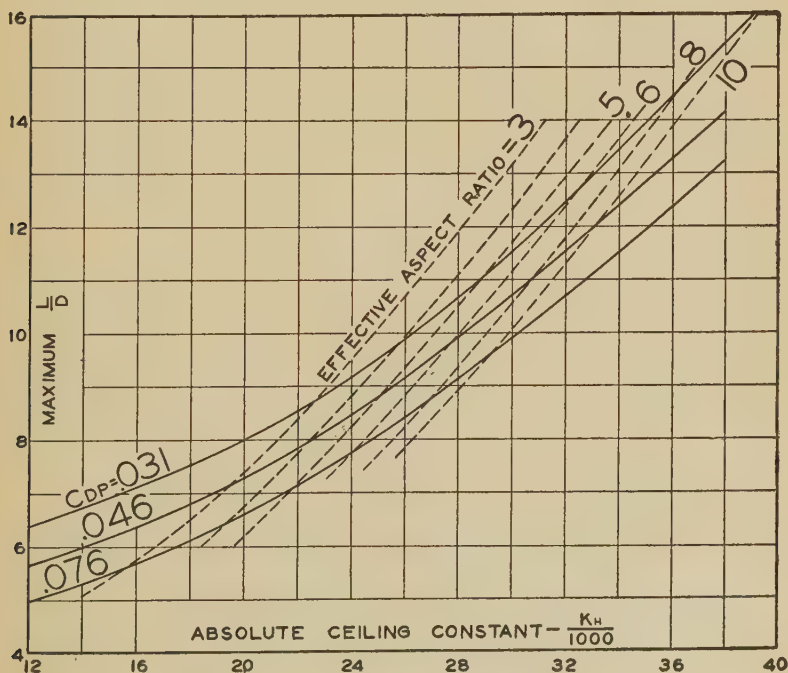


Figure 141. Absolute Ceiling Constant  $K_H$  in Equation (177)

denominator  $\left[ 6.40 \frac{1}{\eta_m} V_S \cdot \frac{W}{Hp.} \right]$ , then pass vertically along this

value to the proper  $\frac{L}{D}$  and then horizontally to the absolute ceiling.

A slightly more reliable method of estimating absolute ceiling is based on the equation

$$H = K_H \cdot \log_{10} \left[ \frac{10,000}{\frac{W}{S} \cdot \left( \frac{W}{\eta Hp.} \right)^2} \right] \quad (177)$$

where  $\frac{W}{S}$  is the wing loading (lbs./sq. ft.),  $\frac{W}{Hp.}$  the power loading (lbs./B.Hp.),  $\eta$  the maximum propeller efficiency, and  $K_H$  a factor which varies with aspect ratio and parasite as shown on Figure 141. This formula has given very satisfactory results.

**Service ceiling.** If the rate of climb decreased linearly with altitude, the service ceiling (at which the rate of climb is 100 ft./min.) would be given by

$$h_S = H \left( \frac{C_0 - 100}{C_0} \right) \quad (99)$$

Owing to the curvature in the rate of climb curve, the actual service ceiling will be slightly below the value given by equation (99). Figure 142 is a plot of  $\Delta h_S$  against absolute ceiling  $H$ ,  $\Delta h_S$  being the difference between the straight line value and the exact value of  $h_S$ .  $\Delta h_S$  is closely approximated by the relation

$$\Delta h_S = 40 \left( \frac{H}{10,000} \right)^3 \quad (178)$$

consequently the exact service ceiling is given by

$$h_S = H \left( \frac{C_0 - 100}{C_0} \right) - 40 \left( \frac{H}{10,000} \right)^3 \quad (179)$$

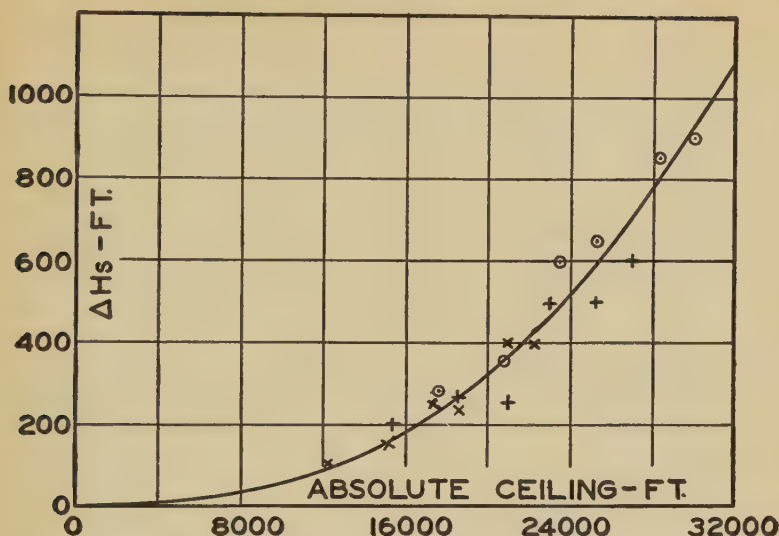


Figure 142. Difference Between Service Ceilings for Linear and for Non-Linear Rate-of-Climb Curves

**Time of climb.** With a linear relation between rate of climb and altitude, the altitude  $h$  climbed in  $T$  minutes is

$$h = H (1 - e^{-\frac{C_0}{H} T}) \quad (101)$$

where  $H$  is the absolute ceiling (ft.), and  $C_0$  the initial rate of climb (ft./min.). A simple relation may be found between the altitude given by equation (101), and the actual altitude for a non-linear rate of climb.

When the ratio of  $\frac{C}{C_0}$  is plotted against  $\frac{h}{H}$  as in Figure 143, the various points (representing about 20 airplanes) lie very near a single curve, which indicates that all climb curves may be similar. Figure 143 may be used for drawing up estimated rate-of-climb curves when the initial rate of climb  $C_0$  and the absolute ceiling  $H$  are known. It may also be used to determine some general relations.

Plotting  $\frac{C_0}{C}$  from Figure 143 against  $\frac{h}{H}$  and integrating the area under the curve gives factors proportional to the times required to climb to any altitude  $h$ . A similar plot of  $\frac{C_0}{C}$  from

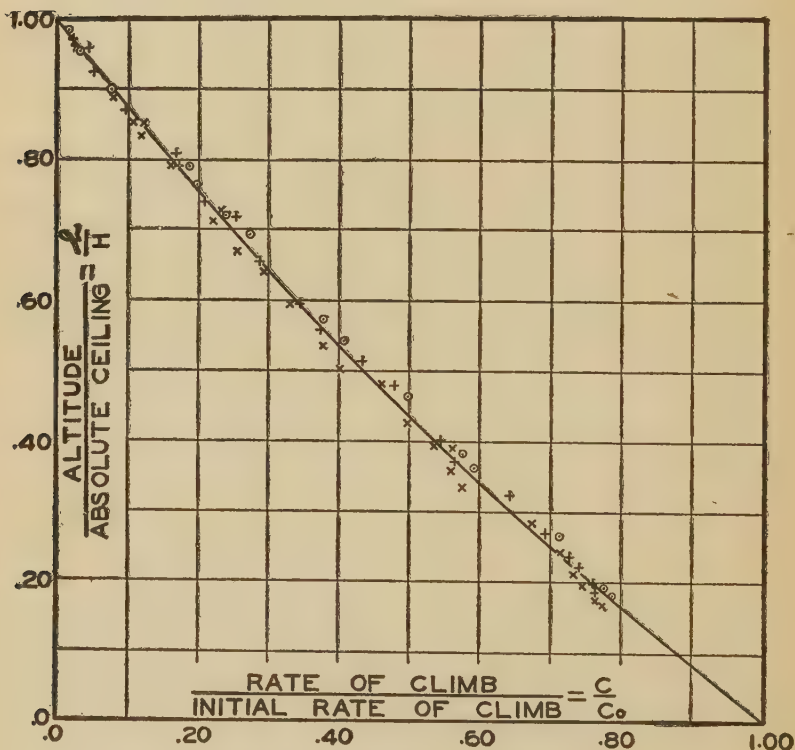


Figure 143. General Rate-of-Climb Curve

equation (101) gives similar factors for linear rate of climb.

The ratio of these factors gives the ratio between the times  $\frac{T_C}{T_S}$  to climb to a given percentage of the absolute ceiling,  $T_C$  being the time for non-linear and  $T_S$  the time for linear rate of climb

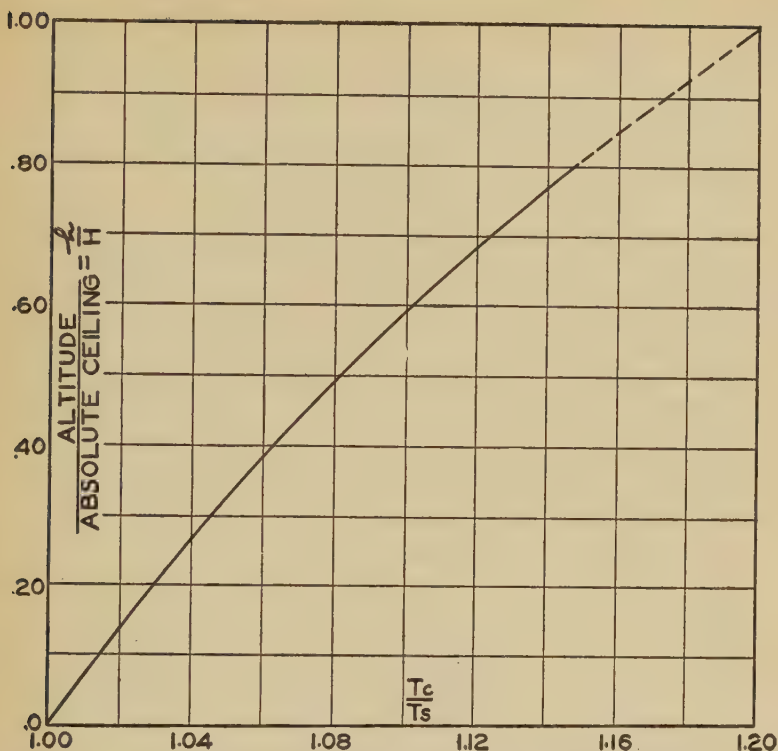


Figure 144. Relation Between the Times to Climb to a Given Altitude with Linear and Non-Linear Rate-of-Climb Curves

against altitude.  $\frac{T_C}{T_S}$  is plotted against  $\frac{h}{H}$  in Figure 144. This figure may be used to determine the value of  $T_C$  from a known or easily determined value of  $T_S$ .

Figure 145 is a plot of  $\frac{T_S}{\left(\frac{H}{C_0}\right)}$  and  $\frac{T_C}{\left(\frac{H}{C_0}\right)}$  against  $\frac{h}{H}$ , using the

data in Table 20. This figure may be used to calculate either the time required to climb to a given altitude or the altitude climbed in a given time, for either a linear or non-linear rate of climb. For example, assume  $H = 20,000$  ft.,  $C_0 = 1,600$  ft./min.,

then  $\frac{H}{C_0} = 12.5$  min. The times to climb to 10,000 ft.,  $\frac{h}{H} = 0.5$ ,

are  $T_S = 0.69 \frac{H}{C_0} = 8.62$  min., and  $T_C = 0.75 \frac{H}{C_0} = 9.37$  min.

The altitudes climbed in 10 minutes are found from  $\frac{T_S}{\left(\frac{H}{C_0}\right)} =$

$\frac{T_C}{\left(\frac{H}{C_0}\right)} = \frac{10}{12.5} = 0.80$ , giving  $\frac{hs}{H} = 0.55$  from which  $hs = 11,000$  ft.,

and  $\frac{hc}{H} = 0.52$  from which  $hc = 10,400$ .

TABLE 20. TIME OF CLIMB FACTORS

| $\frac{h}{H}$ | $\frac{T_S}{\left(\frac{H}{C_0}\right)}$ | $\frac{T_C}{\left(\frac{H}{C_0}\right)}$ | $\frac{T_C}{T_S}$ |
|---------------|------------------------------------------|------------------------------------------|-------------------|
| 0             | 0                                        | 0                                        | 1.000             |
| .10           | .1053                                    | .1068                                    | 1.014             |
| .20           | .2232                                    | .2300                                    | 1.029             |
| .25           | .2875                                    | .2982                                    | 1.037             |
| .30           | .3563                                    | .3723                                    | 1.045             |
| .35           | .4305                                    | .4540                                    | 1.054             |
| .40           | .511                                     | .544                                     | 1.063             |
| .45           | .598                                     | .642                                     | 1.072             |
| .50           | .693                                     | .749                                     | 1.081             |
| .55           | .797                                     | .870                                     | 1.091             |
| .60           | .916                                     | 1.009                                    | 1.102             |
| .65           | 1.051                                    | 1.171                                    | 1.113             |
| .70           | 1.203                                    | 1.352                                    | 1.124             |
| .75           | 1.386                                    | 1.574                                    | 1.136             |
| .80           | 1.609                                    | 1.849                                    | 1.148             |
| .90           | 2.303                                    | 2.703                                    | 1.173             |

The *climb in 10 minutes* is used extensively as a measure of performance. Three charts based on the linear relation have been prepared for the solution of various problems involving this factor. Figure 146 is a plot of  $\frac{h_{10}}{H}$  against  $\frac{C_0}{H}$  from which  $h_{10}$  may be accurately determined. This value may then be



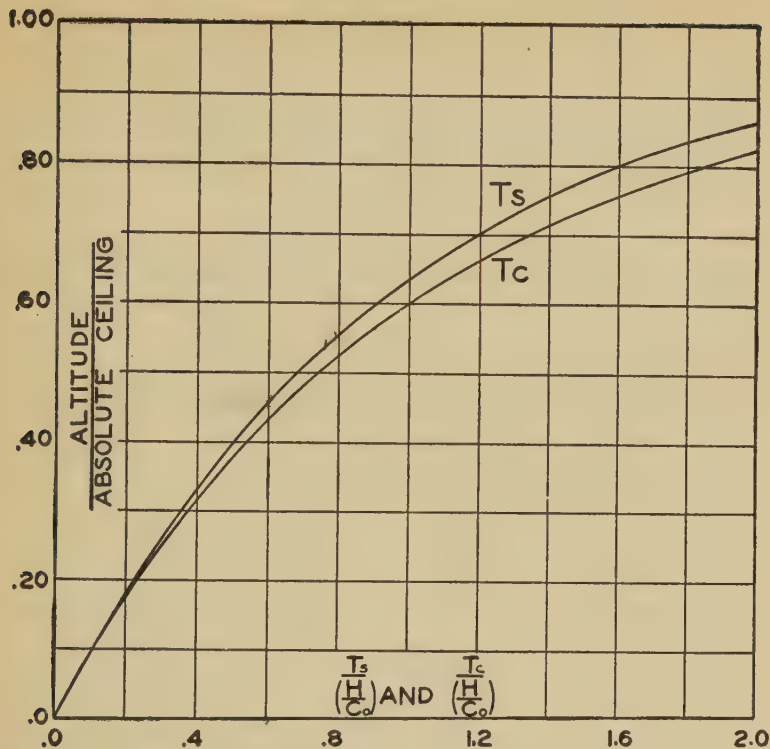


Figure 145. Time to Climb to a Given Altitude.  $T_s$  for Linear,  $T_c$  for Non-Linear Rate of Climb Curves

corrected for the non-linear relation, if desired, using one of the preceding figures. Figure 147 is a plot of  $h_{10}$  against  $H$  and  $C_0$ , from which  $h_{10}$  may be read directly. Figure 148 is a similar plot with the service ceiling  $h_s$  instead of the absolute ceiling  $H$ . The latter figure may be used to determine initial rate of climb when only  $h_{10}$  and  $h_s$  are given.

The climb in 10 minutes is easily obtained from Figure 145, as illustrated by the example on page 242. In general, Figure 145 will be found most convenient for this purpose, but there are many problems for which Figures 146-148 are especially adapted.

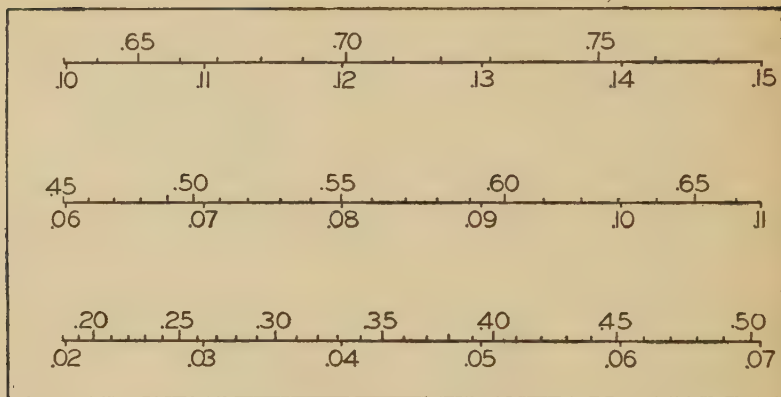


Figure 146. Climb in 10 Minutes, from Absolute Ceiling and Initial Rate of Climb (linear). Values of  $\frac{h_{10}}{H}$  are Above and Values of  $\frac{C_0}{H}$  are Below Base Lines

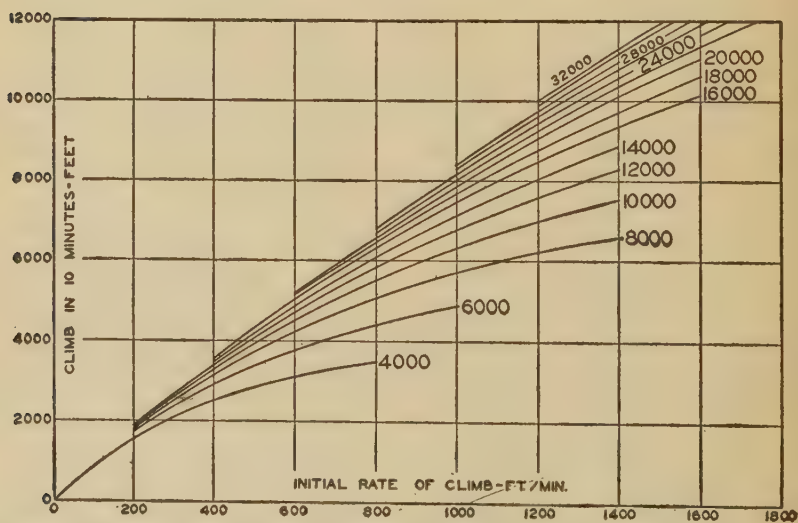


Figure 147. Climb in 10 Minutes from Absolute Ceiling and Initial Rate of Climb (Linear)

**Range and endurance.** The endurance at maximum speed is obtained by dividing the full load by the full throttle fuel consumption in lbs./hr., using the known specific fuel consumption and actual B.Hp. if known, otherwise the rated, or nominal B.Hp. If the specific fuel consumption is not known, it may

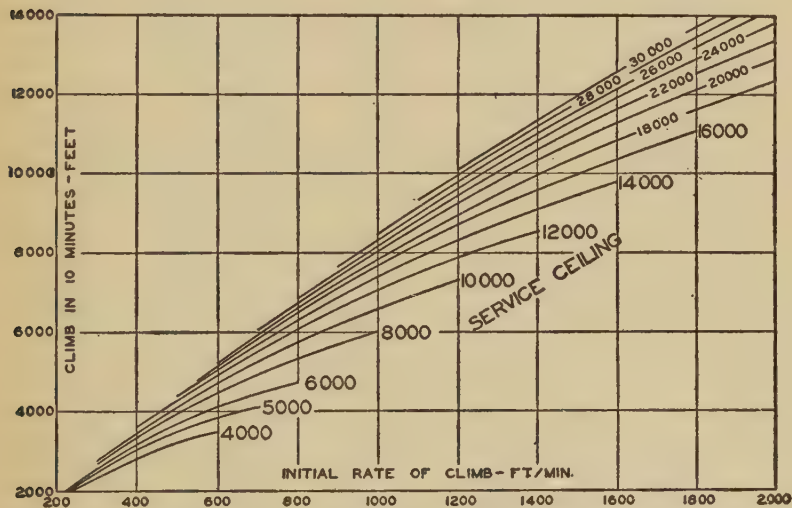


Figure 148. Climb in 10 Minutes from Service Ceiling and Initial Rate of Climb (Linear)

be estimated: for modern engines it varies from 0.46 to 0.55 lbs./B.Hp./hr., depending largely on the treatment the engine receives. An average value of 0.50 will not be far off. The range at maximum speed is obviously the product of the corresponding endurance and speed.

After the endurance and range at maximum speed have been determined, the endurance and range at any other speed are readily found by the factor method<sup>2</sup>. This method is based on the factors

<sup>2</sup> W. S. Diehl, "Three Methods of Calculating Range and Endurance of Airplanes," N. A. C. A. Technical Report No. 234 (1925).

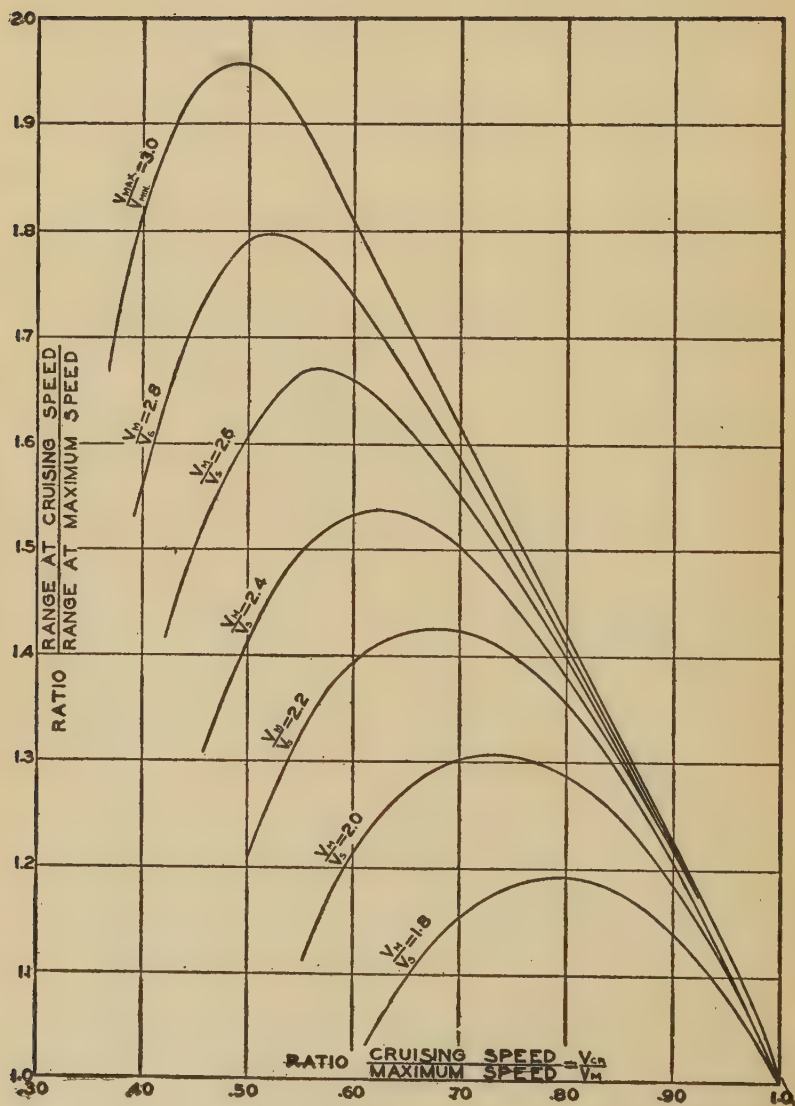


Figure 149. Range Factors in Terms of Speed and Speed Range

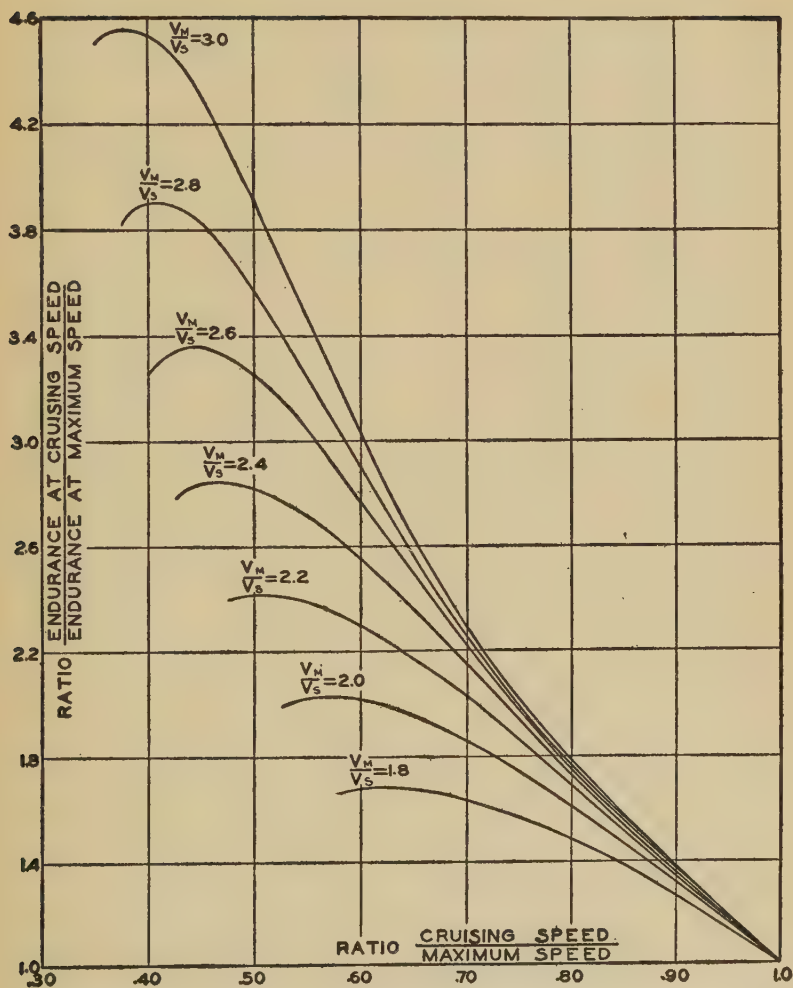


Figure 150. Endurance Factors in Terms of Speed and Speed Range

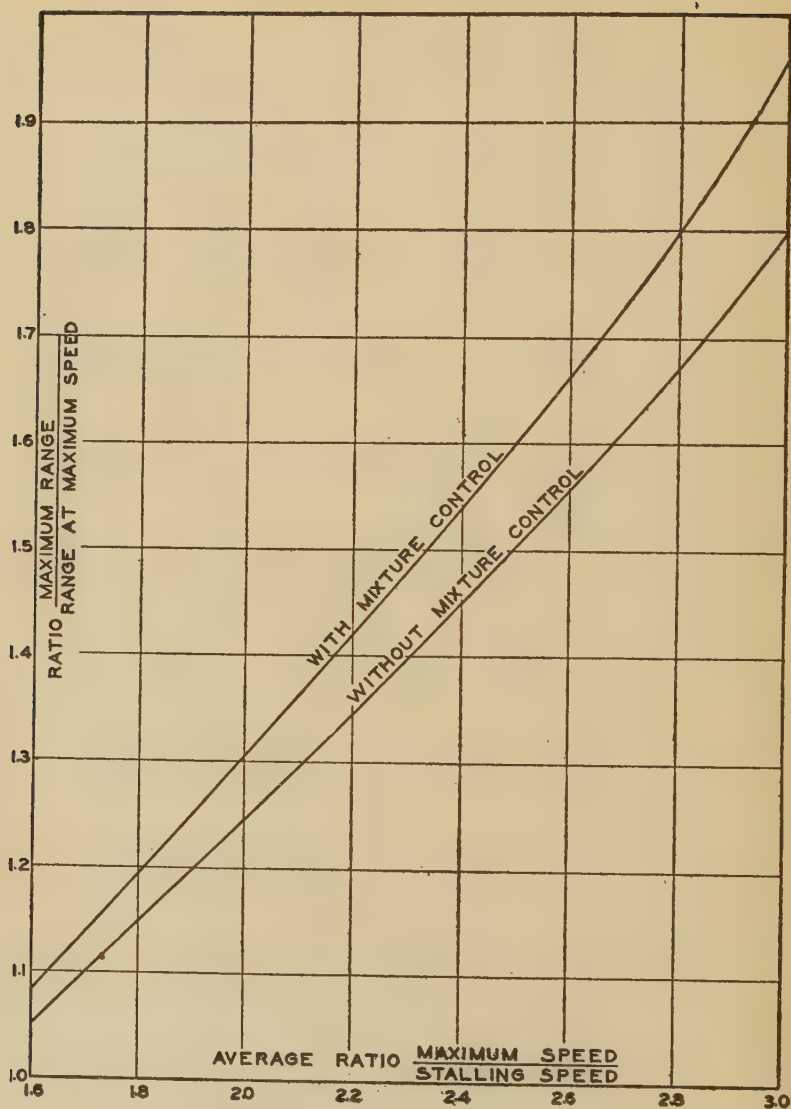


Figure 151. Maximum Range Factors



$$\text{Endurance Factor } F_E = \frac{\text{Endurance at speed } V}{\text{Endurance at max. speed}}$$

and

$$\text{Range Factor } F_R = \frac{\text{Range at speed } V}{\text{Range at max. speed}}$$

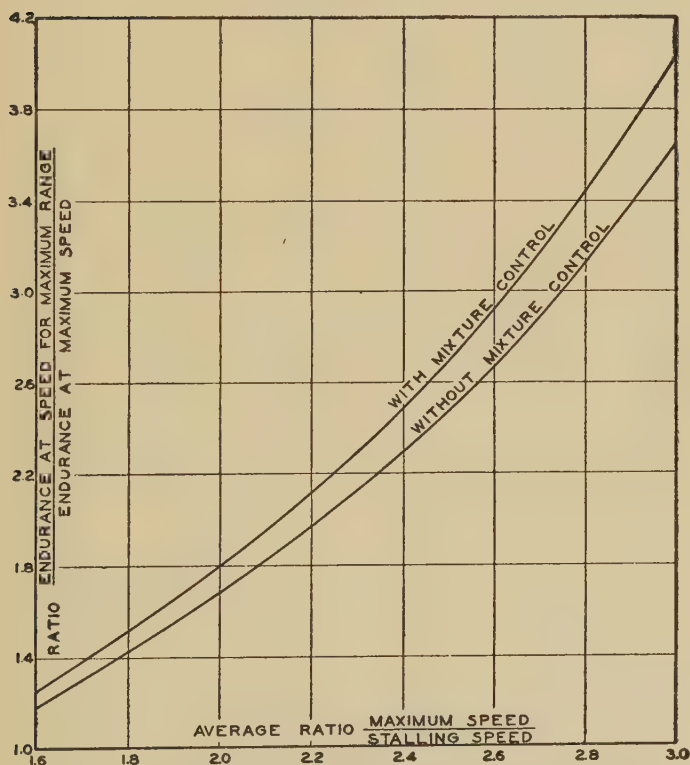


Figure 152. Endurance Factors at Speed for Maximum Range

$F_R$  and  $F_E$  vary with the ratios  $\frac{V_M}{V_S}$  and  $\frac{V_C}{V_M}$ , as shown on Figures 149 and 150. These figures are based on the assumed use of mixture control, the effect of which is shown elsewhere.

In Figure 151 the value of  $F_R$  for maximum range is plotted

against the speed range ratio  $\frac{V_M}{V_S}$ , and in Figure 152 the corresponding values of  $F_E$  are given. These figures enable the maxi-

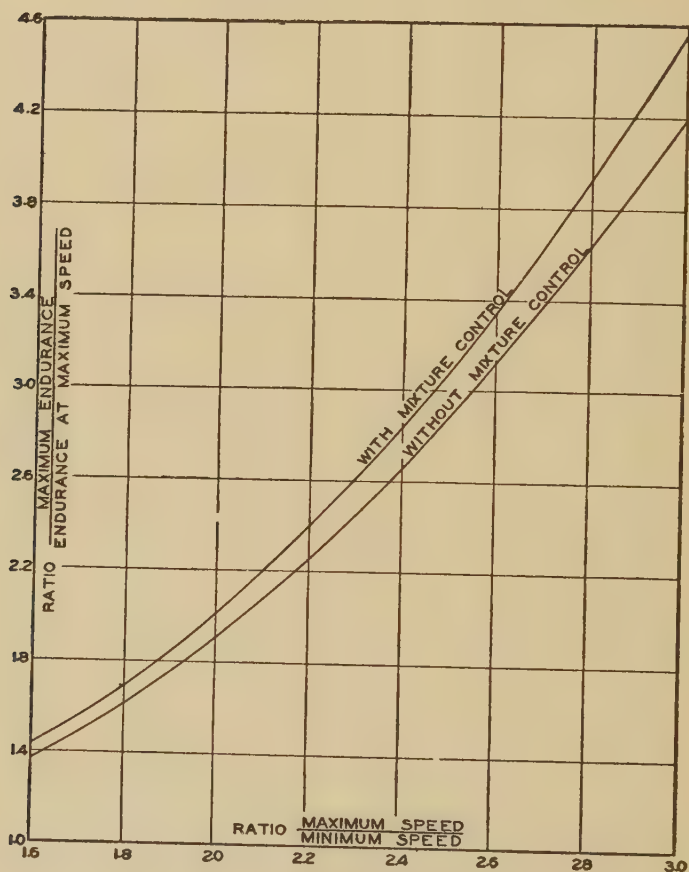


Figure 153. Maximum Endurance Factors

imum range and corresponding endurance to be calculated rapidly.

The maximum endurance is given by the plot of maximum  $F_E$  against  $\frac{V_M}{V_S}$  in Figure 153.

**Example of performance estimation.** Consider the airplane for which performance was calculated in Chapter VII. The characteristics were as follows:

$$\begin{array}{lll} W = 2,100 \text{ lbs.} & S = 290 \text{ sq. ft.} & B.Hp. = 195 \\ \frac{W}{S} = 7.25 & \frac{W}{B.Hp.} = 10.76 & \eta_m = 0.78 \end{array}$$

$$\left(\frac{L}{D}\right)_{max} = 8.2 \quad V_S = 52.6 \text{ mi./hr.}$$

$$\text{Effective aspect ratio} = 4.8$$

*Maximum Speed:* Use equation (114). From Figure 119,  $K = 10.10$ .

$$\frac{V_M}{V_S} = \frac{10.10 (8.2 \times 0.78)^{\frac{1}{2}}}{(52.6 \times 10.76)^{\frac{1}{2}}} = 2.24$$

$$V_M = 2.24 \times 52.6 = 118 \text{ mi./hr.} \quad (116)$$

*Initial rate of Climb:* Use equation (173).  $\frac{V_M}{V_S} = 2.24$ .

From Figure 139,  $K_2 = 0.805$ .

$$\begin{aligned} C_0 &= 33,000 \left[ \frac{0.805 \times 0.78}{10.76} - \frac{(2 \times 52.6 + 118)}{(3 \times 375 \times 8.2)} \right] \\ &= 1,130 \text{ ft./min. (1,100)} \end{aligned}$$

*Absolute Ceiling:* Use equation (177). From Figure 141,

$K_H = 23,600$  for  $\frac{L}{D} = 8.2$ , aspect ratio = 4.8.

$$H = 23,600 \log_{10} \left[ \frac{10,000}{7.25 \times \left(\frac{10.76}{0.78}\right)^2} \right] = 20,200 \text{ ft. (19,600)}$$

*Service Ceiling:* Use equation (179).

$$h_S = 20,200 \left( \frac{1,130 - 100}{1,130} \right) - 40 \left( \frac{20,200}{10,000} \right)^3 = 18,070 \text{ ft. (17,500)}$$

*Climb in 10 Minutes:*

$$\frac{C_0}{H} = \frac{1,130}{20,300} = .056 \quad \left( \frac{10}{\frac{H}{C_0}} \right) = 0.56$$

From Figure 145,  $\frac{h}{H} = 0.41$ .

$$h_{10} = 0.41 \times 20,200 = 8,280 \text{ ft. } (8,000)$$

The values obtained in the exact calculation are given in parentheses alongside the estimated values for comparison.

## CHAPTER XV

### SEAPLANE FLOATS

The naval architect describes seaplane floats with a number of technical words and phrases peculiar to his profession. For the benefit of the student and engineer who is unfamiliar with these terms, a short list of definitions has been prepared, limited to the most frequently used words and phrases.

**Afterbody.** That part of a float between the main step and the stern.

**Bottom.** The area included between chines and keel from bow to stern.

**Bow.** The extreme forward point, or portion of a float.

**Buoyancy.** The displacement (in lbs. of sea water) to a given water line.

**Buoyancy, center of.** The center of gravity of the displaced volume of water.

**Buoyancy, excess.** The difference between the total or submerged and normal or load water line displacements. Usually expressed as a percentage of the normal displacement.

**Chine.** The line of intersection of the bottom with the sides or deck.

**Deadrise.** The angle which each side of the bottom makes with the horizontal, as measured in a transverse plane.

**Deck.** The upper surface between the sides. If the sections are rounded without flat or vertical portion, then all of the upper surface between the chines is called the deck.

**Deck line.** The upper boundary of the float in a side elevation.

**Displacement.** The weight of the sea water displaced to a given water line, or simply the load carried by a float under given conditions.

**Displacement submerged.** The weight of the sea water displaced when the float is completely submerged.

**Draft.** Usually refers to the maximum depth below water surface of any part of the float under given conditions.

**Forebody.** That part of the float between the main step and the bow.

**Keel.** The main longitudinal and continuous strength member located along the bottom and on the center line.

- Keel, false.** A protective member attached along the bottom center line on the outside, to prevent damage from handling or grounding.
- Metacenter.** A point through which the resultant vertical buoyant force passes for all small displacements from the position of equilibrium.
- Metacentric height.** The distance from the center of gravity to the metacenter.
- Porpoising.** Any pronounced pitching oscillation in a moving float.
- Speed, getaway.** The speed at which the entire weight of the seaplane is carried by the wings.
- Speed, hump.** The speed or speeds at which the water resistance is a maximum.
- Sponsons.** Lateral projections added to the sides of a float or hull to increase planing area or transverse stability.
- Spray strips.** Thin longitudinal strips of triangular cross-section attached to the bottom along the chine to "beat down" the spray.
- Squatting.** A pronounced tendency to trim by the stern.
- Step.** A line of discontinuity in a surface. In its usual form, a sudden change in transverse sections.
- Stern.** The extreme rear point, or portion of a float.
- Trim.** The angle of pitch, usually measured between the deck line and the water line.
- Trim by bow.** An angle of trim produced by depressing the bow and raising the stern and measured in the same manner as trim by stern.
- Trim by stern.** An angle of trim produced by raising the bow and depressing the stern and measured from a level position of some reference line, usually the deck line.

**Model tests—corresponding speeds.** The only way to determine the water resistance of a float or hull is by actual test, either full scale or on a model. Since most of the resistance is "wave-making," comparisons must be made on the basis of  $\frac{V^2}{L}$  or  $\frac{V}{\sqrt{L}}$ . That is, the "corresponding speed" is proportional to the square root of the length.

*Froudes' Law of Comparison* states that at corresponding speeds the full-scale wave-making resistance is equal to the model wave-making resistance multiplied by the cube of the linear scale ratio of full size to model.

Letting  $V_1$  = model speed and  $V$  = full-scale speed

$R_1$  = model resistance and  $R$  = full-scale resistance



and  $L = \text{linear scale ratio} \frac{\text{full size}}{\text{model}}$

the following relations hold

Corresponding speeds  $V = V_1 \sqrt{L}$

Resistance at corresponding speeds  $R = R_1 L^3$

If a model is  $\frac{1}{9}$  full scale, i.e., if  $L = 9$ , then the corresponding speeds on the model are one-third of the full-scale speeds, and at corresponding speeds, the full-scale resistance is  $9^3 = 729$  times the model resistance. Since both the displacement  $\Delta$  and the resistance  $R$  vary as the cube of the scale ratio, it follows that the ratio  $\frac{\Delta}{R}$  is the same for model and full-scale at corresponding speeds. Model test data on seaplane floats and hulls are usually given in the form of curves of  $\frac{\Delta}{R}$  against the ratio  $\frac{V}{V_G}$  where  $V_G$  is the "getaway" speed at which the float leaves the water.

A given model test representing a definite full-scale seaplane, also represents the infinite number of full-scale getaway speeds and linear scale ratios giving the same value of model getaway speed. For example, the corresponding getaway speed for a linear scale ratio of 12 and a full-scale getaway speed of 52 knots, is 15 knots. A model tested under these conditions would represent any geometrically similar full-scale getaway but to a different scale ratio, which is obviously  $L = \left(\frac{V_G}{15}\right)^2$ . That is, it would represent a 45 knot getaway to the scale ratio 9, or a 60 knot getaway to the scale ratio 16, etc.

In addition to resistance at various trims, model tests should include, measurements of righting moments at rest, trimming moments underway, planing and spray characteristics. The righting moments at rest give the metacentric heights, while the trimming moments under way bring out any undesirable or uncontrollable tendency to dive or "squat." The planing characteristics are rather general, and include items such as tendency to "porpoise" or "sticking at high speed." The

spray characteristics are observed ordinarily to avoid an arrangement that allows spray to enter the propeller disc, or perhaps more exactly in some cases, to reduce this spray to a minimum.

**Model basin methods.** Since the model basin equipment and general methods of testing seaplane floats at the Washington Navy Yard have been fully described elsewhere,<sup>1</sup> the following résumé has been prepared chiefly for the benefit of aeronautical engineers to whom the original papers are not readily available.

The model basin is simply a tank about 42 ft. in width by 470 ft. in length and 15 ft. in depth at the center. A "carriage" extending across the basin, is driven on carefully aligned tracks by four electric motors using the Ward-Leonard speed control. The carriage is fitted with all of the speed control and dynamometer gear; it weighs approximately 75,000 lbs. Its maximum speed is approximately 18 knots, but testing is very rarely ever done at speeds greater than 15 knots, chiefly on account of the short run available and the arresting difficulties at the higher speeds.

In testing seaplane models, certain assumptions are necessary in order to arrive at practical methods. First, the model is attached to the dynamometer by means of an arm about 8 ft. in length. This arm is hinged at the dynamometer attachment and guided so that it is free to move in a vertical plane. The model attachment to the dynamometer arm is such as to allow the model to pitch freely about the full-scale center of gravity on the "free-to-trim" runs. On the "fixed trim" runs the attitude of the model relative to the arm is fixed. It is assumed that the angular changes due to vertical motion of the model are negligible and this is easily verified. Second, the model displacement at rest is adjusted by means of weights carried on an extra flexible wire cable passing over two pulleys to the

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<sup>1</sup> D. W. Taylor, "The United States Experimental Model Basin," Transactions of the Society of Naval Architects and Marine Engineers, 1900; H. C. Richardson, "The Trend of Flying Boat Development," Journal of the American Society of Naval Engineers, May 1926.

model. These weights also counterbalance the dynamometer arm. Obviously, all of these weights in the system seriously affect any comparison of model oscillations with full scale. Third, it is assumed that the wing lift varies directly as the speed so as to lift the model entirely clear of the water at the "corresponding" model getaway speed. This lift is automatically applied directly to the counterbalance cable by a small vane which is calibrated by trial runs to determine the correct setting. When this setting is once determined, it is not changed during the test although the wing lift is proportional to the angle of attack as well as to the square of the speed. It may be shown, however, that the error introduced by this approximation is negligible, and some allowance is made by taking the getaway speed from 5% to 10% higher than the calculated actual value.

That these assumptions do not seriously affect the validity of test data has been amply proved by a number of comparisons between model and full scale. The best method yet found for an approximate check is in the comparison of predicted and actual maximum loads that can be taken off by a given seaplane, in a calm or in a wind of known velocity. The agreement in every case investigated has been uniformly close, indicating less than 5% difference.

**Typical model basin data.** Table 21 contains the *average* values of  $\frac{\Delta}{R}$  against the ratio  $\frac{V}{V_G}$  for single floats, twin floats, and flying boat hulls as given by Captain H. C. Richardson<sup>2</sup> in his historical paper "Naval Development of Floats for Aircraft." These average values may be used for any general purpose, but too much dependence should not be placed on their use in a performance calculation that indicates a low margin of thrust, because the possible deviation from the average is considerable.

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<sup>2</sup> Transactions of the Society of Naval Architects and Marine Engineers, Vol. 34, 1926.

TABLE 21. AVERAGE RATIO  $\frac{\text{DISPLACEMENT}}{\text{RESISTANCE}}$  FROM MODEL BASIN TESTS

| Speed<br>%<br>Getaway<br>Speed | Average Ratio of $\frac{\Delta}{R}$ |                |       |
|--------------------------------|-------------------------------------|----------------|-------|
|                                | Single<br>Floats                    | Twin<br>Floats | Boats |
| 20                             | 8.05                                | 8.10           | 10.30 |
| 25                             | 6.50                                | 6.60           | 7.40  |
| 30                             | 5.10                                | 5.30           | 6.10  |
| 35                             | 4.30                                | 4.50           | 5.30  |
| 40                             | 4.10                                | 4.10           | 4.90  |
| 45                             | 4.05                                | 4.10           | 4.80  |
| 50                             | 4.30                                | 4.25           | 5.05  |
| 55                             | 4.70                                | 4.65           | 5.25  |
| 60                             | 5.15                                | 5.15           | 5.50  |
| 65                             | 5.55                                | 5.45           | 5.50  |
| 70                             | 5.75                                | 5.50           | 5.35  |
| 75                             | 5.55                                | 5.30           | 5.00  |
| 80                             | 5.00                                | 4.70           | 4.50  |
| 85                             | 4.15                                | 3.90           | 3.90  |
| 90                             | 3.20                                | 3.05           | 3.05  |
| 95                             | 2.00                                | 2.30           | 1.80  |
| 100                            | 0                                   |                | 0     |

**Take-off in a calm.** The resistance due to the floats or hull in a take-off is readily obtained by the following steps:

1. Calculate getaway speed  $V_G$
2. Assume a series of speeds  $V$
3. Find ratio  $\left(\frac{V}{V_G}\right)$  for each speed
4. Read  $\left(\frac{\Delta}{R}\right)$  at each  $\left(\frac{V}{V_G}\right)$  from model basin curves
5. Calculate  $\Delta$  at each speed:  $\Delta = \left(W_1 - \frac{V}{V_G}\right)^2$
6.  $R = \frac{\Delta}{\left(\frac{\Delta}{R}\right)}$

The total resistance is found by adding the air resistance of the airplane to each value of  $R$ . The air resistance is found by

assuming a constant angle of attack in take-off, say  $8^\circ$ , and calculating (or simply estimating) a value of  $\frac{L}{D}$  corresponding to

this angle. The air resistance is  $D = \frac{L_W}{\left(\frac{L}{D}\right)}$  where  $L_W$  is the

wing lift  $= W \left(\frac{V}{V_G}\right)^2$ . If the total resistance and maximum propeller thrust are plotted against speed on the same diagram, the margin of thrust available for acceleration is easily obtained and from this one can obtain the probable take-off performance.

**Take-off in a wind.** The take-off may be made either into or down wind, but the effect of the wind is so great that few seaplanes can take off down wind when the wind velocity  $V_0$  is large. If there is any wind at all, the air speeds and water speeds are no longer equal, and the wing lift no longer follows the curve assumed in making the Model Basin test. For take-off into wind, the wing lifts are greater and for take-off down wind they are less than the assumed values. This is simply equivalent to changing the load on the float at a given speed. Let us consider the effect of changes in load and changes in getaway speed on the values of  $\frac{\Delta}{R}$ . Figure 154 is a plot of  $\frac{\Delta}{R}$  against  $\frac{V}{V_G}$  for an NC type of boat hull with three loads and with constant getaway speed, as obtained in tests at the Washington Navy Yard. The values of  $\frac{\Delta}{R}$  fall upon a single curve, indicating that with constant getaway speed the effect of a 25% change in load carried by the float is negligible. Figure 155 is a plot of  $\frac{\Delta}{R}$  against  $\frac{V}{V_G}$  for an F5 type of hull with three loads and with getaway speed varied according to the load, also from Washington Navy Yard test data. As might be expected, there is a slight decrease in the values of  $\frac{\Delta}{R}$  as the load and getaway



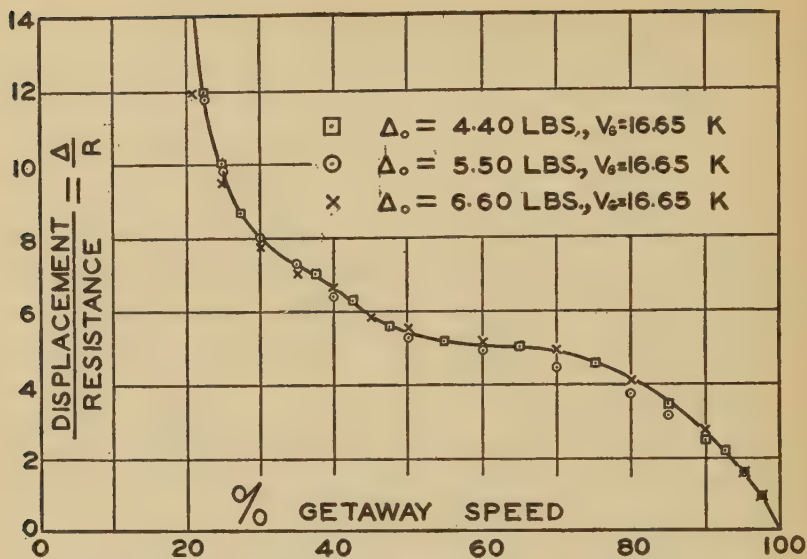


Figure 154. Effect of Variable Load on  $\frac{\Delta}{R}$  with Constant Getaway Speed

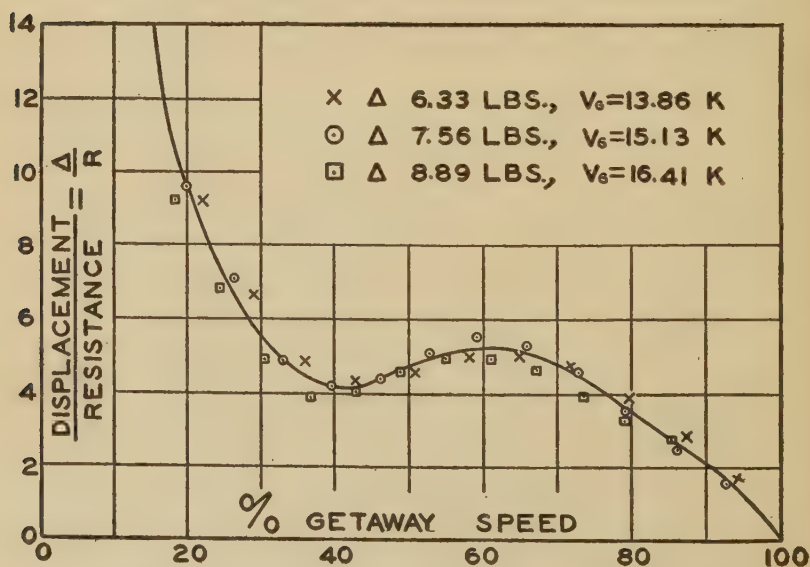


Figure 155. Effect of Variable Load and Getaway Speed on  $\frac{\Delta}{R}$



speed are increased, but the changes are negligible for engineering purposes. Values of  $\frac{\Delta}{R}$  may therefore be read directly from the standard curve, if the ratio  $\frac{V}{V_G}$  is determined from the water speeds. Resistances are found from these values of  $\frac{\Delta}{R}$  and the corresponding displacements. The procedure will be outlined in detail, for clarity.

*Take-Off into Wind:* Denoting wind velocity by  $V_o$  and the getaway air speed by  $V_G$ , the getaway water speed is  $V_{GW} = V_G - V_o$ . If the water speed is  $V_W$ , then the values of  $\frac{\Delta}{R}$  are determined by the ratios of  $\frac{V_W}{V_{GW}}$ , and resistances are obtained as follows:

1. Calculate getaway air speed,  $V_G$
2. Assume value of wind velocity  $V_o$ .
3. Calculate  $V_{GW} = V_G - V_o$ .
4. Assume a series of water speeds,  $V_W$
5. Find ratios  $\frac{V_W}{V_{GW}}$
6. Read  $\frac{\Delta}{R}$  from standard curve, assuming  $\frac{V_W}{V_{GW}}$  equivalent to  $\frac{V}{V_G}$
7. Obtain air speed,  $V = V_W + V_o$ .
8. Find wing lift at each water speed, assuming  $L_W = W \left( \frac{V}{V_G} \right)^2$
9. Load carried by float  $\Delta = W - L_W$
10. Water resistance  $R = \frac{\Delta}{\left( \frac{\Delta}{R} \right)}$
11. Air resistance  $D = \frac{L_W}{\left( \frac{L}{D} \right)}$

*Take-Off Down Wind:* In this case the water speeds are greater than the air speeds and it is necessary to consider  $V_0$  negative.

The method just outlined must be considered as an approximation for use in the absence of more exact data. It gives results which agree quite closely with observed full-scale maximum loads which can be taken off in a wind of known velocity.

Thrust may be assumed to vary linearly with speed, making it necessary to calculate the thrust at two speeds only, one of which may be the static thrust. (See Chapter VI.)

**Metacentric height.** Metacentric height may be defined by considering a floating prism having its c.g. at a point  $G$  and its center of buoyancy at a point  $B$ . A line perpendicular to the water surface and passing through  $B$  will also pass through  $G$ . If the prism be inclined through a small angle  $\theta$  while retaining the same volume of displacement, the center of buoyancy will shift to a point  $B'$ . A vertical through  $B'$  will intersect the original vertical  $BG$  at a point  $M$ , which is called the metacenter. The distance  $GM$  is called the metacentric height.

It is easily shown that the metacentric height is a measure of static stability. Considering a slight inclination  $\theta$  and taking moments about the original center of buoyancy  $B$ , it is seen that the disturbing moment is  $W \cdot \overline{BG} \cdot \sin \theta$  and the righting moment is  $W \cdot \overline{BM} \cdot \sin \theta$ . The total resultant moment is

$$\begin{aligned} M &= W \cdot (BM - BG) \cdot \sin \theta \\ &= W \cdot GM \cdot \sin \theta \end{aligned}$$

and the slope of the resultant moment curve is

$$\frac{dM}{d\theta} = W \cdot GM \cdot \cos \theta$$

from which

$$GM = \frac{dM}{d\theta} \cdot \frac{1}{W \cdot \cos \theta} \quad (\text{for } \theta \text{ in radians})$$

or

$$GM = \frac{57.3}{W} \cdot \frac{dM}{d\theta} \quad (\text{for } \theta \text{ in degrees}) \quad (180)$$

This relation is used to determine metacentric heights from inclination tests on models.

The metacenter may be found by the use of the formula  $BM = I/V$ , where  $I$  is the moment of inertia of the waterplane about its center line and  $V$  is the total volume of displacement (see any book on Naval Architecture).  $I$  varies as  $L^4$ , and  $V$  varies as  $L^3$ , so that  $BM$  and the metacentric height vary directly as the length, or scale ratio.

Approximate metacentric heights for seaplane floats and hulls may be obtained from the empirical formulas given below.

**Metacentric height required.** Analysis of the performance of a great number of seaplanes indicates that satisfactory static stability is obtained when

$$\text{Transverse } GM = \text{longitudinal } GM = 1.4 (\Delta)^{\frac{1}{3}} \quad (181)$$

where  $\Delta$  is the gross weight of the seaplane.

**Transverse metacentric height of twin floats.** It has been shown<sup>3</sup> that with the design proportions in common use, the transverse metacentric height for twin floats is given closely by the empirical formula

$$GM = \frac{K_1 L s^2 B}{\Delta} \quad (182)$$

where  $L$  is the length and  $B$  the beam of each float in ft.,  $s$  the spacing on center lines in ft.,  $\Delta$  the gross weight of the seaplane, and  $K_1$  a constant varying from 17.7 to 20.8 with an average value 19.5. Equation (182) may be used to determine the spacing necessary for stability by substituting the average value of  $GM$  from equation (181). That is

$$s = \left[ \frac{1.4 \Delta^{\frac{1}{3}}}{19.5 LB} \right]^{\frac{1}{2}} = \frac{0.28 \Delta^{\frac{1}{3}}}{\sqrt{LB}} \quad (183)$$

**Longitudinal metacentric height.** It has been shown in N. A. C. A. Technical Note No. 183 that the longitudinal meta-

<sup>3</sup> W. S. Diehl, "Static Stability of Seaplane Floats and Hulls," N. A. C. A. Technical Note No. 183 (1924).

centric height for either single or twin floats is given with sufficient accuracy by the empirical equation

$$GM = \frac{K_2 nBL^3}{\Delta} \quad (184)$$

where  $n$  is the number of floats (i.e., one or two),  $B$  the beam of each float in ft.,  $L$  the length in ft.,  $\Delta$  the gross weight of the seaplane, and  $K_2$  a constant normally varying between 1.90 and 2.40 with an average value of 2.10.

Equation (184) may be used to determine the minimum length of a seaplane float for longitudinal stability by substituting the value of  $GM$  from equation (181)

$$L^3 = \frac{1.4 \Delta^{\frac{1}{3}}}{2.10 nB} = \frac{0.67 \Delta^{\frac{1}{3}}}{nB} \quad (185)$$

**Transverse stability—single floats and flying boats.** Single float seaplanes and flying boats require the use of auxiliary floats on the wing tips in order to secure transverse stability. The usual method is to install the tip floats so that at normal trim and draft they clear the water from 3 in. to 8 in. according to the span, and make the volume large enough to give a righting moment which is greater than twice the upsetting moment when one float is just submerged.

The equation of moments about the C.B. is obviously

$$\Delta_1 l \cos \theta = C \cdot Wh \cdot \sin \theta$$

from which, the wing tip float displacement is

$$\Delta_1 = C \cdot Wl \cdot \tan \theta \quad (186)$$

where  $W$  is the gross weight,  $h$  the height of the C.G. above the center of buoyancy ( $h = BG$ ),  $l$  the distance from the C.B. of the tip float to the C.B. of the main float or hull,  $\theta$  the angle of inclination required to submerge the tip float, and  $C$  a constant greater than 2.0 and preferably between 2.5 and 3.5.

**Section coefficients.** The ratio of maximum cross-section to the product of beam by depth varies according to type as follows:

|                                       | $\frac{A}{BD}$ |
|---------------------------------------|----------------|
| Average single float.....             | .78            |
| Average twin float.....               | .78            |
| Average boat hull, no sponsons.....   | .70            |
| Average boat hull, with sponsons..... | .55            |

These values are useful in making performance estimates. They should be regarded simply as averages, for this or similar purposes.

**Prismatic coefficients.** The ratio of total (submerged) volume to the product of length  $L$ , beam  $B$ , and depth  $D$  is the prismatic coefficient. This coefficient is variable with the type of float or hull, and has the following average values:

| Type                          | Prismatic<br>Coefficient, $C_v$ |
|-------------------------------|---------------------------------|
| Single floats.....            | .70                             |
| Twin floats.....              | .72                             |
| Boat hulls, no sponsons.....  | .50                             |
| Boat hulls with sponsons..... | .32                             |
| Wing tip floats.....          | .55                             |

The submerged displacement in sea water weighing 64 lbs./cu. ft. is given by

$$\Delta_s = 64 C_v LBD \quad (187)$$

$L$ ,  $B$ , and  $D$  being in ft.





## APPENDIX

### STANDARD ATMOSPHERE

The "standard atmosphere" is an arbitrary variation of temperature, pressure, and density, with altitude, which is used for numerous aeronautical purposes, but chiefly as a basis for comparing performance. The standard atmosphere recommended by the National Advisory Committee for Aeronautics<sup>1</sup> and adopted in 1925 by all interested Government Departments for official use in the United States is based on the following assumptions:

Ground temperature,  $t_0 = 15^\circ \text{C.} = 59^\circ \text{F.}$

Isothermal temperature,  $t = -55^\circ \text{C.} = -67^\circ \text{F.}$

Temperature gradient,  $a = .0065^\circ \text{C./m.} = .003566^\circ \text{F./ft.}$

The air is a dry, perfect gas.

The resulting equations are:

$$T = T_0 - ah \quad (188)$$

$$p = Rg_\rho T, \text{ or } \frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right) \frac{T}{T_0} \quad (189)$$

$$h = \frac{p_0}{g\rho_0} \left(\frac{T_m}{T_0}\right) \log_e \left(\frac{p_0}{p}\right) \quad (190)$$

$T_m$  = Harmonic mean temperature

$$= \frac{a h}{\log_e \left(\frac{T_0}{T_0 - ah}\right)} \quad (191)$$

Below the isothermal level (10,769 m. or 35,332 ft.) the following relations exist

<sup>1</sup> See "Standard Atmosphere—Tables and Data," N. A. C. A. Technical Report No. 218 (1925).

$$\frac{T}{T_0} = \left(\frac{p}{p_0}\right)^{0.19} = \left(\frac{\rho}{\rho_0}\right)^{0.235} \quad (192)$$

$$\left(\frac{p}{p_0}\right) = \left(\frac{T}{T_0}\right)^{5.256} = \left(\frac{\rho}{\rho_0}\right)^{1.235} \quad (193)$$

$$\left(\frac{\rho}{\rho_0}\right) = \left(\frac{T}{T_0}\right)^{4.256} = \left(\frac{p}{p_0}\right)^{0.81} \quad (194)$$

$$\left(\frac{p}{p_0}\right)^{0.19} = \left(1 - \frac{a}{T_0} h\right) \quad (195)$$

$$\left(\frac{p}{p_0}\right) = \left(1 - \frac{a}{T_0} h\right)^{5.256} \quad (196)$$

$$\left(\frac{\rho}{\rho_0}\right)^{0.235} = \left(1 - \frac{a}{T_0} h\right) \quad (197)$$

$$\left(\frac{\rho}{\rho_0}\right) = \left(1 - \frac{a}{T_0} h\right)^{4.256} \quad (198)$$

In the foregoing equations, the subscript 0 refers to the standard conditions at sea level.  $h$  is the altitude and  $R$  the gas constant for air.

**Standard atmospheric relations used in performance reduction.** The relation between pressure, temperature, and density ratio, at any altitude is

$$\frac{\rho}{\rho_0} = 0.3789 \frac{p \text{ mm. Hg.}}{[t^{\circ}\text{C.} + 273^{\circ}]} \quad (199)$$

$$= 9.624 \frac{p \text{ in. Hg.}}{[t^{\circ}\text{C.} + 273^{\circ}]} \quad (200)$$

$$= 17.32 \frac{p \text{ in. Hg.}}{[t^{\circ}\text{F.} + 459.4^{\circ}]} \quad (201)$$

The relation between an increment of pressure  $\Delta p = (p_1 - p_2)$ , the average density  $\rho_m$  between  $p_1$  and  $p_2$ , and the increment of altitude  $\Delta h$  is, in metric units:

$$\Delta h = \frac{-13.59 \Delta p \text{ mm. Hg.}}{g \rho_m} \quad (202)$$

and in English units

$$\Delta h = \frac{-70.67 \Delta p \text{ in. Hg.}}{g \rho_m} \quad (203)$$

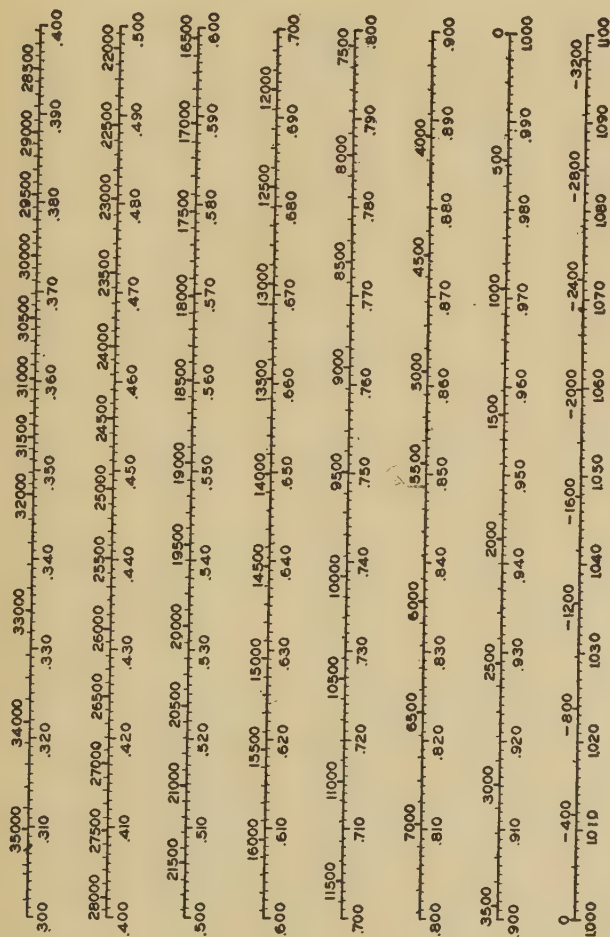


Figure 156. Density Ratio  $\frac{\rho}{\rho_0}$  at any Altitude in Standard Atmosphere. Density ratios are below, and altitudes are above the base lines

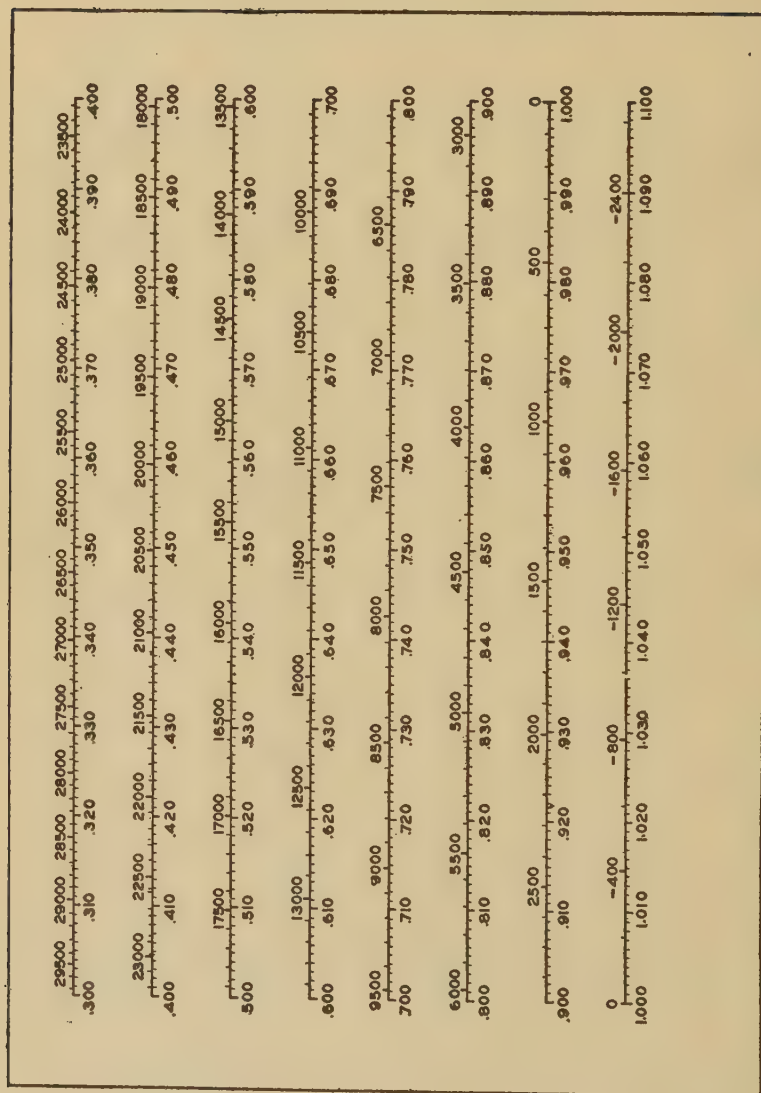


Figure 157. Pressure Ratio  $\frac{p}{p_0}$  at any Altitude in Standard Atmosphere. Pressure ratios are below and  $p_0$  altitudes are above the base lines



Figure 158. Pressure in mm. Hg. at any Altitude in Standard Atmosphere. Pressures are below and altitudes are above the base lines

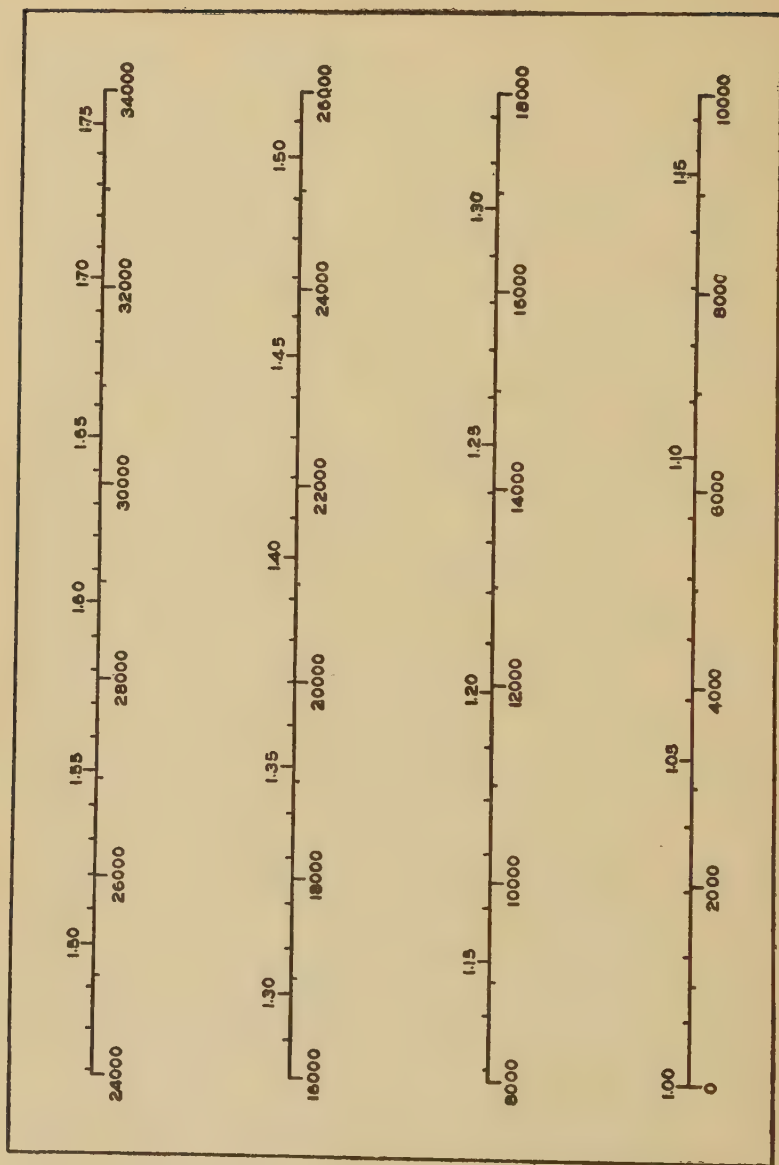


Figure 159.  $\sqrt{\rho_0/\rho}$  at any Altitude in Standard Atmosphere. Values of  $\sqrt{\rho_0/\rho}$  are above and altitudes are below the base lines



Figures 156, 157, and 158 are plots of  $\frac{\rho}{\rho_0}$ ,  $\frac{p}{p_0}$  and  $p$  against altitude in standard atmosphere. Density ratios may be calculated by equation (199), and the corresponding density altitudes found from Figure 156. Pressure altitudes may be read directly from Figure 158.

Figure 159 is a plot of  $\sqrt{\frac{\rho_0}{\rho}}$  against altitude.

**Velocity of sound in air.** The velocity of sound in any gas is

$$a = \sqrt{\frac{Kp}{\rho}} \quad (204)$$

where  $p$  is the pressure,  $\rho$  the density, and  $K$  the ratio of specific heat at constant pressure to that at constant volume. The equation may be written in the form

$$a = \sqrt{K \frac{p_0}{\rho_0} \left(\frac{p}{p_0}\right) \left(\frac{\rho_0}{\rho}\right)} = \sqrt{K \frac{p_0}{\rho_0} \times \frac{T}{T_0}} \quad (205)$$

for air  $K = 1.41$ ,  $p_0 = 2,116$  lbs./sq. ft., and  $\rho_0 = .002378$  lbs. sec.<sup>2</sup>/ft.<sup>4</sup> Substituting these values gives

$$a = \sqrt{1.41 \times \frac{2116}{.002378} \left(\frac{T}{T_0}\right)} = 1,120 \sqrt{\frac{T}{288}} \quad (206)$$

$T$  being in °C. absolute. The variation with temperature is as follows:

|                |       |       |       |       |       |       |
|----------------|-------|-------|-------|-------|-------|-------|
| $T$ , °C.      | -20   | 0     | 10    | 15    | 20    | 30    |
| $a$ , ft./sec. | 1,050 | 1,090 | 1,110 | 1,120 | 1,130 | 1,150 |

## GENERAL CONVERSION FACTORS

The following table of conversion factors departs from the conventional form in that the factors are given to as many significant figures as possible. While in most cases four or five significant figures are sufficient, it has been the author's experience that greater accuracy is often required. It is not intended that the factors be used as given, unless such accuracy is required. For example, the exact conversion factor from cubic inches to cubic centimeters is 16.3871624, but 16.39 or even 16.4 is often close enough.

### Fundamental conversion factors.

$$\begin{aligned} 1 \text{ meter} &= 39.37 \text{ inches (Act of U. S. Congress,} \\ &\quad 28 \text{ July, 1866)} \\ &= 3.2808333 \text{ feet} \end{aligned}$$

$$\begin{aligned} 1 \text{ pound} &= 453.5924277 \text{ grams (International Bu-} \\ &\quad \text{reau of Weights and Measures, July} \\ &\quad 1893) \end{aligned}$$

$$\begin{aligned} \text{Specific weight of dry air with normal CO}_2 \text{ content} \\ \text{at 760 mm. Hg. and } 0^\circ \text{ C.} &= .0012930 \text{ gr./cm.} \end{aligned}$$

$$\begin{aligned} \text{Specific weight of mercury at} \\ {}^\circ\text{C. . . . .} &= 13,595.1 \text{ kg./m}^3 \end{aligned}$$

$$\begin{aligned} \text{Standard gravity (Inter-} \\ \text{national) } g \text{ . . . . .} &= 980.665 \\ &= 32.174 \text{ ft./sec.}^2 \end{aligned}$$

## CONVERSION FACTORS

| MULTIPLY            | By              | To OBTAIN               |
|---------------------|-----------------|-------------------------|
| Atmospheres         | 76.0            | cm. mercury             |
| "                   | 29.9212         | inches mercury          |
| "                   | 33.8985         | feet of water           |
| "                   | 10,332.276      | kilograms per sq. meter |
| "                   | 14.69601        | pounds per sq. in.      |
| "                   | 2,116.225       | pounds per sq. ft.      |
| "                   | 1,013,250.      | bars                    |
| Bars                | 1.0             | dynes per sq. cm.       |
| B. t. u. (mean)     | 777.98          | foot-pounds             |
| "                   | 1,054.8         | joules                  |
| "                   | .25198          | kilogram-calories       |
| "                   | 107.560         | kilogram-meters         |
| Centimeters (cm.)   | .393700         | inches                  |
| "                   | .0328033        | feet                    |
| cm. of mercury      | 5.352391        | inches of water         |
| "                   | .4460326        | feet of water           |
| "                   | .193368         | pounds per sq. in.      |
| "                   | 27.84507        | pounds per sq. ft.      |
| "                   | 135.9510        | kilograms per sq. meter |
| cm. per sec.        | .0328083        | feet per sec.           |
| cubic centimeters   | .000999973      | liters                  |
| "                   | .06102338       | cubic inches            |
| cubic feet          | 1,728.0         | cubic inches            |
| "                   | 1/9             | cubic yards             |
| "                   | 7.480519        | gallons                 |
| "                   | 28,317.017      | cubic centimeters       |
| "                   | 28.31625        | liters                  |
| "                   | .028317017      | cubic meters            |
| cubic feet per min. | .471704         | liters per sec.         |
| "                   | .028317         | cubic meters per min.   |
| cubic feet of water | 62.42833        | pounds                  |
| cubic inches        | 16.3871624      | cubic centimeters       |
| "                   | .0163876        | liters                  |
| "                   | 1/231           | gallons                 |
| cubic meters        | 10 <sup>6</sup> | cubic centimeters       |
| "                   | 61,023.3753     | cubic inches            |
| "                   | 35.3144548      | cubic feet              |
| "                   | 1.307943        | cubic yards             |
| "                   | 264.170         | gallons                 |
| cubic yards         | 27.             | cubic feet              |
| "                   | .76455945       | cubic meters            |
| Degrees (arc)       | .017453292      | radians                 |
| dynes               | .00101972       | grams                   |
| dynes per sq. cm.   | 1.0             | bars                    |
| Ergs                | 1.0             | dyne-centimeters        |

## CONVERSION FACTORS

| MULTIPLY             | By          | To OBTAIN                |
|----------------------|-------------|--------------------------|
| Fathoms              | 6.0         | feet                     |
| "                    | 1.82880     | meters                   |
| feet                 | 12.0        | inches                   |
| "                    | 1/3         | yards                    |
| "                    | 30.4800613  | centimeters              |
| "                    | .3048006    | meters                   |
| feet of water        | .029500     | atmospheres              |
| "                    | .433530     | pounds per sq. in.       |
| "                    | 62.428327   | pounds per sq. ft.       |
| "                    | 304.8006    | kilograms per sq. meter  |
| "                    | .882671     | inches of mercury        |
| "                    | .24199      | centimeters of mercury   |
| feet per min.        | .0113636    | miles per hr.            |
| "                    | .018288     | kilometers per hr.       |
| "                    | .508001     | centimeters per sec.     |
| feet per sec.        | .681818     | miles per hr.            |
| "                    | 1.09728220  | kilometers per hr.       |
| "                    | 30.48006    | centimeters per sec.     |
| "                    | .3048006    | meters per sec.          |
| "                    | .5920858    | knots                    |
| foot-pounds          | .138255     | meter-kilograms          |
| foot-pounds per min. | 1/33,000    | horsepower               |
| foot-pounds per sec. | 1/550       | horsepower               |
| Gallons              | 231.0       | cubic inches             |
| "                    | .133680     | cubic feet               |
| "                    | 3.785332    | liters                   |
| "                    | .832680     | imperial gallons         |
| gallons-Imperial     | 1.20094     | gallons                  |
| "                    | 227.4176    | cubic inches             |
| "                    | 4.5459631   | liters                   |
| grams                | .0647988    | grams                    |
| "                    | 15.43236    | grains                   |
| "                    | .0352739    | ounces                   |
| "                    | .0022046223 | pounds                   |
| "                    | 1,000.0     | milligrams               |
| "                    | .001        | kilograms                |
| "                    | 980.665     | dynes                    |
| gram-calories        | .0039685    | B. t. u.                 |
| gram-centimeters     | 980.665     | ergs                     |
| grams per cm.        | 0.1         | kilograms per meter      |
| "                    | .06719702   | pounds per foot          |
| "                    | .0055914    | pounds per inch          |
| grams per cu. cm.    | 1,000.0     | kilograms per cm.        |
| "                    | 62.42833    | pounds per cu. ft.       |
| Horsepower           | 33,000.0    | foot-pounds per min.     |
| "                    | 550.0       | foot-pounds per sec.     |
| "                    | 76.04039    | kilogram-meters per sec. |
| "                    | .986318     | metric horsepower        |

## CONVERSION FACTORS

| MULTIPLY                | By                        | TO OBTAIN                |
|-------------------------|---------------------------|--------------------------|
| horsepower, metric      | 75.0                      | kilogram-meters per sec. |
| “                       | 1.013872                  | horsepower               |
| horsepower-hours        | 2,545.06                  | B. t. u.                 |
| “                       | 1,980,000.                | foot-pounds              |
| “                       | 273,745.4                 | kilogram-meters          |
| Inches                  | 2.54000508                | centimeters              |
| inches of mercury       | .0334211                  | atmospheres              |
| “                       | 13.5951                   | inches of water          |
| “                       | 1.132925                  | feet of water            |
| “                       | .4911570                  | pounds per sq. in.       |
| “                       | 70.72661                  | pounds per sq. ft.       |
| “                       | 345.3162                  | kilograms per sq. meter  |
| inches of water         | .0735559                  | inches of mercury        |
| “                       | .1868324                  | centimeters of mercury   |
| “                       | .0361275                  | pounds per sq. in.       |
| “                       | 5.202360                  | pounds per sq. ft.       |
| “                       | 25.400051                 | kilograms per sq. meter  |
| Joules                  | 10 <sup>7</sup>           | ergs                     |
| “                       | .7375606                  | foot-pounds              |
| “                       | .1019716                  | kilogram-meters          |
| Kilograms               | 2.20462234                | pounds                   |
| “                       | 35.273957                 | ounces                   |
| “                       | 1,000.0                   | grams                    |
| kilogram-calories       | 3.9685                    | B. t. u.                 |
| “                       | 3,087.4                   | foot-pounds              |
| “                       | 426.85                    | kilogram-meters          |
| kilogram-meters         | 7.2329983                 | foot-pounds              |
| “                       | 9.80665 × 10 <sup>7</sup> | ergs                     |
| kilograms per cu. meter | .06242833                 | pounds per cu. ft.       |
| “                       | .001                      | grams per cu. cm.        |
| kilograms per meter     | .6719702                  | pounds per ft.           |
| kilograms per sq. meter | .00142234                 | pounds per sq. in.       |
| “                       | .2048169                  | pounds per sq. ft.       |
| “                       | .00289590                 | inches of mercury        |
| “                       | .003280833                | feet of water            |
| “                       | 0.1                       | grams per sq. cm.        |
| kilometers              | 3,280.833                 | feet                     |
| “                       | .6213700                  | miles                    |
| “                       | .539553                   | nautical miles           |
| kilometers per hr.      | .9113426                  | feet per sec.            |
| “                       | .6213700                  | miles per hr.            |
| “                       | .2777                     | meters per sec.          |
| “                       | .539553                   | knots                    |
| knots                   | 1.0                       | nautical miles per hr.   |
| “                       | 1.688944                  | feet per sec.            |
| “                       | 1.151553                  | miles per hr.            |
| “                       | 1.853249                  | kilometers per hr.       |
| “                       | .514791                   | meters per sec.          |

## CONVERSION FACTORS

| MULTIPLY              | By          | To OBTAIN               |
|-----------------------|-------------|-------------------------|
| Liters                | 1,000.027   | cubic centimeters       |
| "                     | 61.02503    | cubic inches            |
| "                     | .035315411  | cubic feet              |
| "                     | .264178     | gallons                 |
| "                     | .219975     | imperial gallons        |
| Meters                | 39.37       | inches                  |
| "                     | 3.280833    | feet                    |
| "                     | 1.093611    | yards                   |
| meters per sec.       | 3.280833    | feet per sec.           |
| "                     | 2.2369317   | miles per hr.           |
| "                     | 3.600       | kilometers per hr.      |
| microns               | .0001       | centimeters             |
| miles                 | 5,280.0     | feet                    |
| "                     | 1.609347    | kilometers              |
| "                     | .8683925    | nautical miles          |
| miles per hr.         | 1.46666     | feet per sec.           |
| "                     | .4470409    | meters per sec.         |
| "                     | 1.609347    | kilometers per hr.      |
| "                     | .8683925    | knots                   |
| miles per hr. squared | 2.151111    | feet per sec. squared   |
| mills                 | .001        | inches                  |
| "                     | .025400     | millimeters             |
| Nautical miles        | 6,080.20    | feet                    |
| "                     | 1.1515530   | miles                   |
| "                     | 1,853.2486  | meters                  |
| Ounces                | 1/16        | pounds                  |
| "                     | 28.349527   | grams                   |
| ounces per sq. yd.    | 33.906096   | grams per sq. meter     |
| Poundals              | .0310810    | pounds                  |
| "                     | 13,825.561  | dynes                   |
| pounds                | 453.5924277 | grams                   |
| "                     | .45359243   | kilograms               |
| "                     | 16          | ounces                  |
| "                     | 32.174      | poundals                |
| pounds-feet           | .1382552    | kilogram-meters         |
| pounds per ft.        | 1.4881612   | kilograms per meter     |
| pounds per cu. ft.    | 16.018369   | kilograms per cu. meter |
| "                     | .016018369  | grams per cu. cm.       |
| pounds per cu. in.    | 1,728       | pounds per cu. ft.      |
| "                     | 27.6797424  | grams per cu. cm.       |
| pounds per sq. ft.    | .192220     | inches of water         |
| "                     | 4.8824088   | kilograms per sq. meter |
| pounds per sq. in.    | 2.036009    | inches of mercury       |
| "                     | 2.306645    | feet of water           |
| "                     | .0680457    | atmospheres             |
| "                     | 703.06687   | kilograms per sq. meter |



## CONVERSION FACTORS

| MULTIPLY             | By          | TO OBTAIN            |
|----------------------|-------------|----------------------|
| Quarts (dry)         | 67.20       | cubic inches         |
| “ (liquid)           | 57.75       | cubic inches         |
| Radians              | 57.29578    | degrees (arc)        |
| radians per sec.     | 57.29578    | degrees per sec.     |
| “                    | .159155     | revolutions per sec. |
| “                    | 9.84930     | revolutions per min. |
| revolutions          | 6.283185    | radians              |
| revolutions per min. | .104720     | radians per sec.     |
| Slugs                | 32.174      | pounds               |
| square centimeters   | .1549997    | square inch          |
| “                    | .00107639   | square feet          |
| square feet          | 929.03412   | square centimeters   |
| “                    | .092903412  | square meters        |
| square inches        | 645.162581  | square millimeters   |
| “                    | 6.45162581  | square centimeters   |
| square kilometers    | .3861006    | square miles         |
| square meters        | 10.76386736 | square feet          |
| “                    | 1.1959853   | square yards         |
| square miles         | 2.59000     | square kilometers    |
| square yards         | .8361307    | square meters        |
| Tons, long           | 2,240.      | pounds               |
| “                    | 1,016.047   | kilograms            |
| tons, short          | 2,000.      | pounds               |
| “                    | 907.1849    | kilograms            |
| tons, metric         | 1,000.      | kilograms            |
| “                    | 2,204.622   | pounds               |
| Yards                | 91.44018    | centimeters          |
| “                    | .9144018    | meters               |

## USEFUL FORMULAS

### Powers and Roots.

$$a^{-n} = \frac{1}{a^n}$$

$$a^m \cdot a^n = a^{(m+n)}$$

$$\frac{a^m}{a^n} = a^{(m-n)}$$

$$(a^m)^n = (a^n)^m = a^{mn}$$

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

$$(a^{\frac{1}{m}})^{\frac{1}{n}} = a^{\frac{1}{m \cdot n}}$$

$$(ab)^n = a^n b^n$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} = a^n b^{-n}$$

$$(ab)^{\frac{1}{n}} = a^{\frac{1}{n}} b^{\frac{1}{n}} = \sqrt[n]{ab}$$

### Logarithms.

$$\text{Let } b^x = N \quad (b > 1.0)$$

$$\text{Then: } \log_b N = x$$

$$\log_b b = 1.0$$

$$\log_b 1 = 0$$

$$\log_b 0 = -\infty$$

$$\log_b MN = \log_b M + \log_b N$$

$$\log_b \frac{M}{N} = \log_b M - \log_b N$$

$$\log_b N^r = r \cdot \log_b N$$

$$\log_b \sqrt[s]{N^r} = \frac{r}{s} \cdot \log_b N$$

$$\log_b N = \frac{\log_a N}{\log_a b}$$

$$e = 2.71828183$$

$$\log_e 10 = 2.30258509$$

$$\log_e 2 = 0.69314718$$

$$\log_{10} e = 0.43429448$$

$$\log_e N = \log_e 10 \cdot \log_{10} N$$

## Derivatives.

| FUNCTION      | DERIVATIVE                                                  |
|---------------|-------------------------------------------------------------|
| $x$           | 1                                                           |
| $c$           | 0                                                           |
| $\pm u \pm v$ | $\pm \frac{du}{dx} \pm \frac{dv}{dx}$                       |
| $c \cdot u$   | $c \cdot \frac{du}{dx}$                                     |
| $u \cdot v$   | $u \frac{dv}{dx} + v \frac{du}{dx}$                         |
| $\frac{u}{v}$ | $\frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$             |
| $u^n$         | $nu^{n-1} \frac{du}{dx}$                                    |
| $\log_{10} u$ | $\frac{\log_{10} e}{u} \cdot \frac{du}{dx}$                 |
| $\log_e u$    | $\frac{1}{u} \cdot \frac{du}{dx}$                           |
| $a^u$         | $a^u \cdot \log_e a \cdot \frac{du}{dx}$                    |
| $e^u$         | $e^u \cdot \frac{du}{dx}$                                   |
| $u^v$         | $vu^{v-1} \cdot \frac{du}{dx} + u^v \log_e u \frac{du}{dx}$ |
| $\sin u$      | $\cos u \cdot \frac{du}{dx}$                                |
| $\cos u$      | $-\sin u \cdot \frac{du}{dx}$                               |
| $\tan u$      | $\sec^2 u \cdot \frac{du}{dx}$                              |
| $\cot u$      | $-\csc u \cdot \frac{du}{dx}$                               |

$c$  is any constant.  $u$  and  $v$  are any functions of  $x$ .

## Integrals.

| FUNCTION                         | INTEGRAL                                            |
|----------------------------------|-----------------------------------------------------|
| $a \cdot u^n \cdot du$ . . . . . | $\frac{a \cdot u^{n+1}}{n+1} + c \quad (n \neq -1)$ |
| $\frac{du}{u}$ . . . . .         | $\log_e u + c$                                      |
| $a^u \cdot du$ . . . . .         | $\frac{a^u}{\log_e a} + c$                          |
| $e^{au} \cdot du$ . . . . .      | $\frac{e^{au}}{a} + c$                              |
| $u \cdot dv$ . . . . .           | $uv - \int v \cdot du$                              |

$u$  and  $v$  are any functions of  $x$ .

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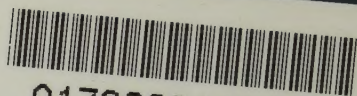
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